SQUARE INTERVALS AND DIVISORS Mm
Elementary theory of the discipline of natural numbers
which regulates the distribution of prime numbers

Filippo Giordano

ABSTRACT

The infinite set of natural numbers is formed by infinite subsets of pairs of quadratic intervals $[n(n-1)+1, n^2]$, $[n^2+1, n(n+1)]$. Each of these pairs of intervals is formed by increasing quantities of elements, all governed by their divisor, of value $\leq n$, closest to n. By bringing together these individual divisors of each element, a scale is formed for each interval, including all values included in the interval [1, n]. By assigning the name Mm (Major than minor) to these particular divisors, we note that between the elements and their divisors Mm, there is a one-to-one group correspondence which always allows for each quadratic interval the presence of at least one element having the trivial divisor 1 , i.e. the presence of at least one prime number, as noted by Oppermann's conjecture of 1882.

By extending Fermat's method of factoring natural numbers, through quadratic number lines, it is confirmed that each of the divisors Mm finds group correspondence with at least one of the elements of the quadratic intervals. The mathematical law that regulates the distribution of prime numbers, therefore, is expressed through the divisors Mm which, from time to time, depending on the occurrences of each quadratic interval, observe some fundamental rules common to all quadratic intervals. Furthermore, by observing the quadratic intervals arranged inside the Spiral of Ulam, one has the opportunity to observe that, in a fascinating way, Nature places all the quadratic intervals, assembling them according to their four different typologies, each in a different cardinal direction: quadratic intervals A of odd n, quadratic intervals B of odd n, quadratic intervals A of even n, quadratic intervals B of even n.

Warning

for reasons of tabular graphic synthesis, I have taken the liberty of representing the divisors Mm by placing them as a subscript of each natural number; therefore, when present, the juxtaposition of the small numbers placed to the right of the natural numbers always represents, only and exclusively, the indication of the specific divisor Mm of this natural number placed to its left. For the correct identification of the Mm divisors, I refer to the explanation contained on pages 2 and 3, as well as to the others which, occasionally, elaborate on some of their characteristics.

I

Quadratic intervals and Oppermann's conjecture

In 1882 Ludwig Oppermann noticed that between every perfect square and its successor there are at least two prime numbers and conjectured that this phenomenon extends to infinity. Delimiting Opperman's conjecture in the limited and closed intervals $[n(n-1)+1, n^2]$, which is given the name of SQUARE INTERVALS A, and in the limited and closed intervals $[(n^2+1), n(n+1)]$ which is given the name of SQUARE INTERVALS B, we obtain a pair of consecutive intervals formed by elements all different from each other, each of which is composed of a quantity of elements equal to the value of n. Since each new perfect square increases its value by one unit with respect to the previous perfect square, it follows that the elements of the two pertinent intervals delimiting the quadratic rooms A and B also constantly grow by one unit each.

By analyzing the divisors of the elements of each quadratic interval A and B belonging to any perfect square, it is evident that by identifying with the name of Mm (acronym of Major of minors) that particular divisor of each natural number constituted by the largest among the minors of all the numerical pairs which divide each natural number and distinguishing the various possible cases as follows:

a) All natural numbers are always divisible by one or more pairs of numbers. Composite numbers always have two or more pairs of divisors. For example, the factor pairs of the number 12 are three:

1x12,
2x6,
3x4.

Taking care to arrange, in such pairs, always the smallest divisor first, it is easy to identify among them the greatest divisor among the minors, which, in this case, is 3 (therefore the divisor Mm is 3).

b) In the case of elements corresponding to square numbers, their divisor Mm always corresponds to its square root. For example, the pairs of divisors of the quadratic number 16 are three:

1x16,
2x8,
4x4.

Among these three pairs it is easy to identify that the major divisor among the minors is 4, square root of 16, therefore divisor Mm.

c) Prime numbers always have only one pair of divisors, one of which is made up of the number itself and the other made up of the number 1. For example, the pair of divisors of the number 17 is
1x17,
therefore the divisor Mm is 1.

we have that each of the elements included in the two intervals considered can be factored by one of the divisors Mm included between 1 and n, given that between three consecutive multiples of n, [n(n-1), n^2, n(n+1)] of which the second is constituted by the perfect square of the same n, each element included between the two extremes always has a divisor Mm between 1 and n.

Example: for n = 5:
Quadratic interval A: 21, 22, 23, 24, 25,
21:3 = 7;
22:2= 11;
23:1 = 23;
24:4 = 6;
25:5 = 5.
Divisors Mm of the elements of the quadratic interval A = 1, 2, 3, 4, 5.

Quadratic interval B: 26, 27, 28, 29, 30.
26:2 = 13;
27:3 = 9;
28:4 = 7;
29:1 = 29;
30:5 = 6.
Divisors Mm of the elements of the quadratic interval B: 1, 2, 3, 4, 5.

Example: for n = 11:
Quadratic interval A: 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121
111:3 = 37;
112:8 = 14;
113:1 = 113;
114:6 = 19;
115:5 = 23;
116:4 = 29;
117:9 = 13;
118:2 = 59;
119:7 = 17;
120:10=12;
121:11=11.
Divisors Mm of the elements of the quadratic interval A: 1, 2, 3, 4, 5, 6, 7,,8, 9, 10, 11.

Quadratic interval B: 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132;
122:2 = 61;
123:3 = 41;
124:4 = 31;
125:5 = 25;
126:6 = 21; 126:7=18; 126:9=14;
127:1 = 127;
128:8 = 16;
129:3 = 43;
130:10=13;
131:1= 131;
132:11=12.
Divisors Mm of the elements of the quadratic interval B: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.
In the quadratic interval B of n=11, since the divisors Mm 6 and 9, which are prime to each other, instead of in two different elements both converge in the same element 126, being the greatest common divisor of 6 and 9 =3, then this common divisor 3 also becomes the divisor Mm of another element of the same quadratic interval and so in the quadratic interval B of n=11, instead of a single element with divisor Mm=3 there are two: 123 and 129. Furthermore, since the divisor Mm 7 (which is of the form 6k+1), is also a divisor of the element 126 which is an even number (of the form 6k); then, since all elements of the form 6k±1 always have divisors of the same form and since in the case of this quadratic interval [122, 132] the elements of this form are 125, 127, 131, while the divisors of the same form are 1 , 5, 7, 11, the number 5 being the only divisor Mm that aligns with the dividend of equal form (125), while 127 and 131 are divisible neither by 7 nor by 11, then the only remaining possible divisor of these elements is the trivial 1.

In this regard it is observed that, generally, in quadratic intervals the quantity of elements of the form 6k±1 is equal to that of the divisors of the same form 6k±1, and this allows the divisor Mm 1, which has the form 6k+1, to always be the exclusive divisor of at least one element of shape 6k±1. It is also observed that all quadratic intervals B of the form 6k±1 (5, 7, 11, 13, 17, 19, ...) although they always have one element of the form 6k±1 less than the divisors of the same form, since l their last element coincides with an even number, whose divisor Mm coincides with the value of n, then the quantity of elements (of the form 6k±1) remaining in the interval (of the form 6k±1) is equivalent to the quantity of the divisors Mm of forms 6k±1, and this ensures that the divisor Mm 1 is always the exclusive divisor Mm of at least one element of each interval.

