# THE THEORY OF PLAFALES. P VS NP PROBLEM SOLUTION

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Abstract. This paper is dedicated to a rigorous review of the theory of plafales, which describes the properties and applications of a new mathematical object. As a consequence of the created theory we give a proof of the equality of complexity classes P and NP.
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## 1. introduction

The first edition of [1] is published in March 2011. The publication's goal is the creation of a new theory in mathematics, where the central object is plafal[1]. The second edition of [2] is published in February 2013. After the report at the 42nd Polish Conference on Mathematics Applications [3], it is necessary to create applications based on the theory of plafales (so-called constructive approach of post-factum power). Here we will provide an appropriate overview of the research and development process of the theory of plafales.

**Finite element method**. There are created mathematical models of serendipity finite elements: a new approach to construction basis and field functions. A quadruple role of the basis functions of serendipity finite elements is structurally shown [4], [5].

**IT** (finite element method as algorithmic support). Due to solving the non-standard Dirichlet boundary value problem [4], using the components of the theory of plafales (to obtain the surface of the temperature field in a three-dimensional space), there is developed a software for testing non-stationary temperature fields [6].

**Cryptography.** In September 2014 there is created a symmetric-key algorithm "ECLECTIC-DT-1" [7]. Algorithm's characteristics: block length is 128 bits, key length is 256 bits, 14 rounds. Algorithm's indicators (upper bounds of practical security): $EDP \leq 2^{-714}$ (against differential cryptanalysis [8]), $ELP \leq 2^{-714}$ (against linear cryptanalysis [9]). In December 2018 the symmetric-key algorithm "STEEL" is created [10] (which is the modification of the algorithm "ECLECTIC-DT-1"). Algorithm's characteristics: block length is 128 bits, key length is 256 bits, 14 rounds. Algorithm's indicators: $EDP \leq 2^{-595}$, $ELP \leq 2^{-595}$.

**Note 1.** *Editions [1], [2] are the basis of the theory of plafales. The theory of plafales has to be taken into consideration starting from this article.*

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[1]The singular form is plafal or plafale. The plural form is plafales.

## 2. CONCEPT OF THE PLAFAL

Now we introduce the following concept. For any simple graph $G(S, E)$[2] let there be a correspondence between each edge and an arbitrary object, and also between each vertex and an arbitrary object[3]. Object's essence is not taken into consideration: an arbitrary set, category, $\infty$-cosmos [12], etc. Designation of plafal in general form is $PF^i_j$, where $i$ is a number of graph's edges, $j$ is a number of graph's vertices[4] (examples of plafales are given in figures 1, 2).

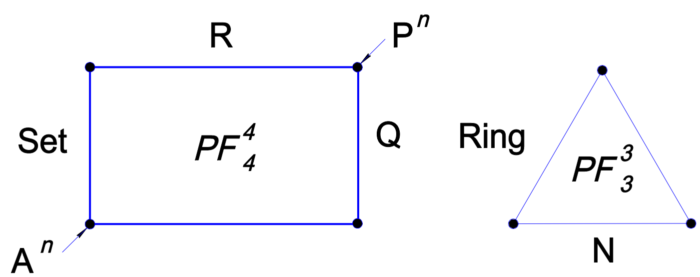


FIGURE 1. $PF^4_4$. Edges: Set (the category of sets), $\mathbb{R}$ is a set of real numbers, $\mathbb{Q}$ is a set of rational numbers, $a \leftrightarrow A$. Vertices: $\mathbb{A}^n$ is an affine space, $\mathbb{P}^n$ is a projective space, $p \leftrightarrow P$ (for two vertices). $PF^3_3$. Edges: $\mathbb{N}$ is a set of natural numbers, Ring (the category of rings).

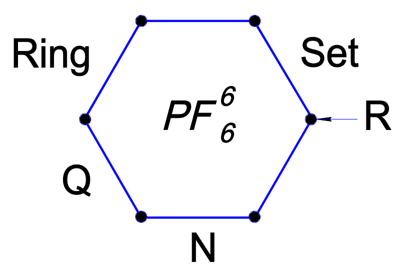


FIGURE 2. $PF^6_6$.

[2]In the general case, $a$ is an edge of $G(S, E)$, $p$ is a vertex of $G(S, E)$. $S$ is a set of vertices, $E$ is a set of edges. $G(S, E)$ does not contain an isolated vertex.

[3]Let us remark that the above correspondences are not the graph's representation (the example of the graph's representation in the finite-dimensional vector space is given in [11]). This representation [11] is a special case of plafal, because instead of $G_a \subseteq V_w + V_q$ ($a \in E, w, q \in S, w \neq q$), for instance, it can be Ring (the category of rings).

[4]**Remark 1.** In the general case, $A$ is an edge of plafal, $P$ is a vertex of plafal. Between the edge $A$ and two vertices $P_1, P_2$, which are connected by $A$, there is not necessarily a logical relation. **Remark 2.** $a \leftrightarrow A$ (when the edge corresponds to itself); $p \leftrightarrow P$ (when the vertex corresponds to itself). **Remark 3.** Generally, we claim that $G(S, E) \equiv G(PF^i_j)$. **Remark 4.** The graph's properties, as a support of the plafal, are preserved.

**Definition 2.1** (Labeled plafal)**.** All or some of edges (vertices) are enumerated[5]. Designation is $PF^{i_k}_{j_l}$, $k$ is a quantity of enumerated edges[6]: $\{\overline{i_1, i_k}\} \xrightarrow{\pi_1} \{\overline{1, i}\}$; $l$ is a quantity of enumerated vertices[7]: $\{\overline{j_1, j_l}\} \xrightarrow{\pi_2} \{\overline{1, j}\}$, (figure 3). Here $\pi_1, \pi_2$ are substitutions.

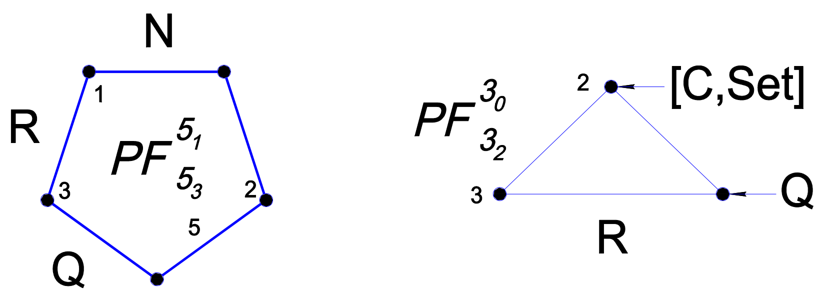


FIGURE 3. $PF^{5_1}_{5_3}$. One edge is enumerated, three vertices are enumerated. $PF^{3_0}_{3_2}$. Two vertices are enumerated.

