Every perfect number, except 6, always has a digital root equal to 1

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ABASTRACT

Euclid, in 300 BC observed that with n = prime number, whenever 2^n –1 corresponds to a further prime number, then (2^n–1)2^(n–1) is a perfect number.

As the research on perfect numbers went on, a curious property of them was noticed: the sum of the single digits of which each perfect number is composed (with the exception of 6), perpetuated until a single digit is reached, always converges to 1. This characteristic leads to the hypothesis that all perfect numbers, including those still unknown, retain this property. But why does the first perfect number not coincide with the root 1? Is it the only exception or will others be discovered later? The glimpse of light that illuminates the conjecture finds an explanation in the cyclical dimensions of numerical systems.

Euclid, in 300 BC observed that with n = prime number, whenever 2n –1 corresponds to a further prime number, then $(2^n-1)(2^n-1)$ is a perfect number. As the research on perfect numbers went on, a curious property of them was noticed: the sum of the single digits of which each perfect number is composed (with the exception of 6), perpetuated until a single digit is reached, always converges to 1. For example, adding the single digits of the fourth perfect number (8128) we have that 8+1+2+8 = 19; in turn 1+9 = 10; finally, 1+0 = 1.

This characteristic leads to the hypothesis that all perfect numbers, including those still unknown, retain this property. However, this conjecture has remained unproven, and any doubt about its veracity is confirmed by the diversity of the first perfect number (the perfect 6 which, precisely, has numerical root = 6).

Doubts and perplexities that coincided with the following questions: Why does the perfect prime not coincide with the root 1? Is it the only exception or will others be discovered later? The glimpse of light that illuminates the conjecture finds an explanation in the cyclical dimensions of numerical systems. By gradually increasing the value of n of the Euclidean formula, from 1 to infinity, infinite hexanumeric cycles are obtained, each of which contains within it, numbers which respectively converge into the following roots: 1, 6, 1, 3, 1, 9.

$(2^n - 1)(2^{n-1})$ = product → Numerical root

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$(2^1 - 1)(2^{1-1})$	= 1, →	numerical root 1
$(2^2 - 1)(2^{2-1})$	= 6, →	numerical root 6
$(2^3 - 1)(2^{3-1})$	= 28, →	numerical root 1
$(2^4 - 1)(2^{4-1})$	= 120, →	numerical root 3
$(2^5 - 1)(2^{5-1})$	=496, →	numerical root 1
$(2^6 - 1)(2^{6-1})$	=2016,→	numerical root 9

$(2^7 - 1)(2^{7-1})$	= 8128,	numerical root 1
$(2^8 - 1)(2^{8-1})$	= 32640,	numerical root 6
$(2^9 - 1)(2^{9-1})$	=130816,	numerical root 1
$(2^{10} - 1)(2^{10-1})$	= 523776,	numerical root 3
$(2^{11} - 1)(2^{11-1})$	= 2096128,	numerical root 1
$(2^{12} - 1)(2^{12-1})$	= 8386560,	numerical root 9

And so on, ad infinitum.

Within the first 2 hexa-numeric cycles of the Euclidean formula shown above, the first 4 perfect numbers are found, determined by n having a value equal to the prime numbers 2, 3, 5 and 7.

It can be noted that all numbers corresponding to an odd n value have a numerical root of 1, while all those corresponding to an even n value have, alternatively, a numerical root of 3 and its multiples (6-3-9). 6 (the only perfect number to have a digital root other than 1) is in fact the product of the two factors having the only exponent n that is even prime, i.e. 2, $(2^2-1)(2^{2-1}) = 2x3 = 6$.

Since the necessary and indispensable condition for 2^n-1 to be a prime number is the preliminary one that n corresponds to a prime number and since all prime numbers, with the exception of 2, are odd, then all perfect numbers since they are determined by the Euclidean formula having n = odd number they always keep (with the exception of n = 2) the digital root 1.