Examples:
Quadratic interval B of n=5; elements [26, 30]; dividers Mm 6k±1 →1, 5;
combinations of dividers Mm of shape 6k±1 → 29-$_1$ – **30-**$_5$

Quadratic interval B of n = 7; elements [50, 56]; dividers Mm 6k±1 →1, 5, 7;
combinations of dividers Mm of shape 6k±1 → 53-$_{1}$ – 55-$_{5}$ – **56-**$_{7}$

Quadratic interval B of n = 11; elements [122, 132]; dividers Mm 6k±1 →1, 5, 7, 11;
combinations of dividers Mm of shape 6k±1 → 127-$_{1}$ – 125-$_{5}$ – 131-$_{1}$ – 126-$_{7}$ – **132-**$_{11}$

Quadratic interval B of n=13; elements [170, 182]; dividers Mm 6k±1 →1, 5, 7, 11, 13;
combinations of dividers Mm of shape 6k±1 → 173-$_{1}$ – 175-$_{5}$-7, 179-$_{1}$ -176-$_{11-8}$ – **182-**$_{13}$
In fact whenever in the quadratic intervals A or B, there are elements with different divisors Mm which converge there, then other elements of the same quadratic room assume divisors Mm which are greatest common divisors of the divisors Mm which converge in the same element. For example, in the quadratic interval B of n=10 formed by the elements included in the interval [101, 110], since in the element 105 the divisors Mm 3, 5, 7 converge, prime among them, then other elements of the same quadratic room assume the divisor Mm 1 as the greatest common divisor of the numbers 3, 5, 7. Moreover, since this quadratic room has an element of shape 6k±1 more (101, 103, 107, 109) than the divisors Mm of the same shape (1, 5, 7), then in this quadratic interval 4 elements are formed which have their exclusive divisor Mm 1, i.e. the prime numbers 101, 103, 107, 109.

This is the first precious rule common to all quadratic intervals, since the elements that form part of it have an overall discipline which guarantees the presence of all divisors Mm, from 1 to n, in all quadratic intervals (like a football or rugby team which guarantees the presence of all roles). When some element sometimes concentrates different roles on itself (ie has several prime divisors) then the team leaves more room for the prime elements. In practice, thanks to the phenomenon of the repeated confluence of some divisors Mm with single elements belonging to each quadratic interval, a sort of non-one-to-one correspondence is established between single elements and single divisors Mm, but a one-to-one group correspondence between all the elements and all divisors (all elements put together guarantee the presence of all divisors Mm between 1 and n).

If the one-to-one correspondence between each element of the quadratic intervals with a different divisor Mm were perpetual (and therefore the phenomenon of confluences of different divisors Mm in some elements did not exist) then, the phenomenon of rarefaction of prime numbers be more rapid, being the single presence of prime numbers in each quadratic interval constantly double that of perfect squares (one prime element in range A and one prime element in range B) since including 1 as divisor Mm of the only prime number of each quadratic interval, would cause a linear

ratio of presence of prime numbers to natural numbers equal to 1+1+1/2+1/2+1/3+1/3 +1/4+1/4+ ... +1/n+1/n and this would mean that for each oblong number n(n+1) there would always be counted a quantity of prime numbers equal to 2n (example, for n=20, then from 2 to 420 there would be only 39 prime numbers (apart from 1), instead of 81, while for n=100 from 2 to 10100 there would be only 199 prime numbers, instead of 1240.
Instead, to this natural tendency must be added the other natural tendency of the confluences which are the cause of the replicas of the prime numbers; trend, the latter, which, despite being fluctuating, remains constant and being generated by repetitive mathematical phenomena allows Gauss's formula, which splendidly quantifies the rarefaction of prime numbers, to always remain valid even for the regions of natural numbers still today unexplored.

II
SQUARE NUMBER LINES
mathematical extensions of Pierre de Fermat's factoring method

Quadratic number lines are mathematical structures that collimate and extend Pierre de Fermat's factorization method which is based on the representation of a number as the difference between two squares. In fact, quadratic number lines use and extend Fermat's idea in a methodical and extensive way by associating it with quadratic intervals. The quadratic number lines have their birth point (corresponding to the zero point of the line) each in a different perfect square, so that, since the perfect squares are infinite, the quadratic lines that form from them are also infinite. Their function, strictly connected to that of the quadratic intervals, is to filter ordered sequences of numbers, all placed at quadratic distances from the respective zero point, whose divisors Mm, starting from each perfect square, follow one another in perfect scalar order from n (with n having value corresponding to the root of the perfect square).

Below is a diagram of the first 110 natural numbers grouped according to the quadratic intervals A and B of n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

QUADRATIC INTERVAL A																					QUADRATIC INTERVAL B	
N=1										1	-											2
N=2									3	4	-										5	6
N=3								7	8	9	-									10	11	12
N=4							13	14	15	16	-								17	18	19	20
N=5						21	22	23	24	25	-							26	27	28	29	30
N=6					31	32	33	34	35	36	-						37	38	39	40	41	42
N=7				43	44	45	46	47	48	49	-					50	51	52	53	54	55	56
N=8			57	58	59	60	61	62	63	64	-				65	66	67	68	69	70	71	72
N=9		73	74	75	76	77	78	79	80	81	-			82	83	84	85	86	87	88	9	90
N=10	91	92	93	94	95	96	97	98	99	100	-		101	102	103	104	105	106	107	108	109	110

Highlighted in red: Perfect squares   Highlighted in red: Oblong numbers

Since n is the divisor always identified with a perfect square element = n^2, applying the divisibility method of Pierre de Fermat, we have that:
the elements placed at a distance n^2-1 from the perfect square, since 1 =1^2, the divisors of these elements are (n-1), (n+1).
Example: n^2 =100;
n= 10,
100 – 1= 99;
99 = (10-1)(10+1) = 9x11;

the elements placed at distance n-4 from the perfect square, since 4 =2^2, the divisors of these elements are (n-2), (n+2).
Example: n^2 =100;
n= 10,
100–4 = 96;
96 = (10-2)(10+2) = 8x12;

the elements placed at distance n-9 from the perfect square, since 9 =3^2, the divisors of these elements are (n-3), (n+3).
Example: n^2 =100;
n= 10,
100–9 = 91;
91 = (10-3)(10+3) = 7x13;

the elements placed at distance n-16 from the perfect square, since 16 =4^2, the divisors of these elements are (n-4), (n+4).
Example: n^2 =100;
n= 10,
100–16 = 84;
84 = (10-4)(10+4) = 6x14;

thus continuing up to n-(n-1), [example: 10-(10-1)=1], bearing in mind that the corresponding elements up to n-(10-2) are always composite numbers, example: [10-(10-2)=10-8=2] with consequent divisors: [(10-8)(10+8)=2x18=36] while, instead, the elements corresponding to n-(n-1), can coincide with composite numbers or be prime numbers [example: 10-(10-1)=10-9=1] with consequent divisors (10-9)(10+9) =1x19.

Example: n^2 =100;
n= 10,

100 – 81= 19;
19 = (10- 9) (10+9) = 1x19.

Of each of the pairs of divisors of the elements placed at quadratic distances from the respective perfect squares which fall within the sphere of the elements of the quadratic intervals, between the two, the one having value ≤ n assumes the function of divisor Mm.
Within the elements placed in each quadratic interval A of the previous table, all divisors Mm of each interval which have distance 1 from n, are aligned in a single vertical column; in the same way, in the fourth column preceding the perfect square, the divisors Mm of the elements placed at a distance of 2^2 from the perfect square are aligned; and so on also the other divisors placed at a distance of 9, 16, 25, …, m^2 from the respective perfect squares.