Let's give a constructive example of using of the theory of plafales in cryptography [10]. For a byte $\{b_7b_6b_5b_4b_3b_2b_1b_0\}$ there is the correspondence that forms the plafal[8]: $b_{8-k} \leftrightarrow i_k, k \in M, M = \{\overline{1, 8}\}$.

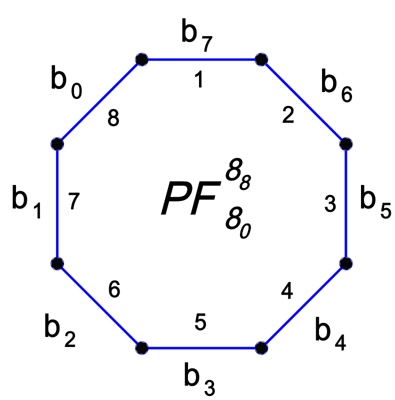


FIGURE 4. $PF^{8_8}_{8_0}$. Eight edges are enumerated.

[5]The camoufleur makes a graph labeling (see section 10).
[6]If $k < i$, then $\pi_1$ is an injection; if $k = i$, then $\pi_1$ is a bijection.
[7]If $l < j$, then $\pi_2$ is an injection; if $l = j$, then $\pi_2$ is a bijection.
[8]Edges $\{\overline{i_1, i_k}\}$ are sequential from north to northwest.

## 3. The category of plafales

**Definition 3.1.** Given plafales $PF_j^i$ and $PF_{j'}^{i'}$, a plafal morphism $g$:
$\langle G(PF_j^i), PF_j^i\rangle \xrightarrow{g=\langle f,\Psi\rangle} \langle G(PF_{j'}^{i'}), PF_{j'}^{i'}\rangle$ is called an ordered pair of maps for which the following conditions hold:
(1) $G(PF_j^i) \xrightarrow{f} G(PF_{j'}^{i'})$, $f$ is a graph morphism [13];
(2) $PF_j^i \xrightarrow{\Psi} PF_{j'}^{i'}$: $P \xrightarrow{\psi_{t'}} P'$ and $A \xrightarrow{\psi_{t''}} A'$; $P, A \in PF_j^i$, $P', A' \in PF_{j'}^{i'}$, $\Psi = \{\psi_t\}_{t\in\{\overline{1,(i+j)}\}}$ is a family of maps.

Example is given in figures 5, 6.

Plafales and plafal morphisms (as defined in def. 3.1) form the category **Plafales**, together with the componentwise compositions $\langle f, \Psi\rangle \circ \langle f', \Psi'\rangle = \langle f \circ f', \Psi \circ \Psi'\rangle$ and identities $id_{PF_j^i} = \langle id_{G(PF_j^i)}, id_P, id_A\rangle$.

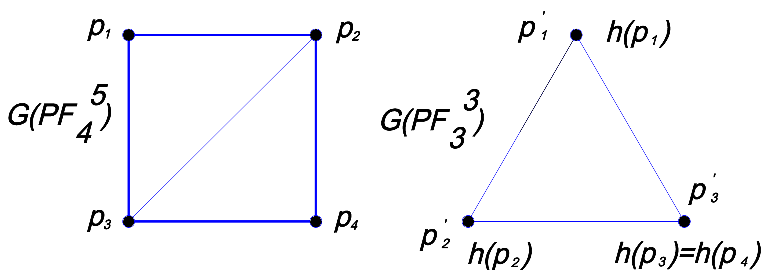


FIGURE 5. Condition 1. The image of $G(PF_4^5)$ under the strict homomorphism is $G(PF_3^3)$.

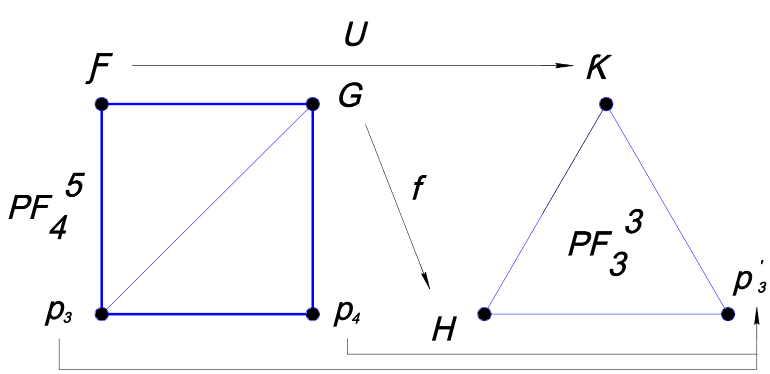


FIGURE 6. Condition 2. $U$ is an accessible cosmological functor between $\infty$-cosmoi which are accessible simplicially enriched categories; $f$ is a homomorphism of groups.

**Claim 3.2.** *In **Plafales**, there're no initial and terminal objects.*

*Proof.* By definition 3.1 and due to the concept of the plafal, we have the following: the initial object must be $\langle \widehat{0}_{G(PF_j^i)}, \widehat{0}_C \rangle$[9], the terminal object must be $\langle \widehat{1}_{G(PF_j^i)}, \widehat{1}_C \rangle$[10]. **a. Initial object.** Assume the converse. Consider the single plafal with the following configuration[11]: $A \leftrightarrow \mathrm{Ob}_{\mathrm{Set}}$ and $P \leftrightarrow \mathrm{Ob}_{\mathrm{Ring}}$. Thus, we get two initial objects $\langle \widehat{0}_{G(PF_j^i)}, \widehat{0}_C = \varnothing \rangle$ and $\langle \widehat{0}_{G(PF_j^i)}, \widehat{0}_C = \mathbb{Z} \rangle$[12]. Contradiction. **b. Terminal object.** In SiLlStG (The Category of Simple Loopless Graphs with Strict Morphisms) [14], there's no terminal object. This completes the proof of claim 3.2. □

Given two plafales $PF_1$ and $PF_2$, a product $PF^{prod} = PF_1 \times PF_2$ is defined by the graph product $G = G(PF_1) \times G(PF_2)$ [14] and existence of $P' \times P''$ (in accordance with $G$), $P' \in PF_1$, $P'' \in PF_2$; and for edges $A_j \in PF^{prod}$ we have the following: $A_j \leftrightarrow a$. Certainly, $G$ is a simple graph and does not contain an isolated vertex.

Given two plafales $PF_1$ and $PF_2$, a coproduct $PF^{cprod} = PF_1 \oplus PF_2$ is defined by the graph coproduct $G = G(PF_1) \oplus G(PF_2)$ (the disjoint union of graphs) [14] and existence of $\coprod$ for $P, A \in PF_1, P', A' \in PF_2$.

**Claim 3.3.** *In **Plafales**, the product of any two plafales does not always exist.*

*Proof.* Consider the discrete category on the two-object set $\{B, A\}$ (this is most easily seen by thinking of $\{B, A\}$ as a discrete poset). The product $B \times A$ does not exist (the proof is trivial). Consider two plafales $PF_1$ and $PF_2$ with the following configurations: $P^{'} \leftrightarrow B, P^{''} \leftrightarrow A$, $P' \in PF_1$, $P'' \in PF_2$. Then $B \times A$ does not exist at the $P^{'} \times P^{''}$. This completes the proof of claim 3.3. □

**Claim 3.4.** *In **Plafales**, the coproduct of any two plafales does not always exist.*

*Proof.* Consider the category of fields. $P \leftrightarrow \mathbb{Q}, P^{'} \leftrightarrow \mathbb{Z}/p\mathbb{Z}$, $P \in PF_1, P' \in PF_2$. Then $P \oplus P'$ does not exist. This completes the proof of claim 3.4. □

**Claim 3.5.** ***Plafales** fails to have the following properties:*
*(S1) NNO (a natural numbers object).*
*(S2) AC (the axiom of choice).*
*(S3) A subobject classifier is two-valued.*

*Proof.* SiLlStG (The Category of Simple Loopless Graphs with Strict Morphisms) fails to have the properties $(S1), (S2), (S3)$ [14]. This completes the proof of claim 3.5. □

**Corollary 3.6.** *Limits and colimits fail to exist, exponentiation with evaluation fails to exist, **Plafales** is not a topos. The proof is omitted.*

**Remark 5.** If the graph product $G$ is a lexicographical product or of other types, then $PF^{prod}$ will be defined in an analogous way.

[9] $\widehat{0}_{G(PF_j^i)}$ is the initial object in the Categories of Graphs [14].
[10] $\widehat{1}_{G(PF_j^i)}$ is the terminal object in the Categories of Graphs [14].
[11] See remark 2.
[12] $\widehat{0}_C = \mathbb{Z}$ is the ring of integers.

## 4. A CANVAS OF THE PLAFALES

**Definition 4.1.** A canvas of the plafales $PF_r^U$ is an algebraic surface of the first order (plane) which contains $r \in \mathbb{Z}_+ \cup \{0\}$ plafales[13]. If $r = 0$, then $PF_0^U$ is called an empty canvas of the plafales. ${}^pPF_j^i$ (${}^pPF_{j_l}^{i_k}$) is a plafal with an assigned number, $1 \leq p \leq r$.

**Definition 4.2.** $PF^p$ is an adherent point of $PF_r^U$. Types: limit point $PF^{pl}$; isolated point $PF^{pis}$. $S(PF^p)$ is the set of all adherent points. $S(PF^{pl}) \subset S(PF^p)$ is the set of all limit points. $S(PF^{pis}) \subset S(PF^p)$ is the set of all isolated points.

**Definition 4.3.** Interior region of the plafal $PF^I$ is the set of all points located inside of plafal. Exterior region of the plafal $PF^E$ is the set of all points located outside of plafal[14].

**Definition 4.4.** $PF^{Ip}$ is an adherent point of $PF^I$. Types: interior limit point $PF^{Ipl}$; interior isolated point $PF^{Ipis}$. $PF^{Ep}$ is an adherent point of $PF^E$. Types: exterior limit point $PF^{Epl}$; exterior isolated point $PF^{Epis}$.

### 4.1. **Special points.**

**Definition 4.5.** Special limit point (imaginary point) $PF^{spl}$ is a type of $PF^{pl}$[15] for which the following correspondence holds:

$$PF^{spl} \leftrightarrow \begin{bmatrix} u_1 \cdot c_1(t) & W_i^{spl} \cdot c_1(t) \end{bmatrix}, \tag{4.1}$$

$u_1$ is a unique parameter[16], $W_i^{spl} = (m_i(x,y,t) \pm 1)$ is a characteristic function[17], $\begin{bmatrix} u_1 \cdot c_1(t) & W_i^{spl} \cdot c_1(t) \end{bmatrix}$ is a state matrix, $c_1(t) = 1$ (see section 5).

**Definition 4.6.** Special isolated point $PF^{spis}$ is a type of $PF^{pis}$[18] for which the following correspondence holds:

$$PF^{spis} \leftrightarrow \begin{bmatrix} u_2 \cdot c_2(t) & W_i^{spis} \cdot c_2(t) \end{bmatrix}, \tag{4.2}$$

$u_2$ is a unique parameter[19], $W_i^{spis} = (n_i(x,y,t) \pm 1)$ is a characteristic function[20], $\begin{bmatrix} u_2 \cdot c_2(t) & W_i^{spis} \cdot c_2(t) \end{bmatrix}$ is a state matrix, $c_2(t) = 1$ (see section 5).

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[13] The other types of algebraic surfaces are not considered in this paper. $PF_r^U$ is $\mathbb{R}^2$ equipped with the metric topology. The induced topology on $\mathbb{R}^2$ defines on $R(K) = \bigcup_{I \in K} \triangle_I \subset \mathbb{R}^2$ the structure of compact space, $\triangle_I = \text{conv}(e_i \mid i \in I,\ I \subset \{1,2\})$ is a simplex spans the vectors $e_1, e_2$. This compact space $|K|$ is the geometric realization of a 1-dimensional simplicial complex $K = G(S,E)$ [15].

[14] $|K|$ is the Jordan curve, $PF^I = \text{Int}\ |K|$, $PF^E = \text{Ext}\ |K|$.

[15] $PF^{spl} \notin |K|$, $S(PF^{spl})$ is the set of all imaginary points.

[16] $u_1 \in \{a \in \mathbb{R},\ \text{color}, \ldots\}$, $u_1$ is a same for $S(PF^{spl})$.

[17] The coordinates of $PF^{spl}$ are $(x,y)$, $t$ is a time. $\forall PF^{spl}\ \exists!\ W_i^{spl}$; $W_1^{spl}, \ldots, W_k^{spl}$ is a collection of characteristic functions. In the general case, $W_i^{spl} = (m_i(x,y,t) \pm 1)$. If $PF^{spl} \in PF^I$, then $W_i^{spl} = (m_i(x,y,t)+1)$. If $PF^{spl} \in PF^E$, then $W_i^{spl} = (m_i(x,y,t)-1)$.

[18] $PF^{spis} \notin |K|$, $S(PF^{spis})$ is the set of all special isolated points.

[19] $u_1 \neq u_2$, $u_2$ is a same for $S(PF^{spis})$.

[20] $\forall PF^{spis}\ \exists!\ W_i^{spis}$; $W_1^{spis}, \ldots, W_l^{spis}$ is a collection of characteristic functions. In the general case, $W_i^{spis} = (n_i(x,y,t) \pm 1)$. If $PF^{spis} \in PF^I$, then $W_i^{spis} = (n_i(x,y,t)+1)$. If $PF^{spis} \in PF^E$, then $W_i^{spis} = (n_i(x,y,t)-1)$.

**Definition 4.7.** Flickering point $PF^{spf}$ is a point of $PF_r^U$ for which the following correspondence holds[21]:

$$PF^{spf} \leftrightarrow \begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix}, \tag{4.3}$$

$u_3 = (u_1 \cdot c_1(t) + u_2 \cdot c_2(t))$ [22], $W_i^{spf} = (h_i(x,y,t) \pm 1) = (W_i^{spl} \cdot c_1(t) + W_i^{spis} \cdot c_2(t))$ is a characteristic function[23], $\begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix}$ is a state matrix.

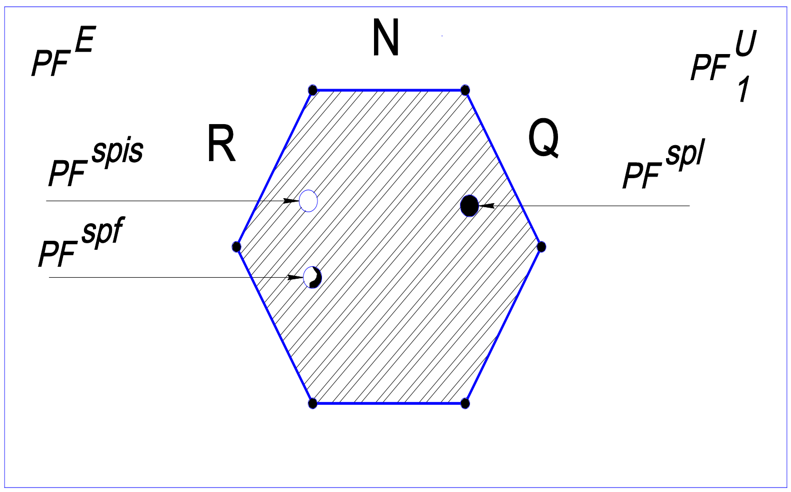


FIGURE 7. The canvas of the plafales $PF_1^U$: interior region of the plafal, exterior region of the plafal, special limit point, special isolated point, flickering point.

## 5. ABSOLUTE TRANSITIONS

**Definition 5.1.** For given $PF^{spl}$ and $PF^{spis}$ let an absolute transition $t^{aslis}$: $\langle (x,y), PF^{spl} \rangle \xrightarrow{t^{aslis} = \langle p, h^{aslis} \rangle} \langle (x,y), PF^{spis} \rangle$ be an ordered pair of maps for which the following conditions hold:

(1) $(x,y) \xrightarrow{p} (x,y)$ (i.e. $PF^{spl}$ and $PF^{spis}$ have the same coordinates);

(2) $PF^{spl} \xrightarrow{h^{aslis}} PF^{spis}$.

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[21] $PF^{spf} \notin |K|$, $S(PF^{spf})$ is the set of all flickering points. $PF^{spf}$ is an intermediate point in the transition between $PF^{spl}$ and $PF^{spis}$ (see section 5), i.e. $PF^{spf}$ is a superposition of the states $PF^{spl}$ and $PF^{spis}$, where $c_i(t)$ are the probabilities of the states $PF^{spl}$ and $PF^{spis}$ at the $PF^{spf}$.

[22] $c_2(t) = 1 - c_1(t),\ 0 < c_1(t) < 1$.

[23] $\forall PF^{spf}\ \exists!\ W_i^{spf}$; $W_1^{spf}, \ldots, W_s^{spf}$ is a collection of characteristic functions. In the general case, $W_i^{spf} = (h_i(x,y,t) \pm 1)$. If $PF^{spf} \in PF^I$, then $W_i^{spf} = (h_i(x,y,t)+1)$. If $PF^{spf} \in PF^E$, then $W_i^{spf} = (h_i(x,y,t) - 1)$.

5.1. **There's a transition from the special limit point to the special isolated point without returning back.**

$$PF^{spl} \xrightarrow{h^{aslis}=f^{asfis}\circ g^{aslf}} PF^{spis},\ T_1 \le t \le T_2{}^{24}:$$

$$PF^{spl} \xrightarrow{g^{aslf}} PF^{spf} \xrightarrow{f^{asfis}} PF^{spis}. \tag{5.1}$$

Using (4.1), (4.2), (4.3) in (5.1), we get (5.2), (5.3).

$$\begin{gathered}
PF^{spl} \xrightarrow{g^{aslf}} PF^{spf} : \begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix} = \begin{bmatrix} u_1 \cdot c_1(t) & W_i^{spl} \cdot c_1(t) \end{bmatrix} \cdot g^{aslf} = \\
= \begin{bmatrix} u_1 \cdot c_1(t) & W_i^{spl} \cdot c_1(t) \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{W_i^{spl}\cdot c_1(t)}{u_1\cdot c_1(t)} \\ \frac{u_2\cdot c_2(t)}{W_i^{spl}\cdot c_1(t)} & \frac{W_i^{spis}\cdot c_2(t)}{W_i^{spl}\cdot c_1(t)} \end{bmatrix}, \\
c_1(t) = \begin{cases} 1,\ t \le T_1, \\ \frac{T_2-t}{T_2-T_1},\ T_1 < t < T_2, \end{cases} \quad c_2(t) = \begin{cases} 0,\ t \le T_1, \\ \frac{t-T_1}{T_2-T_1},\ T_1 < t < T_2, \end{cases} \\
\lim_{t\to T_1} c_1(t) = 1,\ \lim_{t\to T_1} c_2(t) = 0.
\end{gathered} \tag{5.2}$$

$$\begin{gathered}
PF^{spf} \xrightarrow{f^{asfis}} PF^{spis} : \begin{bmatrix} u_2 \cdot c_2(t) & W_i^{spis} \cdot c_2(t) \end{bmatrix} = \begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix} \cdot f^{asfis} = \\
= \begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix} \cdot \begin{bmatrix} \frac{u_2\cdot c_2(t)}{u_3} & 0 \\ 0 & \frac{W_i^{spis}\cdot c_2(t)}{W_i^{spf}} \end{bmatrix}, \\
c_1(t) = \begin{cases} \frac{T_2-t}{T_2-T_1},\ T_1 < t < T_2, \\ 0,\ t \ge T_2, \end{cases} \quad c_2(t) = \begin{cases} \frac{t-T_1}{T_2-T_1},\ T_1 < t < T_2, \\ 1,\ t \ge T_2, \end{cases} \\
\lim_{t\to T_2} c_1(t) = 0,\ \lim_{t\to T_2} c_2(t) = 1.
\end{gathered} \tag{5.3}$$

**Definition 5.2.** For given $PF^{spis}$ and $PF^{spl}$ let an absolute transition $t^{asisl}$: $\langle (x,y), PF^{spis}\rangle \xrightarrow{t^{asisl}=\langle p, h^{asisl}\rangle} \langle (x,y), PF^{spl}\rangle$ be an ordered pair of maps for which the following conditions hold:
(1) $(x,y) \xrightarrow{p} (x,y)$ (i.e. $PF^{spis}$ and $PF^{spl}$ have the same coordinates);
(2) $PF^{spis} \xrightarrow{h^{asisl}} PF^{spl}$.

5.2. **There's a transition from the special isolated point to the special limit point without returning back.**

$$PF^{spis} \xrightarrow{h^{asisl}=f^{asfl}\circ g^{asisf}} PF^{spl},\ T_1 \le t \le T_2{}^{25}:$$

$$PF^{spis} \xrightarrow{g^{asisf}} PF^{spf} \xrightarrow{f^{asfl}} PF^{spl}. \tag{5.4}$$

[24] $PF^{spl} \to PF^{spf}$ at time $T_1$, $PF^{spf} \to PF^{spis}$ at time $T_2$.
[25] $PF^{spis} \to PF^{spf}$ at time $T_1$, $PF^{spf} \to PF^{spl}$ at time $T_2$.

Using (4.1), (4.2), (4.3) in (5.4), we get (5.5), (5.6).

$$PF^{spis} \xrightarrow{g^{asisf}} PF^{spf} : \begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix} = \begin{bmatrix} u_2 \cdot c_2(t) & W_i^{spis} \cdot c_2(t) \end{bmatrix} \cdot g^{asisf} =$$
$$= \begin{bmatrix} u_2 \cdot c_2(t) & W_i^{spis} \cdot c_2(t) \end{bmatrix} \cdot \begin{bmatrix} \frac{\frac{u_1 \cdot c_1(t)}{u_2 \cdot c_2(t)}}{\frac{u_2 \cdot c_2(t)}{W_i^{spis} \cdot c_2(t)}} & \frac{W_i^{spl} \cdot c_1(t)}{u_2 \cdot c_2(t)} \\ & 1 \end{bmatrix},$$
$$c_1(t) = \begin{cases} 0, \ t \leq T_1, \\ \frac{t-T_1}{T_2-T_1}, \ T_1 < t < T_2, \end{cases} \quad c_2(t) = \begin{cases} 1, \ t \leq T_1, \\ \frac{T_2-t}{T_2-T_1}, \ T_1 < t < T_2, \end{cases}$$
$$\lim_{t \to T_1} c_1(t) = 0, \ \lim_{t \to T_1} c_2(t) = 1. \tag{5.5}$$

$$PF^{spf} \xrightarrow{f^{asfl}} PF^{spl} : \begin{bmatrix} u_1 \cdot c_1(t) & W_i^{spl} \cdot c_1(t) \end{bmatrix} = \begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix} \cdot f^{asfl} =$$
$$= \begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix} \cdot \begin{bmatrix} \frac{u_1 \cdot c_1(t)}{u_3} & 0 \\ 0 & \frac{W_i^{spl} \cdot c_1(t)}{W_i^{spf}} \end{bmatrix},$$
$$c_1(t) = \begin{cases} \frac{t-T_1}{T_2-T_1}, \ T_1 < t < T_2, \\ 1, \ t \geq T_2, \end{cases} \quad c_2(t) = \begin{cases} \frac{T_2-t}{T_2-T_1}, \ T_1 < t < T_2, \\ 0, \ t \geq T_2, \end{cases}$$
$$\lim_{t \to T_2} c_1(t) = 1, \ \lim_{t \to T_2} c_2(t) = 0. \tag{5.6}$$

5.3. **There're the transitions from the special limit point to the special isolated point and instantaneous returning back to the special limit point.** Combining (5.1) – (5.6), we obtain (5.7).

$$\begin{cases} PF^{spl} \xrightarrow{h^{aslis} = f^{asfis} \circ g^{aslf}} PF^{spis}, \ T_1 \leq t \leq T_2, \\ PF^{spis} \xrightarrow{h^{asisl} = f^{asfl} \circ g^{asisf}} PF^{spl}, \ T_2 \leq t \leq T_3. \end{cases} \tag{5.7}$$

5.4. **There're the transitions from the special isolated point to the special limit point and instantaneous returning back to the special isolated point.** Combining (5.1) – (5.6), we obtain (5.8).

$$\begin{cases} PF^{spis} \xrightarrow{h^{asisl} = f^{asfl} \circ g^{asisf}} PF^{spl}, \ T_1 \leq t \leq T_2, \\ PF^{spl} \xrightarrow{h^{aslis} = f^{asfis} \circ g^{aslf}} PF^{spis}, \ T_2 \leq t \leq T_3. \end{cases} \tag{5.8}$$

5.5. **There're $n \leq \infty$ transitions from the special limit (isolated) point to the special isolated (limit) point.** Using (5.7), (5.8), we obtain (5.9).

$$\begin{cases} PF^{spl}(PF^{spis}) \to PF^{spis}(PF^{spl}), \ T_i \leq t \leq T_{i+1}, \\ PF^{spis}(PF^{spl}) \to PF^{spl}(PF^{spis}), \ T_{i+2} \leq t \leq T_{i+3}, \\ |T_{j+1} - T_j| = \gamma, \ \gamma \in \mathbb{Z}_+ \cup \{0\}. \end{cases} \tag{5.9}$$

**Claim 5.3.** *$PF^{spl}$, $PF^{spis}$ and morphisms $t^{aslis}, t^{asisl}$ (in accordance with (5.9)) form the category* ***Plafales-AT****.*

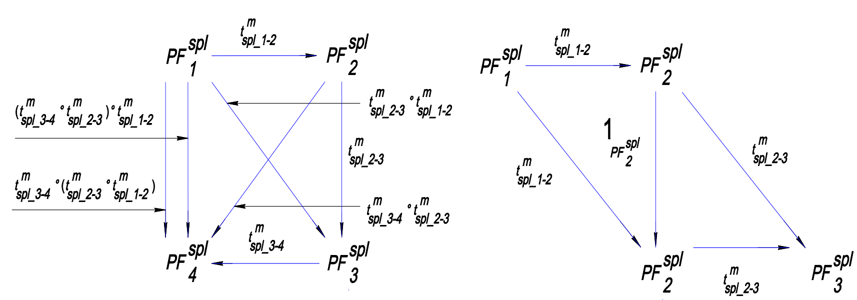
*Proof.* $id = id_{PF^{spl}} = id_{PF^{spis}} = \langle (x,y), \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rangle$, $t^{aslis} \circ id = t^{aslis}$, $id \circ t^{aslis} =$ $= t^{aslis}$, $t^{asisl} \circ id = t^{asisl}$, $id \circ t^{asisl} = t^{asisl}$. ∎

## 6. MOVING TRANSITIONS

**Definition 6.1.** For given $PF_1^{spl}, PF_2^{spl} \in S(PF^{spl})$ let a moving transition $t_{spl}^m$: $\langle (x,y), PF_1^{spl}\rangle \xrightarrow{t_{spl}^m = \langle p, h_{spl}^m\rangle} \langle (x',y'), PF_2^{spl}\rangle$ be an ordered pair of maps for which the following conditions hold:
(1) $(x,y) \xrightarrow{p} (x',y')$ (i.e. $PF_1^{spl}$ and $PF_2^{spl}$ have different coordinates[26]);
(2) $PF_1^{spl} \xrightarrow{h_{spl}^m} PF_2^{spl}$: $\begin{bmatrix} u_1 \cdot c_1(t) & W_2^{spl} \cdot c_1(t)\end{bmatrix} = \begin{bmatrix} u_1 \cdot c_1(t) & W_1^{spl} \cdot c_1(t)\end{bmatrix} \cdot h_{spl}^m =$
$= \begin{bmatrix} u_1 \cdot c_1(t) & W_1^{spl} \cdot c_1(t)\end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & \frac{W_2^{spl} \cdot c_1(t)}{W_1^{spl} \cdot c_1(t)}\end{bmatrix}$.

**Claim 6.2.** *$S(PF^{spl})$ and morphisms $t_{spl}^m$ form the category **Plafales-MT-spl**.*

*Proof.* $id = \langle (x,y), \begin{bmatrix} 1 & 0 \\ 0 & 1\end{bmatrix}\rangle$. We get the following commutative diagrams:




∎

**Definition 6.3.** For given $PF_1^{spis}, PF_2^{spis} \in S(PF^{spis})$ let a moving transition $t_{spis}^m$:
$\langle (x,y), PF_1^{spis}\rangle \xrightarrow{t_{spis}^m = \langle p, h_{spis}^m\rangle} \langle (x',y'), PF_2^{spis}\rangle$ be an ordered pair of maps for which the following conditions hold:
(1) $(x,y) \xrightarrow{p} (x',y')$ (i.e. $PF_1^{spis}$ and $PF_2^{spis}$ have different coordinates);
(2) $PF_1^{spis} \xrightarrow{h_{spis}^m} PF_2^{spis}$: $\begin{bmatrix} u_2 \cdot c_2(t) & W_2^{spis} \cdot c_2(t)\end{bmatrix} = \begin{bmatrix} u_2 \cdot c_2(t) & W_1^{spis} \cdot c_2(t)\end{bmatrix} \times$
$\times\, h_{spis}^m = \begin{bmatrix} u_2 \cdot c_2(t) & W_1^{spis} \cdot c_2(t)\end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & \frac{W_2^{spis} \cdot c_2(t)}{W_1^{spis} \cdot c_2(t)}\end{bmatrix}$.

**Claim 6.4.** *$S(PF^{spis})$ and morphisms $t_{spis}^m$ form the category **Plafales-MT-spis**. The proof is omitted.*

**Definition 6.5.** For given $PF_1^{spf}, PF_2^{spf} \in S(PF^{spf})$ let a moving transition $t_{spf}^m$: $\langle (x,y), PF_1^{spf}\rangle \xrightarrow{t_{spf}^m = \langle p, h_{spf}^m\rangle} \langle (x',y'), PF_2^{spf}\rangle$ be an ordered pair of maps for which the following conditions hold:

[26] In particular, $x = x'$ or $y = y'$.

(1) $(x,y) \xrightarrow{p} (x',y')$ (i.e. $PF_1^{spf}$ and $PF_2^{spf}$ have different coordinates);

(2) $PF_1^{spf} \xrightarrow{h^m_{spf}} PF_2^{spf}$: $\begin{bmatrix} u_3 & W_2^{spf} \end{bmatrix} = \begin{bmatrix} u_3 & W_1^{spf} \end{bmatrix} \cdot h^m_{spf} = \begin{bmatrix} u_3 & W_1^{spf} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & \frac{W_2^{spf}}{W_1^{spf}} \end{bmatrix}$.

**Claim 6.6.** *$S(PF^{spf})$ and morphisms $t^m_{spf}$ form the category* ***Plafales-MT-spf****. The proof is omitted.*

*By definitions 6.1, 6.3, we have the following:*

**Definition 6.7.** For given $p_1, p_2 \in S(PF^{splis})$[27] let a moving transition $t^m_{splis}$: $\langle (x,y), p_1 \rangle \xrightarrow{t^m_{splis} = \langle p, h^m_{splis} \rangle} \langle (x',y'), p_2 \rangle$ be an ordered pair of maps for which the following conditions hold:

(1) $(x,y) \xrightarrow{p} (x',y')$ (i.e. $p_1$ and $p_2$ have different coordinates);

(2) $p_1 \xrightarrow{h^m_{splis}} p_2$[28].

**Claim 6.8.** *$S(PF^{splis})$ and morphisms $t^m_{splis}$ form the category* ***Plafales-MT-splis****. The proof is omitted.*

*By definitions 6.1, 6.5, we have the following:*

**Definition 6.9.** For given $p_1, p_2 \in S(PF^{splf})$[29] let a moving transition $t^m_{splf}$: $\langle (x,y), p_1 \rangle \xrightarrow{t^m_{splf} = \langle p, h^m_{splf} \rangle} \langle (x',y'), p_2 \rangle$ be an ordered pair of maps for which the following conditions hold:

(1) $(x,y) \xrightarrow{p} (x',y')$ (i.e. $p_1$ and $p_2$ have different coordinates);

(2) $p_1 \xrightarrow{h^m_{splf}} p_2$[30].

**Claim 6.10.** *$S(PF^{splf})$ and morphisms $t^m_{splf}$ form the category* ***Plafales-MT-splf****. The proof is omitted.*

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[27] $S(PF^{splis}) = S(PF^{spl}) \cup S(PF^{spis})$.

[28] $PF^{spl} \xrightarrow{h^m_{splis}} PF^{spis}$ : $\begin{bmatrix} u_2 \cdot c_2(t) & W_i^{spis} \cdot c_2(t) \end{bmatrix} = \begin{bmatrix} u_1 \cdot c_1(t) & W_i^{spl} \cdot c_1(t) \end{bmatrix} \cdot h^m_{splis} =$
$= \begin{bmatrix} u_1 \cdot c_1(t) & W_i^{spl} \cdot c_1(t) \end{bmatrix} \cdot \begin{bmatrix} \frac{u_2 \cdot c_2(t)}{u_1 \cdot c_1(t)} & 0 \\ 0 & \frac{W_i^{spis} \cdot c_2(t)}{W_i^{spl} \cdot c_1(t)} \end{bmatrix}$;
$PF^{spis} \xrightarrow{h^m_{splis}} PF^{spl}$ : $\begin{bmatrix} u_1 \cdot c_1(t) & W_i^{spl} \cdot c_1(t) \end{bmatrix} = \begin{bmatrix} u_2 \cdot c_2(t) & W_i^{spis} \cdot c_2(t) \end{bmatrix} \cdot h^m_{splis} =$
$= \begin{bmatrix} u_2 \cdot c_2(t) & W_i^{spis} \cdot c_2(t) \end{bmatrix} \cdot \begin{bmatrix} \frac{u_1 \cdot c_1(t)}{u_2 \cdot c_2(t)} & 0 \\ 0 & \frac{W_i^{spl} \cdot c_1(t)}{W_i^{spis} \cdot c_2(t)} \end{bmatrix}$; if $p_1 = p_2 = PF^{spl}$, we have $h^m_{splis} =$
$= h^m_{spl}$; if $p_1 = p_2 = PF^{spis}$, we have $h^m_{splis} = h^m_{spis}$.

[29] $S(PF^{splf}) = S(PF^{spl}) \cup S(PF^{spf})$.

[30] $PF^{spl} \xrightarrow{h^m_{splf}} PF^{spf}$ : $\begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix} = \begin{bmatrix} u_1 \cdot c_1(t) & W_i^{spl} \cdot c_1(t) \end{bmatrix} \cdot h^m_{splf} =$
$= \begin{bmatrix} u_1 \cdot c_1(t) & W_i^{spl} \cdot c_1(t) \end{bmatrix} \cdot \begin{bmatrix} \frac{u_3}{u_1 \cdot c_1(t)} & 0 \\ 0 & \frac{W_i^{spf}}{W_i^{spl} \cdot c_1(t)} \end{bmatrix}$;
$PF^{spf} \xrightarrow{h^m_{splf}} PF^{spl}$ : $\begin{bmatrix} u_1 \cdot c_1(t) & W_i^{spl} \cdot c_1(t) \end{bmatrix} = \begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix} \cdot h^m_{splf} =$
$= \begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix} \cdot \begin{bmatrix} \frac{u_1 \cdot c_1(t)}{u_3} & 0 \\ 0 & \frac{W_i^{spl} \cdot c_1(t)}{W_i^{spf}} \end{bmatrix}$; if $p_1 = p_2 = PF^{spl}$, we have $h^m_{splf} = h^m_{spl}$; if $p_1 =$
$= p_2 = PF^{spf}$, we have $h^m_{splf} = h^m_{spf}$.

*By definitions 6.3, 6.5, we have the following:*

**Definition 6.11.** For given $p_1, p_2 \in S(PF^{spisf})$[31] let a moving transition $t^m_{spisf}$: $\langle(x,y),p_1\rangle \xrightarrow{t^m_{spisf}=\langle p,h^m_{spisf}\rangle} \langle(x',y'),p_2\rangle$ be an ordered pair of maps for which the following conditions hold:
(1) $(x,y) \xrightarrow{p} (x',y')$ (i.e. $p_1$ and $p_2$ have different coordinates);
(2) $p_1 \xrightarrow{h^m_{spisf}} p_2$[32].

**Claim 6.12.** *$S(PF^{spisf})$ and morphisms $t^m_{spisf}$ form the category **Plafales-MT-spisf**. The proof is omitted.*

*By definitions 6.1, 6.3, 6.5, 6.7, 6.9, 6.11, we have the following:*

**Definition 6.13.** For given $p_1, p_2 \in S(PF^{splisf})$[33] let a moving transition $t^m_{splisf}$: $\langle(x,y),p_1\rangle \xrightarrow{t^m_{splisf}=\langle p,h^m_{splisf}\rangle} \langle(x',y'),p_2\rangle$ be an ordered pair of maps for which the following conditions hold:
(1) $(x,y) \xrightarrow{p} (x',y')$ (i.e. $p_1$ and $p_2$ have different coordinates);
(2) $p_1 \xrightarrow{h^m_{splisf}} p_2$.

**Claim 6.14.** *$S(PF^{splisf})$ and morphisms $t^m_{splisf}$ form the category **Plafales-MT-splisf**. The proof is omitted.*

**Corollary 6.15.** ***Plafales-MT-spl**, **Plafales-MT-spis**, **Plafales-MT-spf**, **Plafales-MT-splis**, **Plafales-MT-splf**, **Plafales-MT-spisf** are the subcategories of **Plafales-MT-splisf**. The proof is streightforward.*

*By definitions 5.1, 5.2, 6.7, we have the following:*

**Claim 6.16.** *$S(PF^{splis})$ and morphisms $t^{aslis}, t^{asisl}, t^m_{splis}$ form the category **Plafales-splis**. The proof is left to the reader.*

**Corollary 6.17.** ***Plafales-MT-splis** and **Plafales-AT** are the subcategories of **Plafales-splis**. The proof is streightforward.*

**Claim 6.18.** *A singleton $\{x\}$ with $id_{\{x\}}$ is a skeleton of **Plafales-MT-splisf** and **Plafales-splis**. The proof is trivial.*

## 6.1. **An ensemble of the special points (SP).**

**Definition 6.19.** An ensemble of the special points $PF^{ens}_{sp} \subset S(PF^{splisf})$ is a plane curve with the following configuration: each point is a special point.

---

[31] $S(PF^{spisf}) = S(PF^{spis}) \cup S(PF^{spf})$.

[32] $PF^{spis} \xrightarrow{h^m_{spisf}} PF^{spf}$ : $\begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix} = \begin{bmatrix} u_2 \cdot c_2(t) & W_i^{spis} \cdot c_2(t) \end{bmatrix} \cdot h^m_{spisf} =$
$= \begin{bmatrix} u_2 \cdot c_2(t) & W_i^{spis} \cdot c_2(t) \end{bmatrix} \cdot \begin{bmatrix} \frac{u_3}{u_2 \cdot c_2(t)} & 0 \\ 0 & \frac{W_i^{spf}}{W_i^{spis} \cdot c_2(t)} \end{bmatrix}$;
$PF^{spf} \xrightarrow{h^m_{spisf}} PF^{spis}$ : $\begin{bmatrix} u_2 \cdot c_2(t) & W_i^{spis} \cdot c_2(t) \end{bmatrix} = \begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix} \cdot h^m_{spisf} =$
$= \begin{bmatrix} u_3 & W_i^{spf} \end{bmatrix} \cdot \begin{bmatrix} \frac{u_2 \cdot c_2(t)}{u_3} & 0 \\ 0 & \frac{W_i^{spis} \cdot c_2(t)}{W_i^{spf}} \end{bmatrix}$; if $p_1 = p_2 = PF^{spis}$, we have $h^m_{spisf} = h^m_{spis}$; if $p_1 =$ $= p_2 = PF^{spf}$, we have $h^m_{spisf} = h^m_{spf}$.

[33] $S(PF^{splisf}) = S(PF^{spl}) \cup S(PF^{spis}) \cup S(PF^{spf})$.

6.2. **Dynamical system of the SP.**

**Definition 6.20.** A dynamical system of the SP is a tuple $S^{\star} = \langle T, S(PF^{splisf}), \Phi\rangle$, $(T \times S(PF^{splisf})) \supseteq S^{*} \xrightarrow{\Phi=\langle\Psi,\Upsilon\rangle} S(PF^{splisf})$, $\Psi$ is a family of maps of the points $(x, y)$ on the plane $PF^{U}_{r}$, $\Upsilon = \{\Upsilon', h^{m}_{splisf}\}$ is a family of maps[34], $S(PF^{splisf})$ is a phase space, $t \in T$.

As an example, let $S(PF^{splisf}) = PF^{ens}_{sp}$ be an unit circle, the position of the point on the unit circle is determined by the angle $\varphi$, dynamical system with discrete time is determined by $\Psi = \Psi(\varphi) = 2 \cdot \varphi \pmod{2\pi}$. Therefore, we have

$$S^{*} \xrightarrow{\Phi=\langle\Psi(\varphi),\Upsilon'\rangle} S(PF^{splisf}).$$

**Claim 6.21.** *$S^{\star}$ with $\Upsilon = \{h^{m}_{splisf}\}$ can be determined by $\Upsilon'$.*

*Proof.* It is sufficient to consider the single $h^{m}_{splisf}$ between two points $p_1, p_2$ (as defined in def. 6.13). Therefore, we have

$$\langle (x, y), p_1(t_1)\rangle \xrightarrow{\Phi=\langle p, h^{m}_{splisf}\rangle} \langle (x', y'), p_2(t_2)\rangle.$$

Thus we have $S^{\star} = \langle T = \{t_1, t_2\}, \{p_1, p_2\}, \Phi\rangle$. It is easily shown that $S^{\star}$ can be determined by

$$\begin{cases} \langle (x, y), p_1\rangle \xrightarrow{\langle p, id\rangle} \langle (x, y), p_1\rangle, \ t \le t_1, \\ \langle (x', y'), p_2\rangle \xrightarrow{\langle p', id\rangle} \langle (x', y'), p_2\rangle, \ t_2 \le t. \end{cases}$$

□

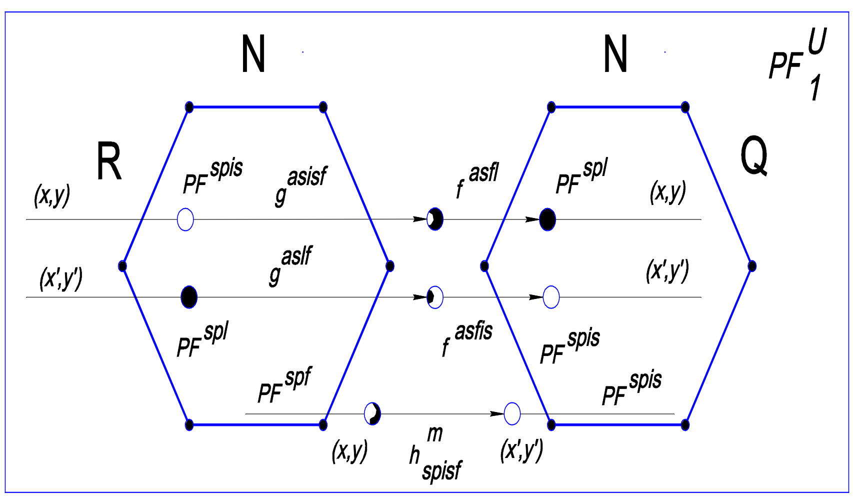


FIGURE 8. Absolute transitions. Moving transitions.

[34] $\Upsilon' = \{id = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, h^{aslis}, h^{asisl}, f^{asfis}, f^{asfl}, g^{asisf}, g^{aslf}\}$. See sections 5, 6.

## 7. PLAFALES OPERATIONS

In this section we will consider the basic plafales operations.

7.1. **Elementary operations.** 1. Vertex deletion ${}^{p}PF_{j_l}^{i_k} - P$; 2. Edge deletion ${}^{p}PF_{j_l}^{i_k} - A$; 3. Edge addition ${}^{p}PF_{j_l}^{i_k} + A$; 4. Subplafal contraction ${}^{p}PF_{j_l}^{i_k} \backslash {}^{p}PF_{j_l}^{\prime i_k}$; 5. