Quantum X-entropy in Generalized Quantum Evidence Theory

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Abstract

In this paper, a new quantum model of generalized quantum evidence theory is proposed. Besides, a new quantum X-entropy is proposed to measure the uncertainty in generalized quantum evidence theory.

Keywords: Generalized quantum evidence theory, Quantum X-entropy

1. A new quantum model of GQET

Definition 1.1 Let $\{|\Phi\rangle = \{ |\phi_1\rangle, \ldots, |\phi_j\rangle, \ldots, |\phi_m\rangle \}$ be a QFOD. A set of basis events is defined:

$$BE = \{|\emptyset\rangle, |\phi_1\rangle, \ldots, |\phi_j\rangle, \ldots, |\phi_m\rangle \},$$

where $|\emptyset\rangle$ is an unknown event.

Definition 1.2 A vector representation of a basis event is defined:

$$|e_z\rangle = [\eta_0, \eta_1, \ldots, \eta_g, \ldots, \eta_m]^T, \quad \eta_g = \begin{cases} 1, & g = z, \\ 0, & g \neq z. \end{cases}$$

Definition 1.3 A pure quantum state of proposition $|\psi_i\rangle$ is defined:

$$|\psi_i\rangle = \sum_{c} \lambda_i |e_c\rangle,$$
where $\lambda_i^z$ is a complex number with $\sum_t |\lambda_i^z|^2 = 1$.

**Definition 1.4** A density operator of $|\psi_i\rangle$ is defined as:

$$\rho_i = |\psi_i\rangle\langle\psi_i|.$$  \hfill (4)

**Definition 1.5** The density operator of a GQBBA is defined as:

$$\rho_{Q_M} = \sum_i Q_M(|\psi_i\rangle)\rho_i.$$  \hfill (5)

2. The proposed quantum X-entropy

**Definition 2.1** The quantum X-entropy is defined as:

$$X(Q_{M}) = -\tr\left(\rho_{Q_M} \log \frac{\rho_{Q_M}}{d}\right),$$  \hfill (6)

where $d$ denotes eigenvectors of $\rho_{Q_M}$.

Let $E_w$ and $d_w$ be eigenvalues and eigenvectors of $\rho_{Q_M}$, respectively. The quantum X-entropy is also defined as:

$$X(Q_{M}) = -\sum_w E_w \log \frac{E_w}{d_w}.$$  \hfill (7)