**Casimir effect and the stability of the electrons**

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**Abstract**

The stability of an electron is affected by electrostatic self-repulsion of the charge, attraction of opposite charge/s, Casimir effect, kinetic, and zero-point energies. Investigating the energy balance of these affects it can be shown that the point charge electron loses its stability at the Bohr’s radius and forms a static surface charge around the proton. First principle calculations also show that for individual atoms the formed surface charge around the nucleus is stable as long as the absorbed destabilizing energies are below the one-dimensional Casimir energy. Thus in neutral atoms the Casimir effect stabilizes the surface charge electron shell. For individual atoms the Casimir effect is active on the entire surface area of the electron shell. In this case the one-dimensional Casimir energy should be equivalent with the ionization energy. If the neighboring atoms shielding the Casimir effect then the energy required removing an electron should be reduced in proportion to the active surface area of the atom. On the surface of a metal about half of the atomic surface is shielded by the neighboring atoms. Thus the energy requiring removing an electron should be half of the ionization energy. In the bulk of the metal, where the Casimir effect is completely shielded by the neighboring atoms, the electron shell should be unstable. These predictions are consistent with experiments, since the ionization energy is the same as the one-dimensional Casimir energy, the measured energies of the work function are about half of the ionization energy, and the band gaps of metals are zero.

**Keywords:** atom models, Casimir effect, electron, electron stability, zero-point energy, photoelectric effect, ionization energy, work function
1. Introduction
Recent study has been shown that the point charge electrons transform to surface charge in the vicinity of the nucleus (Garai, 2017). The transformation of charge can be described as:

\[ n q_e = 4\pi r_{\text{shell}}^2 \sigma_{\text{shell}} \]  \hspace{1cm} (1)

where \( n \) is the number of electrons in the shell, \( q_e \) is the elementary charge of the electron, \( r_{\text{shell}} \) is the radius of the electron shell, and \( \sigma_{\text{shell}} \) is the surface charge of the same shell. The formed surface charge electron shell around the nucleus does not rotate. It can be derived from the first principles that the speed of the propagating wave in the static surface charge electron in Hydrogen atom is \( \alpha c \), where \( c \) is the speed of light in free space, and \( \alpha \) is the fine structure constant defined as:

\[ \alpha = \frac{1}{4\pi\varepsilon_o \frac{e^2}{\hbar c'}} \]  \hspace{1cm} (2)

where \( \varepsilon_o \) is the permittivity of free space, \( e \) is the elementary charge, and \( \hbar \) is the reduced Planck constant. This static surface charge model with propagating waves on its surface offers physical explanation for quantum mechanics, and also explains the observed uniformity and stability of the atoms. For the Hydrogen atom gives the correct zero angular momentum at the ground state, and the right ionization energy. The model is consistent with the laws of classical electromagnetism, indicating that these laws should remain valid at atomic scale.

The physical reason of the transformation of the point charge electron to a surface charge in the vicinity of the nucleus and vice versa is not known and investigated in this study.

2. Contributing factors to the stability of the electron
The physical phenomena affecting the stability of the electrons are reviewed here. The energy contributions to the stability of the electrons are derived.

2.1 Classical electromagnetism
The surface charge electron model can explain all the features of the atoms without violating electromagnetism. Thus, classical electromagnetism remains valid in the vicinity of the nucleus. One of the supporting arguments is exposed here.

Electron deflected by an atomic nucleus emits electromagnetic radiation, which is known as Bremsstrahlung (Fig. 1).

![Fig. 1 The electron decelerates in the vicinity of an opposite charge, emitting a Bremsstrahlung radiation. No emission of the neutral atoms is observed, which is contradict with the point charge orbiting electron model. In order to be consistent with electromagnetism the electron/s in an atom must be static. Static electron/s in an atom can only exist if the electrons form a surface charge around the nucleus.](image-url)
Current atomic models assume that the orbiting electrons in the atoms do not emit radiation. The observed Bremsstrahlung radiation contradict with this assumption. In order to be consistent with the law of electromagnetism, the electron in the vicinity of the nucleus must be static. Static charge around the nucleus can be stable only if the charge is symmetrically distributed surface charge around the nucleus (Fig. 2).

The validity of classical electromagnetism at atomic scale allows calculating the energies of self-repulsion of the electron charge, and the attraction of an opposite charge. The energy relating to self-repulsion of a single electron \( E_{\text{rep}}^e \) can be calculated as:

\[
E_{\text{rep}}^e = \frac{1}{4\pi \epsilon_0} \frac{e^2}{d_e}
\]

where \( d_e \) is the diameter of the electron shell. The self-repulsion of the charge has destabilizing effect on the stability of the electron in both point and surface charge form.