Divisors QUADRATIC INTERVALS A											
	n-3					n-2			n-1	n	-
	↓					↓			↓	↓	-
										1$_{-1}$	-
									3$_{-1}$	**4**$_{-2}$	-
								7	**8**$_{-2}$	**9**$_{-3}$	-
							13	**14**	**15**$_{-3}$	**16**$_{-4}$	-
						21$_{-3}$	**22**	**23**	**24**$_{-4}$	**25**$_{-5}$	-
					31	**32**$_{-4}$	**33**	**34**	**35**$_{-5}$	**36**$_{-6}$	-
				43	44	**45**$_{-5}$	46	47	**48**$_{-6}$	**49**$_{-7}$	-
			57	58	59	**60**$_{-6}$	61	62	**63**$_{-7}$	**64**$_{-8}$	-
		73	74	75	76	**77**$_{-7}$	78	79	**80**$_{-8}$	**81**$_{-9}$	-
	91$_{-7}$	92	93	94	95	**96**$_{-8}$	97	98	**99**$_{-9}$	**100**$_{-10}$	-
111	**112**$_{-8}$	113	114	115	116	**117**$_{-9}$	118	119	**120**$_{-10}$	**121**$_{-11}$	-

The numbers placed in subscript represent the divisors Mm of the elements

Given that through this procedure the other quadratic distances m are placed outside the quadratic intervals, in order to find the exact location of the divisors Mm of all the elements that form the quadratic intervals, an integrative method is applied. Continuing in decreasing order the search for the divisors Mm smaller than those already found, using the same method, first the elements external to the quadratic intervals are found and, subsequently, from this element one proceeds in the opposite direction, adding several times to the external element found, the value of the corresponding divisor, until, falling within the quadratic interval, all the elements of the interval which fall within the cadence of the divisor are reached.

Once these elements have been reached, it is necessary to proceed with the exact placement of the divisor Mm to the relevant element bearing in mind that, when the divisor Mm odd within the quadratic interval finds two of its consecutive multiples, of which obviously one is even and the other is odd , the natural allocation of the divisor

Mm is to refer to the odd element, since the even element is naturally destined to assume an even divisor Mm.
However, if the divisor Mm finds as its multiple a single element that is multiple of its even form then it is to be assigned to this even element, albeit in conjunction with another even divisor of the element itself. Example: quadratic interval of n=11, elements [111, 121]. Considering that the 4 largest divisors Mm of the interval (11, 10, 9, 8) have already been found, we proceed with the search for the element having the next decreasing divisor Mm, 7. The relevant element outside the interval is the number 105 since corresponding to $11^2 - 4^2$. In fact 105 is divisible by 7. By adding the value of the divisor 7 to the element 105 we obtain the value 112, which is part of the quadratic interval A of n=11 and, by adding 7 again, we obtain the value 119. However, since 112 is an even number which already has the divisor Mm =8, and since 119 is odd and is not divisible by numbers greater than 7, then 7 is the natural divisor Mm of 119. The proof is that the other divisor of 119 is 17, which, like 7, is also a prime number.

Proceeding to scale, the other out-of-range elements that form part of the quadratic number line of n = 121 are: 96 with divisor 6; 85 with divisor 5; 72 with divider 4; 57 with divisor 3; 40 with divider 2. Subsequently, adding to 96 (element outside the quadratic interval of n = 11, placed at a distance of -25 from 121) several times the divisor Mm 6, (96+6+6+6) one reaches element 114 of the interval quadratic which, is not already combined with another divisor Mm greater than 6 and, therefore, the divisor Mm of 114 is 6.
Subsequently, adding to 85 several times the divisor Mm 5, (85+5+5+5+5+ 5+5) the element 115 of the quadratic interval is reached which, is not already combined with another divisor Mm greater than 5.
Subsequently, by adding the divisor Mm 4 to 72 several times, the element 116 of the quadratic interval is reached which, is not already associated with another divisor Mm greater than 4.
Subsequently, by adding the divisor Mm 3 to 57 several times, one reaches 'element 111 of the quadratic interval which, is not already matched to another divisor Mm greater than 3.
Subsequently adding the divisor Mm 2 to 40 several times, one reaches the element 118 of the quadratic interval which, is not already matched to another divisor Mm greater than 2. Finally, since the element 113 has not been reached by any of the divisors Mm greater than 1, then it is prime number element.

40_2	57_3	72_4	85_5	96_6	105_7		111	112_8	113	114	115	116	117_9	118	119	120_{10}	121_{11}
↓	↓	↓	↓	↓	↓+7		→	~~7~~	→	→	→	→	→	→	119_7		
↓	↓	↓	↓	→+6	→		→	→	→	114_6							
↓	↓	↓	→+5	→	→		→	→	→	→	115_5						
↓	↓	→+4	→	→	→		→	→	→	→	→	116_4					
↓	→3	→	→	→	→		111_3		→								
→2	→		→		→		→		→		→		→	118_2			
							3	8	1	6	5	4	9	2	7	10	11

With the same procedure, proceeding to scale, following the same rules applied in the quadratic interval A, the divisors Mm are combined from the quadratic interval A to the quadratic interval B.

Continuing to give the examples with the quadratic interval B of n = 11, 11 is added to the element 121 (perfect square with divisor Mm 11) and the last element of the quadratic room B is reached, 132 (oblong number) attributing to it the divisor itself mm 11;

to the element 120, which has divisor Mm 10, we add 10 and we reach the element of the quadratic interval B = 130 by attributing equal divisor 10;

to the element 117, which has divisor Mm 9, we add 9 and we reach the element of the quadratic interval B = 126, attributing equal divisor 9;

to element 112, which has divisor Mm 8, one adds 8+8 (because adding only one 8 one would remain in the quadratic interval a) and one reaches the element of the quadratic interval B =128, attributing to it divisor 8;

to the element 119, which has divisor Mm 7, we add 7 and we reach the element of the quadratic interval B =126 attributing to it, (although 126 is already matched with the divisor Mm=9) the divisor 7 since this divisor Mm, neither before nor after the element 126 finds other elements that are multiples of it positioned in the same quadratic interval B, given that 126+7=133, an element outside the quadratic interval B of n=11. In this case the divisor 7, finding only one element of the quadratic interval B its multiple (126) which already has another divisor greater than 7, is defined as the "confluent" divisor Mm.

to the element 114, which has divisor Mm 6, one adds 6+6 (because adding only one 6 one remains in the quadratic interval a) and one reaches the element of the quadratic interval B =126, attributing equal divisor 6. Not the position 126+6=132 is considered since this element already has divisor Mm =11, root of the perfect square 121; also the divisor Mm 6, for the same reason as the divisor 7, is defined as the divisor Mm confluent.

to the element 115, which has divisor Mm 5, one adds 5+5 (because adding only one 5 one remains in the quadratic interval a) and one reaches the element of the quadratic interval B =125, attributing divisor 5. Not consider the position 125+5=130 since this element already has a divisor Mm =10, multiple of 5;

to element 116, which has divisor Mm 4, one adds 4+4 (because adding only one 4 one remains in the quadratic interval a) and one reaches the element of the quadratic interval B =124, attributing divisor 4. Not they consider the positions 124+4=128 since this element already has a divisor Mm =8, multiple of 4, and not even 124+4+4=132, since this element has a divisor Mm= 11, root of the perfect square 121;

to element 111, which has divisor Mm 3, one adds 3+3+3+3 (because adding 3 once, twice, three times, one remains in the quadratic interval a) and one reaches the element of the quadratic interval B =123, attributing equal divisor 3. The position 126 is not considered, since this element already has both 6 and 9 as its divisors which are both multiples of 3. Instead, the element 123+3+3=129 is considered since it does not combined with other Mm dividers, being only a multiple of 3;

to the element 118, which has divisor Mm 2, one adds 2+2 and thus the element of the quadratic interval B = 122 is reached, attributing equal divisor 2. The positions 124, 126, 128, 130, reachable, are not considered by the divisor Mm 2, since these elements already have other different divisors Mm all multiples of 2. The element 132 is not considered because it is an oblong number which always has as its divisor Mm the root of the perfect square of reference (11).

Having concluded the search for all divisors Mm >1, (2, 3, 4, 5, 6, 7, 8, 9, 10, 11) the remaining elements (127, 131), not reached by these divisors, are certainly prime numbers since the divisor Mm=1 is the only one among all the divisors that reaches all the elements one after the other, therefore the only one capable of reaching the elements that are not multiples of numbers greater than 1. In summary, the divisors Mm of the elements of the interval [122, 132] are those registered as subscripts of the elements.

122_{-2}	123_{-3}	124_{-4}	125_{-5}	126_{-6-7-9}	127_{-1}	128_{-8}	129_{-3}	130_{-10}	131_{-1}	132_{-11}

The one-to-one group correspondence between elements and divisors Mm assigns three different divisors confluent in element 126 (6, 7, 9). The divisors Mm 6 and 9, placed on the same element 126, since they are both multiples of the divisor 3 and because they are positioned on the same element, cause the replication of their common divisor 3 on another element of the group and therefore the divisor 3, as well as on the normal its element, it is positioned on another element (123 and 129).

The divisor Mm 7, also positioned on the element 126, since it is a divisor of the shape 6k±1 positioned on the element of the shape 6k, leaves empty an element of the shape 6k±1 of the interval which is therefore occupied by the single divisor of shape 6k±1 capable of reaching all elements of any shape, i.e. the divisor Mm 1. Therefore the divisor Mm 1, in addition to its guaranteed element of shape 6k±1, also becomes a divisor of another element of the same shape. So that the two excesses of divisors of the element 126 (7, 9) are compensated by the double presence of the divisors 1 and 3, respectively submultiples of 7 and 9.

These are the main rules governing the distribution of the divisors Mm within the quadratic intervals A and B, however there are still others, including two other different methods for the search for the Mm factors integrating those obtainable with the Pierre de Fermat method, to which a study of several other dozens of pages has been dedicated which are not listed here in order not to weigh down this theoretical synthesis.

Here is a summary table of the first 11 quadratic intervals A including the Fermat divisors placed at quadratic distances m from the roots n, which coincide with the divisors Mm, and the other divisors Mm obtained with the integrative method.

D i v i s o r s of quadratic intervals A											
	n-3					**n-2**			**n-1**	**n**	-
	↓					↓			↓	↓	-
										$\mathbf{1}_{-1}$	-
									$\mathbf{3}_{-1}$	$\mathbf{4}_{-2}$	-
								7_{-1}	$\mathbf{8}_{-2}$	$\mathbf{9}_{-3}$	-
							13_{-1}	14_{-2}	$\mathbf{15}_{-3}$	$\mathbf{16}_{-4}$	-
						$\mathbf{21}_{-3}$	22_{-2}	23_{-1}	$\mathbf{24}_{-4}$	$\mathbf{25}_{-5}$	-
					31_{-1}	$\mathbf{32}_{-4}$	33_{-3}	34_{-2}	$\mathbf{35}_{-5}$	$\mathbf{36}_{-6}$	-
				43_{1}	44_{4}	$\mathbf{45}_{-5}$	46_{2}	47_{1}	$\mathbf{48}_{-6}$	$\mathbf{49}_{-7}$	-
			57_{3}	58_{2}	59_{1}	$\mathbf{60}4\text{-}5_{-6}$	61_{1}	62_{2}	$\mathbf{63\text{-}}_{7}$	$\mathbf{64}_{-8}$	-
		73_{1}	74_{2}	75_{5}	76_{4}	$\mathbf{77}_{-7}$	78_{6}	79_{1}	$\mathbf{80}_{-8}$	$\mathbf{81}_{-9}$	-
	$\mathbf{91}_{-7}$	92_{4}	93_{3}	94_{2}	95_{5}	$\mathbf{96}_{-8}$	97_{1}	98_{2}	$\mathbf{99}_{-9}$	$\mathbf{100}_{-10}$	-
111_{3}	$\mathbf{112}_{-8}$	113_{1}	114_{6}	115_{5}	116_{4}	$\mathbf{117}_{-9}$	118_{2}	119_{7}	$\mathbf{120}_{-10}$	$\mathbf{121}_{-11}$	-

And here is a summary table of the divisors Mm of the elements of the first 11 quadratic intervals A and B, arranged according to the natural order of their elements. The divisors obtainable with Fermat's method are those indicated in red.

Divisors quadratic intervals A												Divisors quadratic intervals B										
	n-3					n-2			n-1	n	-					n-2				n-1		N
	↓					↓			↓	↓	-					↓				↓		↓
										1	-											1
									1	2	-										1	2
								1	2	3	-									2	1	3
							1	2	3	4	-								1	2-3	1	4
						3	2	1	4	5	-							2	3	4	1	5
					1	4	3	2	5	6	-						1	2	3	4-5	1	6
				1	4	3-5	2	1	6	7	-					2-5	3	4	1	6	5	7
			3	2	1	4-5-6	1	2	7	8	-				5	6	1	4	3	2-7	1	8
		1	2	5	4	7	6	1	8	9	-			2	1	4-6-7	5	2	3	8	1	9
	7	4	3	2	5	6-8	1	2	9	10	-		1	6	1	8	3-5-7	2	1	4-9	1	10
3	8	1	6	5	4	9	2	7	10	11	-	2	3	4	5	6-7-9	1	8	3	10	1	11

The numbers appearing in the table reflect the divisors Mm of the elements of the first 11 pairs of quadratic rooms in the strict order of appearance of their dividing elements which, starting from 1, reach 132 following their horizontal order. Thus the numbers lined up under the letter n of the quadratic rooms A and B indicate the specific name of the quadratic rooms which each time corresponds to the root of the corresponding element that we virtually have to see lined up under the first n (1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121) while the same vertical sequence of the divisors Mm placed under the second n refer to the extreme elements of the quadratic rooms B, i.e., respectively, 2, 6, 12, 20, 30, 42 , 56, 72, 90, 110, 132.

From which it can be deduced that Quadratic intervals, divisors Mm and numerical quadratic lines, elementary mathematical entities hitherto unknown, synergistically allow to obtain the divisors Mm of all the composite numbers that are part of each "square interval" and consequently, by exclusion, also of prime numbers. Furthermore, since each natural number is part of a single set called "square interval", it can be deduced that the synergy of the three entities allows the automatic factorization of each natural number.

It is obvious that, dealing with large numbers, the mathematical logic that regulates the distribution of prime numbers is one thing and the computational difficulty of correct combinations between elements and divisors Mm is another thing. Currently humanity does not have computers capable of performing these tasks quickly but when it does, it will know which path to use.

The following table shows a map of the quadratic intervals A and B where all the elements of the first 11 values of n alternate. The subscript numbers of the elements represent the divisors Mm of the elements. In each of these intervals the one-to-one correspondence of the group between elements and divisors Mm is noted since all the divisors Mm, comprised between 1 and n, are distributed in all the elements that form the intervals.

1-$_{1}$											1-A
2-$_{1}$											1-B
3-$_{1}$	4-$_{2}$										2-A
5-$_{1}$	6-$_{2}$										2-B
7-$_{1}$	8-$_{2}$	9-$_{3}$									3-A
10-$_{2}$	11-$_{1}$	12-$_{3}$									3-B
13-$_{1}$	14-$_{2}$	15-$_{3}$	16-$_{4}$								4-A
17-$_{1}$	18-$_{2-3}$	19-$_{1}$	20-$_{4}$								4-B
21-$_{3}$	22-$_{2}$	23-$_{1}$	24-$_{4}$	25-$_{5}$							5-A
26-$_{2}$	27-$_{3}$	28-$_{4}$	29-$_{1}$	30-$_{5}$							5-B
31-$_{1}$	32-$_{4}$	33-$_{3}$	34-$_{2}$	35-$_{5}$	36-$_{6}$						6-A
37-$_{1}$	38-$_{2}$	39-$_{3}$	40-$_{4-5}$	41-$_{1}$	42-$_{6}$						6-B
43-$_{1}$	44-$_{4}$	45-$_{3-5}$	46-$_{2}$	47-$_{1}$	48-$_{6}$	49-$_{7}$					7-A
50-$_{2}$	51-$_{3}$	52-$_{4}$	53-$_{1}$	54-$_{6}$	55-$_{5}$	56-$_{7}$					7-B
57-$_{3}$	58-$_{2}$	59-$_{1}$	60-$_{4-5-6}$	61-$_{1}$	62-$_{2}$	63-$_{7}$	64-$_{8}$				8-A
65-$_{5}$	66-$_{6}$	67-$_{1}$	68-$_{4}$	69-$_{3}$	70-$_{2}$	71-$_{1}$	72-$_{8}$				8-B
73-$_{1}$	74-$_{2}$	75-$_{5}$	76-$_{4}$	77-$_{7}$	78-$_{6}$	79-$_{1}$	80-$_{8}$	81-$_{9}$			9-A
82-$_{2}$	83-$_{1}$	84-$_{4-6-7}$	85-$_{5}$	86-$_{2}$	87-$_{3}$	88-$_{8}$	89-$_{1}$	90-$_{9}$			9-B
91-$_{7}$	92-$_{4}$	93-$_{3}$	94-$_{2}$	95-$_{5}$	96-$_{6-8}$	97-$_{1}$	98-$_{2}$	99-$_{9}$	100-$_{10}$		10-A
101-$_{1}$	102-$_{6}$	103-$_{1}$	104-$_{8}$	105$_{3-5-7}$	106-$_{2}$	107-$_{1}$	108-$_{4}$-$_{9}$	109-$_{1}$	110-$_{10}$		10-B
111-$_{3}$	112-$_{8}$	113-$_{1}$	114-$_{6}$	115-$_{5}$	116-$_{4}$	117-$_{9}$	118-$_{2}$	119-$_{7}$	120-$_{10}$	121-$_{11}$	11-A
122-$_{2}$	123-$_{3}$	124-$_{4}$	125-$_{5}$	126$_{6-7-9}$	127-$_{1}$	128-$_{8}$	129-$_{3}$	130$_{10}$	131-$_{1}$	132-$_{11}$	11-B

It should be noted that the sum of the divisors Mm of each quadratic interval, taken individually, i.e. not considering any duplications, always corresponds to the triangular number corresponding to the value of each n, i.e. to the root of the perfect square corresponding to the intervals. For example, the sum of the divisors Mm of the elements forming the quadratic interval A of n= 6 (root of the perfect square 36) is 1+ 2+ 3+ 4+ 5+ 6 =21 (which is the sixth triangular number)

Quadratic intervals	Elements	Sum of divisors Mm	Triangular number corresponding
1 – A	1	1	1
1 – B	2	1	1
2 – A	3, 4	1+2	3
2 – B	5, 6	1+2	3
3 – A	7, 8, 9	1+2+3	6
3 – B	10, 11, 12	1+2+3	6
4 – A	13, 14, 15, 16	1+2+3+4	10
4 – B	17, 18, 19, 20	1+2+3+4	10
5 - A	21, 22, 23, 24, 25	1+2+3+4+5	15
5 – B	26, 27, 28, 29, 30	1+2+3+4+5	15
6 – A	31, 32, 33, 34, 35, 36	1+2+3+4+5+6	21
6 – B	37, 38, 39, 40, 41, 42	1+2+3+4+5+6	21
7 – A	43, 44, 45, 46, 47, 48, 49	1+2+3+4+5+6+7	28
7 – B	50, 51, 52, 53, 54, 55, 56	1+2+3+4+5+6+7	28

Numbers in red are elements with duplicate divisor Mm, not considered for the sum

If in any quadratic interval the prime number, which has divisor Mm=1, were missing, then the sum of the divisors Mm of the elements would not coincide with the triangular number and, consequently, the sum of the divisors of the interval containing their numbers doubled , lacking the element having divisor Mm 2 = 2p equally it would not coincide with a triangular number and so on for prime numbers tripled, quadrupled, etc.

Growth trend of the quantity of prime numbers thanks to the phenomenon of the confluences of the divisors Mm in some elements.

N values of the quadratic intervals	Intervals considered From... to...	In the first column, quantity of prime numbers if the phenomenon of confluences of divisors were not present. Alongside, the effective quantity of prime numbers existing in the intervals.	
Da 01 a 05	2 – 30	5x2 = 10	5x2 = 10
da 06 a 13	31 – 182	8x2 = 16	8x4 = 32
da 14 a 24	183 – 600	11x2 =22	11x6 = 66 (+1) = 67
da 25 a 36	601 – 1332	12x2 =24	12x9 = 108
Da 37 a 48	1333 – 2352	12x2 =24	12x11=132
Da 49 a 60	2353 – 3660	12x2 =24	12x13,5 =162
Da 61 a 72	3661 – 5256	12x2 =24	12x15,5= 186
Da 73 a 84	5257 – 7140	12x2 = 24	12x18 (+1) = 217
Da 84 a 96	7141 – 9312	12x2 = 24	12x20 (-2) =238

This key to understanding the distribution of prime numbers in the context of the natural numbers, while obviously not altering their objective constant rarefaction, offers us a more precise picture of the real mathematical dynamics affecting all natural numbers starting from their set aggregations within the quadratic intervals initially organized through a one-to-one correspondence between elements and divisors Mm and which, proceeding towards large numbers, by virtue of the natural confluences of the divisors in some elements, favor the emergence of an increasing mass of numbers first which considerably slows down the rarefaction to which they would be destined.

The analysis of the recurrences that determine the constant growth of the prime numbers is confirmed by the tests carried out on hundreds of consecutive quadratic intervals, as well as, on a sample basis, for even greater values of n which show a slightly fluctuating trend between consecutive but homogeneous values of n when compared to larger distances; mathematical phenomenon that arises from misalignments between the dividend elements of the shape $6k\pm1$, whose quantity remains constant, and their divisors Mm of the same shape $6k\pm1$ whose quantity of the same shape is equal to that of the dividends but whose misalignment between the parts is due to the increasing quantity of prime numbers present for each quadratic interval. However, it is evident that the constant growth of the presence of prime numbers within the quadratic intervals, given the constant and linear increase of the composite elements, reflects a proportion between primes and compounds which tends to a continuous rarefaction of the prime numbers.

In other words, the natural phenomenon of rarefaction, which is caused by the multiples of prime numbers (if all numbers of the form $6k \pm 1$ were always prime, the ratio between primes and composites would always be equivalent to 1/3, as effectively occurs in the interval 1, 24) undergoes two opposite tendencies: on the one hand the natural overall rarefaction and on the other the equally natural tendency of the constant growth of the quantity of prime numbers within the quadratic rooms. From which it can be deduced that the second tendency, which extrinsics a constant increase of the prime numbers within the quadratic rooms, slows down the first.

Dwelling on particular aspects of the quadratic intervals A and B, one realizes that they can be divided into typologies depending on the quantity of elements from which they are formed. The in-depth analysis of these typologies allows us to verify that in them mathematical phenomena are cyclically repeated which cause the recurrence of the confluences of the divisors Mm and the consequent growth of the presence of prime numbers in their respective quadratic intervals. These reiterated characteristics, differently articulated, which affect the twelve different types of quadratic rooms (already described in a large separate study entitled: "Quadratic rooms and divisors Mm, the unknown discipline of natural numbers that regulates the distribution of prime numbers") since they cannot be summarized in a few lines, they are obviously excluded from this summary. To give an example: all quadratic intervals B [n2+1, n(n+1)] of n even, with $n>2$ (i.e. the quadratic intervals B of $n=4, 6, 8, 10$, etc.) always have at least two prime numbers. The motivation is caused precisely in the mathematical reiterations of confluences of divisors Mm.

Quantity of prime numbers within quadratic intervals

Value of n	Prime numbers in A	Prime n. in B	Value of n	Prime n. in A	Prime numbers in B
1	1	1	**2**	1	1
3	1	1	**4**	1	2
5	1	1	**6**	1	2
7	2	1	**8**	2	2
9	2	2	**10**	1	4
11	1	2	**12**	2	2
13	3	3	**14**	2	2
15	2	4	**16**	2	4
17	3	1	**18**	4	2
19	4	3	**20**	3	3
21	4	4	**22**	3	4
23	3	2	**24**	4	4
25	5	4	**26**	6	4
27	3	4	**28**	4	4
29	5	4	**30**	4	4
31	4	5	**32**	5	5
33	4	6	**34**	4	4
35	5	5	**36**	5	7
37	2	3	**38**	6	6
39	6	6	**40**	5	8
41	4	5	**42**	6	5
43	4	7	**44**	5	4
45	7	6	**46**	7	7
47	3	6	**48**	7	7
49	8	6	**50**	4	6
51	5	5	**52**	10	9
53	7	7	**54**	5	7
55	6	6	**56**	5	7
57	5	7	**58**	10	6
59	7	8	**60**	8	8
61	8	7	**62**	6	7
63	10	8	**64**	7	9
65	5	11	**66**	5	7
67	8	8	**68**	7	10
69	7	8	**70**	5	11
71	6	8	**72**	7	7
73	8	7	**74**	10	10
75	7	11	**76**	7	12
77	10	4	**78**	10	9
79	9	11	**80**	12	6
81	7	9	**82**	11	9
83	10	10	**84**	10	8
85	9	9	**86**	7	8
87	13	11	**88**	11	8
89	10	8	**90**	10	9
91	11	10	**92**	10	8
93	11	13	**94**	10	10
95	11	9	**96**	12	10
97	11	14	**98**	8	12
99	11	12	**100**	9	11

From the analytical reading of table 2 (Amount of prime numbers within the quadratic intervals) it can be seen that up to the value of n=17 in all quadratic intervals both A and B there is a constant presence of prime numbers which oscillates from one at four; starting from the quadratic interval A of n = 18 the prime numbers always become at least two; fluctuating again upwards until finally becoming at least three starting from the interval B of the value of n= 37; at least 4 from interval B of n=47, at least 5 from interval A of n=51, and at least 8 from interval B of n=86.

Thus, fluctuating, the prime numbers present in the quadratic intervals A and B of n = 1000, (table 3) composed of one thousand + one thousand elements, become 65 in the interval A and 75 in the interval B, while, still rising to the value of n =10,000 become 533 in interval A and 551 in interval B, with an average proportion equal to 5.42% which highlights the fact that, despite the continuous growth of prime numbers present in the respective quadratic intervals, the effective proportion of prime numbers compared to the totality of natural numbers tends to decrease, which is quite obvious since everything is related to the increasingly massive presence of natural numbers within quadratic intervals.

Value of n	Prime numbers in A	Prime n. in B	**Value of n**	Prime n. in A	Prime numbers in B
150	13	15	**200**	20	20
250	19	22	**300**	22	32
350	31	29	**400**	35	33
450	33	36	**500**	40	33
550	33	50	**600**	39	41
650	47	52	**700**	55	56
750	55	65	**800**	58	60
850	63	65	**900**	72	69
950	61	67	**1.000**	65	75
1.500	102	88	**2.000**	133	137
2.500	157	166	**3.000**	187	164
3.500	210	205	**4.000**	251	228
4.500	255	269	**5.000**	271	315
10.000	533	551	**20.000**	1029	986

As explained above, i.e. due to the increasing presence in the quadratic intervals of the confluences of divisors Mm, which cause the increasing presence of prime numbers in the quadratic intervals, I believe that Oppermann's conjecture can be considered proved also because Oppermann's conjecture is limited to affirm that in each of the intervals that precede and follow the perfect squares there is always the presence of at least one prime number, while the growing phenomenon of the confluences of the divisors Mm and the consequent replicas of the prime numbers demonstrates the natural tendency of the prime numbers to a constant growth of their quantity, within the quadratic intervals (see tables 2 and 3).

Furthermore, in the light of the internal properties of the Sets called Quadratic Intervals, assumed by virtue of the divisors Mm, it does not seem rash to me to consider that they are to all intents and purposes to be considered perpetual sub-orders of the natural numbers of which they constitute ever larger intervals , autonomously governed by the divisors Mm, which, among other things, reveal the arcane law that regulates the distribution of prime numbers. In this regard it is important to grasp a particular aspect, namely that the present "elementary theory" is not to be understood as an evolution of modern mathematics but as its basis, being the mathematical edifice that houses the natural numbers so far suspended on a cultural void that has always left unsatisfied the question: "Is there a mathematical law that regulates the distribution of prime numbers?" A cultural void that the theory of quadratic intervals and Mm” divisors fill. A basis supported by elementary mathematical reasoning which, for this specific reason, could have been understood several centuries ago, since it is not necessary for its demonstration to resort to the modern evolutions of mathematics, being, on the contrary, necessary to make a historical regression such as that of quadratic system used by Nature, compared to the decimal system subsequently introduced by man.

By virtue of the elementary mathematical logic that within the Sets called quadratic rooms always distributes elements having the trivial 1 as their natural and exclusive divisor, these being natural Sets appendages of each perfect square and the perfect squares being infinite, it becomes elementary to conceive the idea of the mathematical reason why prime numbers are consequently destined to be infinite too.

III
Spiral of Ulam, "squared intervals" topography

A different view of the Spiral of Ulam, compared to the common view of it that has been recorded up to now by those who have studied it, shows a formidable confirmation of the validity of the theory of quadratic intervals and of the divisors Mm, which, like tesserae of a mosaic, fit perfectly into the grid randomly created by the Polish mathematician and physicist Stanislaw Ulam in 1963.
A new and original key to reading the natural numbers, which decodes the extraordinary map of natural numbers that is obtained from it thanks to the particular properties that the Sets of quadratic intervals acquire which are functional to them. Observing the Spiral of Ulam, segmented into four equal parts by the four different background colours, one can notice all the quadratic intervals which branch out in an orderly manner in the four cardinal directions, starting from the south center of the table with the first two intervals, composed of a single element each (1, 2) and successively covering, in continuous rotation, the four cardinal directions and turning, always following the same order, at each interval:

north: 3, 4, (interval elements A of n=2);
west: 5, 6, (interval B elements of n=2);
south: 7, 8, 9, (interval A elements of n=3);
ext: 10, 11, 12 (interval B elements of n=3);

north: 13, 14, 15, 16, (interval elements A of n=4);
west: 17, 18, 19, 20, (interval B elements of n=4);
south: 21, 22, 23, 24, 25, (interval elements A of n=5);
est: 26, 27, 28, 29, 30, (interval B elements of n=5);

north: 31, 32 ,33, 34, 35, 36, (interval elements A of n=6);
west: 37, 38, 39, 40, 41, 42, (interval B elements of n=6);
south: 43, 44, 45, 46, 47, 48, 49, (interval elements A of n= 7);
est: 50, 51, 52, 53, 54, 55, 56, (interval B elements of n=7)…

and so on to infinity,

so:
all intervals A of even n lie north,
all intervals B of even n lie to the west,
all intervals A of odd n lie to the south,
all intervals B of n odd lie to the south.

The Spiral of Ulam

324	323	322	321	320	319	318	317	316	315	314	313	312	311	310	309	308	307
257	256	255	254	253	252	251	250	249	248	247	246	245	244	243	242	241	306
258	197	196	195	194	193	192	191	190	189	188	187	186	185	184	183	240	305
259	198	145	144	143	142	141	140	139	138	137	136	135	134	133	182	239	304
260	199	146	101	100	099	098	097	096	095	094	093	092	091	132	181	238	303
261	200	147	102	065	064	063	062	061	060	059	058	057	090	131	180	237	302
262	201	148	103	066	037	036	035	034	033	032	031	056	089	130	179	236	301
263	202	149	104	067	038	017	016	015	014	013	030	055	088	129	178	235	300
264	203	150	105	068	039	018	005	004	003	012	029	054	087	128	177	234	299
265	204	151	106	069	040	019	006	001	002	011	028	053	086	127	176	233	298
266	205	152	107	070	041	020	007	008	009	010	027	052	085	126	175	232	297
267	206	153	108	071	042	021	022	023	024	025	026	051	084	125	174	231	296
268	207	154	109	072	043	044	045	046	047	048	049	050	083	124	173	230	295
269	208	155	110	073	074	075	076	077	078	079	080	081	082	123	172	229	294
270	209	156	111	112	113	114	115	116	117	118	119	120	121	122	171	228	293
271	210	157	158	159	160	161	162	163	164	165	166	167	168	169	170	227	292
272	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	291
273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290

This interpretation of the Spiral of Ulam, in my personal way of seeing the universe of natural numbers, is a beacon that illuminates the foundations of the mathematical edifice since, properly investigated, it reveals itself to be a mine of information regarding various aspects of the factorization of numbers and, among these aspects, emerges the possibility of generalizing Fermat's factorization method through continuous, uninterrupted lines of factorizable numbers, which cross the table of the Spiral of Ulam which, obviously, is represented here only for the first values but which can be extended, with suitable means, up to where one wants.
The following table traces a network of numbers of the Ulam Spiral which conveniently brings together the elements that can be factorizable with divisors Mm using Pierre de Fermat's method within each quadratic interval A (because each of these intervals refers to the perfect square, placed as the last element of the range

itself). These divisors Mm can be formulated a priori since they are always placed on elements of the intervals placed at fixed distances from the perfect squares.

Elements: $(n^2-1^2) \rightarrow$ Divisors Mm: (n-1).
Example : 17^2-1^2 = Element 288, divisor 16

Elements: $(n^2-2^2) \rightarrow$ Divisors Mm: (n-2).
Example: 17^2-2^2 = Element 285, divisor 15

Elements: $(n^2-3^2) \rightarrow$ Divisors Mm: (n-3).
Example: 17^2-3^2 = Element 280, divisor 14

Elements: $(n^2-4^2) \rightarrow$ Divisors Mm: (n-4).
Example: 17^2-4^2 = Element 273, divisor 13

484	483	482	481	480	479	478	477	476	475	474	473	472	471	470	469	468	467	466	465	464	463
401	400	399	398	397	396	395	394	393	392	391	390	389	388	387	386	385	384	383	382	381	462
402	325	324	323	322	321	320	319	318	317	316	315	314	313	312	311	310	309	308	307	380	461
403	326	257	256	255	254	253	252	251	250	249	248	247	246	245	244	243	242	241	306	379	460
404	327	258	197	196	195	194	193	192	191	190	189	188	187	186	185	184	183	240	305	378	459
405	328	259	198	145	144	143	142	141	140	139	138	137	136	135	134	133	182	239	304	377	458
406	329	260	199	146	101	100	099	098	097	096	095	094	093	092	091	132	181	238	303	376	457
407	330	261	200	147	102	065	064	063	062	061	060	059	058	057	090	131	180	237	302	375	456
408	331	262	201	148	103	066	037	036	035	034	033	032	031	056	089	130	179	236	301	374	455
409	332	263	202	149	104	067	038	017	016	015	014	013	030	055	088	129	178	235	300	373	454
410	333	264	203	150	105	068	039	018	005	004	003	012	029	054	087	128	177	234	299	372	453
411	334	265	204	151	106	069	040	019	006	001	002	011	028	053	086	127	176	233	298	371	452
412	335	266	205	152	107	070	041	020	007	008	009	010	027	052	085	126	175	232	297	370	451
413	336	267	206	153	108	071	042	021	022	023	024	025	026	051	084	125	174	231	296	369	450
414	337	268	207	154	109	072	043	044	045	046	047	048	049	050	083	124	173	230	295	368	449
415	338	269	208	155	110	073	074	075	076	077	078	079	080	081	082	123	172	229	294	367	448
416	339	270	209	156	111	112	113	114	115	116	117	118	119	120	121	122	171	228	293	366	447
417	340	271	210	157	158	159	160	161	162	163	164	165	166	167	168	169	170	227	292	365	446
418	341	272	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	291	364	445
419	342	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	363	444
420	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	443
421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442

The quadratic intervals A and B give shape to a dress (the spiral of Ulam) which, up to now, despite the multiple trails of prime numbers identified in it, has not been adequately understood. In fact, the numerical body of the Spiral of Ulam, in addition to the trails of prime numbers, which represent a marginal phenomenon, contains a heart that pulsates within the Sets A and B made up of quadratic intervals and by pulsating it always supplies new life to other new Sets. The new lymph is constituted by the prime numbers which, being born in each Set, constitute the origin of their respective multiples. So that each new Set always includes a complete chain of divisors which, starting from the trivial 1, arrives at the value of n.

The study of the Spiral of Ulam, following the filter of the divisors Mm of the quadratic intervals, allows us to discover further characteristics concerning the natural location of these dividers within each of the four sections of the Spiral. Proceeding from the inside of the Spiral outwards, a line of numerical elements rises from each of the sections (highlighted in green) whose respective divisors represent the average of the divisor value of the elements of the Set to which they belong.

In the case of Sets that refer to N of even value, the median element of the Sets forming part of the ordinate vertical line is divisible by a number equal to n/2 (if n=4 the median element of the Set is divisible by 2 ; if n=6 the median element of the Set is divisible by 3, etc.). In the case of Sets that refer to n of odd value, when the elements of the ordinate vertical line that crosses them belong to Sets A then the value of the divisor of the considered elements corresponds to (n-1)/2 (if n=5 l median element of the Set is divisible by (5-1)/2 = 2; if n=7 the median element of the Set is divisible by (7-1)/2 = 3, etc. When the elements belong to Sets B then the divisor of the elements considered corresponds to (n+1)/2 (if n=5 the median element of the Set is divisible by (5+1)/2 = 3; if n = 7, the median element of the Set is divisible by (7+1)/2 = 4.

So that of the numerical lines of the following table highlighted in green, arranged vertically with respect to Sets A and arranged horizontally with respect to Sets B, the respective divisors are: 2, 3, 4, 5, 6, ,7 ,8, 9, 10, etc.

10-$_{2}$ – 27-$_{3}$ – 52-$_{4}$ – 085-$_{5}$ – 126-$_{6}$ – 175-$_{7}$ – 232-$_{8}$ – 297-$_{9}$ – 370-$_{10}$ – 451-$_{11}$ – ecc.

14-$_{2}$ – 33-$_{3}$ – 60-$_{4}$ – 095-$_{5}$ – 138-$_{6}$ – 189-$_{7}$ – 248-$_{8}$ – 315-$_{9}$ – 390-$_{10}$ – 473-$_{11}$ – ecc.

18-$_{2}$ – 39-$_{3}$ – 68-$_{4}$ – 105-$_{5}$ – 150-$_{6}$ – 203-$_{7}$ – 264-$_{8}$ – 333-$_{9}$ – 410-$_{10}$ – 495-$_{11}$ – ecc.

22-$_{2}$ – 45-$_{3}$ – 76-$_{4}$ – 115-$_{5}$ – 162-$_{6}$ – 217-$_{7}$ – 280-$_{8}$ – 351-$_{9}$ – 430-$_{10}$ – 517-$_{11}$ – ecc.

484	483	482	481	480	479	478	477	476	475	474	473	472	471	470	469	468	467	466	465	464	463
401	400	399	398	397	396	395	394	393	392	391	390	389	388	387	386	385	384	383	382	381	462
402	325	324	323	322	321	320	319	318	317	316	315	314	313	312	311	310	309	308	307	380	461
403	326	257	256	255	254	253	252	251	250	249	248	247	246	245	244	243	242	241	306	379	460
404	327	258	197	196	195	194	193	192	191	190	189	188	187	186	185	184	183	240	305	378	459
405	328	259	198	145	144	143	142	141	140	139	138	137	136	135	134	133	182	239	304	377	458
406	329	260	199	146	101	100	099	098	097	096	095	094	093	092	091	132	181	238	303	376	457
407	330	261	200	147	102	065	064	063	062	061	060	059	058	057	090	131	180	237	302	375	456
408	331	262	201	148	103	066	037	036	035	034	033	032	031	056	089	130	179	236	301	374	455
409	332	263	202	149	104	067	038	017	016	015	014	013	030	055	088	129	178	235	300	373	454
410	333	264	203	150	105	068	039	018	005	004	003	012	029	054	087	128	177	234	299	372	453
411	334	265	204	151	106	069	040	019	006	001	002	011	028	053	086	127	176	233	298	371	452
412	335	266	205	152	107	070	041	020	007	008	009	010	027	052	085	126	175	232	297	370	451
413	336	267	206	153	108	071	042	021	022	023	024	025	026	051	084	125	174	231	296	369	450
414	337	268	207	154	109	072	043	044	045	046	047	048	049	050	083	124	173	230	295	368	449
415	338	269	208	155	110	073	074	075	076	077	078	079	080	081	082	123	172	229	294	367	448
416	339	270	209	156	111	112	113	114	115	116	117	118	119	120	121	122	171	228	293	366	447
417	340	271	210	157	158	159	160	161	162	163	164	165	166	167	168	169	170	227	292	365	446
418	341	272	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	291	364	445
419	342	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	363	444
420	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	443
421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442

In addition to these highlighted lines, there are many other lines with the same characteristics, i.e. uninterrupted lines of elements whose divisors Mm follow each other in perfect ascending order. These lines are discovered gradually as, by enlarging the Spiral in the four cardinal directions, the quadratic intervals contain more and more elements and proceeding outwards, new lines of elements are always discovered which have their divisors Mm in an orderly natural sequence.

Bibliografhy

-Marcus du Sautoy, L'enigma dei numeri primi, Rizzoli, Milano 2004.

-Umberto Eco, Contenti di arrivare primi, L'Espresso, Roma 2004.

-Piergiorgio Odifreddi, La matematica del Novecento, Einaudi. Edizione speciale per Panorama, 2007.

-John Derbyshire: L'ossessione dei numeri primi, Bernard Riemann e il principale problema irrisolto della matematica, 2009, edizione speciale per il mensile Le Scienze.

-Filippo Giordano, La ragione dei Primi, Cosmografica, Roma 2009.

-Kitty Ferguson, La musica di Pitagora, Longanesi. 2010 Edizione speciale per il mensile Le Scienze".

-Enrique Graciàn, I numeri primi, un lungo cammino verso l'infinito. RBA Italia S.r.l. 2011.

-Keith Devlin: I numeri magici di Fibonacci, l'avventurosa scoperta che cambiò la storia della matematica, Corriere della sera, 2014.

-Piergiorgio Odifreddi, Il museo dei numeri, Da zero verso l'infinito, storie dal mondo della matematica, Rizzoli 2014.

-Antonio Rufian Lizana, Gauss, una rivoluzione nella teoria dei numeri, RBA Italia Srl, 2017.

-Nelo Maestre Blanco, Paradossi e assiomi, la matematica e i suoi fondamenti. Hachette Fascicoli srl, Milano 2019.

-Francisco Javier Mateos Maroto, In principio era il numero. L'unanità impara a contare, Hachette Fascicoli srl, Milano 2019.

-Filippo Giordano, Stanze quadratiche e divisori Mm, l'ignota disciplina dei numeri naturali, Youcanprint 2020.

-Clara Isabel Grima Ruiz e Alberto Màrquez Pérez, La privacy digitale & I teoremi di Fermat, RBA, Milano 2020.

-Filippo Giordano, Spirale di Ulam, la straordinaria mappa dei sotto ordini dei numeri naturali, Youcanprint 2020.

-Filippo Giordano, Rette numeriche quadratiche, le impalcature matematiche che guidano la fattorizzazione, Youcanprint 2021.

-Filippo Giordano, Primi di Mersenne e numeri perfetti. Esplorazione di una rarità numerica, Youcanprint, Lecce 2022.

-Filippo Giordano, Fattorizzazione dei numeri naturali, evoluzione del metodo di Fermat e nuove scoperte, Youcanprint 2022.

-Filippo Giordano, Terne Pitagoriche Primitive, Youcanprint, Lecce 2022.

-Filippo Giordano, Formazione e rarefazione dei numeri primi, Youcanprint, Lecce 2022.