In the vicinity of an outside opposite charge, like proton, the electron is subject electrostatic attraction. The energy of the attraction \( E_{\text{attr}}^p \) between one proton and one electron according to classical electrostatic is:

\[
E_{\text{attr}}^p = \frac{1}{4\pi \epsilon_0} \frac{e^2}{d_{e-p}}
\]

where \( d_{e-p} \) is the distance between the proton and the electron. The electrostatic attraction between the proton and electron has destabilizing affect on a point charge electron, and stabilizing affect on the surface charge electron.

### 2.2 Casimir effect

Based on quantum field theory, the empty space is filled with fluctuating electromagnetic waves, with all possible wavelengths. The presence of the electromagnetic waves in empty space means that empty space contains a certain amount of energy. Pressure is induced between two parallel conducting mirror plates facing into each other because the waves longer than the distance between the plates create pressure on the two faces of the mirror. Thus, the pressure outside of the plates will be higher than inside, resulting in the attraction of the two mirror plates. This effect is known as the Casimir Effect, naming after the Dutch physicist, who predicted
the existence of vacuum pressure in 1948. Half a century later the predicted Casimir force between two surfaces has been experimentally verified (Lamoreaux, 1997; Mohideen and Roy, 1998), and confirmed by many experiments since then (e.g., Klimchitskaya et al., 2009). The Casimir energy for two hollow hemispheres \( E_{Co} \) can be calculated as (Garai, 2022):

\[
E_{Co} \approx \frac{\pi^3 \hbar c}{1440d}
\]

(5)

where \( d \) is the diameter of the hemispheres. Approximating the multiplier of the previous equation (5) as:

\[
\frac{\pi^3}{1440} \approx 3\alpha
\]

(6)

gives the Casimir energy acting on an electron \( E_{Co}^e \) as:

\[
E_{Co}^e \approx 3\alpha\hbar c \frac{1}{d_e}
\]

(7)

The Casimir energy has stabilizing contribution to both point and the surface charge electrons.

2.3 Kinetic energy

In order to form static electron shell around the nucleus the kinetic energy of the point charge electron has to be transferred. The possessed kinetic energy of a point-charge electron works against the surface charge transformation. Thus, the kinetic energy of the electron has a stabilizing effect on the point charge electron. On the other hand, the kinetic energy transferred to a surface charge helps to destabilize the electron. Based on classical physics the kinetic energy of a charge \( E_{kin}^e \) can be calculated as:

\[
E_{kin}^e = \frac{1}{2} m_e v^2
\]

(8)

where \( v \) is the velocity, and \( m_e \) is the mass of the electron.

2.4 Zero-point energy

Zero-point energy \( E_{ZP}^e \) is the lowest possible energy that a quantum mechanical system may have. Based on Heisenberg uncertainty principle this energy for the electron can be calculated as:

\[
E_{ZP}^e \approx \frac{\hbar^2}{2m_e a^2}
\]

(9)

where \( a \) is the uncertainty in the position of the electron. The zero-point energy is a destabilizing factor for both point and surface charge electrons.

3. Stability requirements

Summing the stabilizing and the destabilizing energies can determine the stability of an electron. The stability requirements for point and surface charges are discussed separately.

3.1 Point charge electron

Outside of the atom the electron can be depicted as a point charge. The destabilizing affects on this point charge electron are the electrostatic self-repulsion of the elementary charge, the attraction of an opposite charge/s (proton), and the zero-point energy of the charge. The stability of a point charge electron is ensured by the Casimir effect, which exerts pressure on the surface of the electron. The possessed kinetic energy of the point charge electron works against the transformation to surface
charge since the transformation requires capturing the point charge. The stability requirement for the point charge electron can be given then as:

\[ E_{C}^{e-p} + E_{kin}^{e-p} > E_{rep}^{e-p} + E_{attr}^{e-p} + E_{ZP}^{e-p} \]  

(10)

The superscript e-p refers to point charge electron. The transformation from point charge to surface charge electron occurs when the overall destabilizing energies are equal or exceed the stabilizing energies. This condition allows calculating the distance where this destabilization occurs. Based on the equilibrium condition between the stabilizing and destabilizing energies, the point charge electron lose its stability in the vicinity of a proton at the distance (d):

\[ d = \frac{4\pi \varepsilon_0 \hbar^2}{m_e e^2} = a_o \]  

(11)

This distance is the same as the Bohr’s radius (a_o). Thus, the point charge electron in the vicinity of a proton loses its stability at the Bohr’s radius. The destabilized charge under the attraction of the proton forms a static surface charge around the proton or nucleus.

### 3.2 Surface charge electron

The stability requirement of the formed surface charge electron around the nucleus is slightly differs from the point charge electron, because the attraction of the nucleus in this case works for the stability of the electron shell. Additionally, when the surface charge destabilized, the Casimir effect remains active in two dimensions during the transformation. Thus, the contribution of Casimir effect to the stability of a surface charge electron is effective in one-dimension.

\[ E_{C}^{e-s} = E_{C-1D}^{e} = \frac{1}{3} E_{C}^{e} \]  

(12)

where superscript e-s refers to surface charge electron. The stability requirement of the surface charge electron in an atom can be given then as:

\[ E_{C}^{e-s} + E_{attr}^{e-s} > E_{rep}^{e-s} + E_{ZP}^{e-s} + E_{Y}^{e-s} + E_{kin}^{e-s} \]  

(13)

where \( E_{Y}^{e-s} \) is the absorbed photon energy.

In the atoms the distance between the positive charge and the surface charge of the electron is constant. In terms of the Hydrogen atom this is equal with the Bohr’s radius, resulting that the self-repulsion, and zero-point energies are identically compensating the attraction of the proton.

\[ E_{attr}^{e-s} = E_{rep}^{e-s} + E_{ZP}^{e-s} \]  

(14)

The stability requirement for the surface charge electron surrounding the nucleus can be reduced then as:

\[ E_{C}^{e-s} > E_{Y}^{e-s} + E_{kin}^{e-s} \]  

(15)

The stability of the outer shell electron with zero kinetic energy is lost, when the energy of the absorbed photon is equal or exceeds the one-dimensional or effective Casimir energy of the surface charge electron. For the Hydrogen atom the calculated one-dimensional Casimir energy is the same as the ionization energy of the Hydrogen atom. It is speculated that this relationship can be generalized to all elements as:

\[ E_{C}^{e-s} = IE \]  

(16)

where IE is the ionization energy of the element. The schematic figure of the transition is shown in Figure 3.
Fig. 3 Schematic figure of the transition of an electron from point to surface charge and vice versa in the vicinity of the nucleus is shown. For the Hydrogen atom the transition from point to surface charge occurs at the Bohr’s radius. In reverse the transformation from surface to point charge occurs at the diameter of the Bohr’s atom.

4. Photoelectric effect

The illumination of the surface of metals by light results in electron emission above certain energies. Hertz discovered this photoelectric effect in 1887, which was explored experimentally by Philipp Lenard in 1902. The characteristic features of the photoelectric effect are shown in Figure 4.

a./ no electron is emitted until the incident light frequency is below some cut off or threshold frequency (ν₀),
b./ the number of the emitted electrons is proportional with the intensity of light,
c./ the kinetic energy of the emitted photoelectrons (KE₀) is independent from intensity,
the kinetic energy of the emitted photoelectrons is proportionally increases with frequency, where the proportionality factor is \( h \), or the Planck constant.

Many features of the photoelectric effect could not be explained by classical physics. In order to comply with experiments, Einstein suggested that the light, or any electromagnetic radiation, should be considered as the stream of photons with the energy \( (E_\nu) \), given as:

\[
E_\nu = h \nu ,
\]

where \( h \) is the Planck constant and \( \nu \) is the frequency of light. Einstein (1905, 1906) predicted that the maximum kinetic energy of the emitted electrons \( (K_{\text{max}}) \) should be:

\[
K_{\text{max}} = \nu h - \Phi
\]

where \( \Phi \) is the work function of the photosensitive surface, representing the minimum energy required to free an electron from the metal. He also stated that the intensity of the incident radiation should affect only the number and not the energy of the ejected photoelectrons. Millikan verified these predictions experimentally in 1916.

Einstein physical model, single photon directly delivering the entire energy to the electron, successfully explains almost all of the observed features of the photoelectric effect. However, one problem, which is not exposed in the literature, and still remains an open question, is that the energy required removing an electron from the outer shell of a neutral atom should be the first ionization energy. The energies of the work function for all of the photoelectron emitter elements are below the ionization energies of the same elements (Fig. 5).

\[\text{If } IE > \Phi + KE \text{ electron should not be emitted}\]

\[
h = \text{Planck's constant} \\
IE = \text{ionization energy} \\
KE = \text{kinetic energy of the photo electrons} \\
E_\nu = \text{energy of the incident photon} \\
\Phi = \text{work function} \\
\nu = \text{frequency of the incident light} \\
\nu_0 = \text{threshold frequency}
\]

**Fig. 5** Photoelectrons from the surface of the metal can be removed much lower energy than the ionization energy.

The experimentally defined first ionization energies of the 61 elements are 1.39-2.61 times higher than the energy of the work function.

Even more extreme, the electrons are delocalized, and can move freely inside the metal. Thus, the removal of an electron from the valence shell requires no energy in the bulk of a metallic solid.
How and why electrons can be freed with less energy than the ionization one is discussed.

5. Shielding the Casimir effect from the surface of the atom

It has been shown that an electron in the valence shell loses its stability when the absorbed photon and kinetic energies exceed the one-dimensional Casimir energy. It is speculated that if the Casimir effect is not active on the entire surface of the atom, then electrons can be removed at lower energies than the ionization one. If this hypothesis is correct, then the energy required to remove an outer shell electron from a neutral atom should be proportional to the active surface of the atom on which the Casimir effect is acting. The stability condition of the surface charge electrons, given in Eq. 15-16, it should be modified as:

\[ E_{\text{atom}}^e = A_{\text{active}}^\text{atom} E_{\text{C}}^e > E_{\gamma}^{e-c} + E_{\text{kin}}^{e-c} \]  \hspace{1cm} (19)

where \( A_{\text{active}}^\text{atom} \) is the proportion of the active surface area of the atom, exposed to the Casimir effect. Thus, the stability of an electron in the valence shell is also the function of the exposed surface of the atom on which the Casimir effect is active. On the surface of a solid metal approximately only half of the surface of the atom is exposed to the Casimir effect because the neighboring atoms in the bulk of the metal shield the inner parts of the surface of the atom/s. Thus, the removal of an electron from the valence shell predictably should be about half of the energy of the ionization. This prediction is consistent with experiments (Fig. 6).

![Fig. 6](image)

**Fig. 6** Ionization energies and the work functions of the elements with closed packing crystal structure are shown as a function of atomic radii. The ratio between the ionization energy and the work function is averaging around two with no correlation to the atomic radii. Thus, the ratio is independent from the ionization energy and remains the same for the same crystal structure.

Inside a metal the proportion of the effective surface area is zero, since the atoms are completely shielded by the neighboring atoms. Thus, the Casimir effect is entirely blocked, and ineffective in the bulk of the metal. Lacking the Casimir effect inside the metal results in the destabilization of the outer shell electrons even in the absent of any photon absorption energy. The electrons inside the metal; therefore, are unstable
and can move it freely forming the “sea of electrons” (Drude, 1900/a-b; Sommerfeld, 1928).

6. Conclusions
The stability, and the transformation from point to surface charge and vice versa of electrons are investigated in this study. The energies defining the stability of the electrons are the self-repulsion of the charge, the attraction of opposite charge/s on the electron, the Casimir effect, the zero-point energy, and the kinetic energy of the charge. Separating the energies to stabilizing and destabilizing ones in a one electron-proton system it is shown that the point charge electron loses its stability and transforms to a static surface charge around the nucleus at the distance equal to the Bohr’s radius.

The static surface charge electron in an atom remains stable as long as the sum of the energies of the attraction, and the one-dimensional Casimir effect exceeds the sum of the destabilizing energies, which are the self-repulsion, the zero-point, and the absorbed photon and kinetic energies. It can be calculated from the first principle that the static surface charge electron loses its stability in the Hydrogen atom, when the absorbed energies are the same or exceeds the one-dimensional Casimir energy, which is equal with the ionization energy.

It was speculated that if the Casimir effect is not active, because shielded by the neighboring atoms, then the electron can be removed at lower energy than the ionization energy. The reduction in the ionization energy should be proportional to the shielded area of the atom. On the surface of a metal the Casimir effect is active on about half of the surface of the atom. Thus, the destabilizing energy should be about half of the ionization energy. This prediction is consistent with observed work function of the photoelectric effect, which is about half of the ionization energy. Inside a metal where the entire surface of the atoms is shielded, the electrons are unstable, and can be detached without any additional photon energy absorption. This prediction is consistent with the known “electron sea” model of metals, or zero band energies of metals.

Declaration of competing interest
The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References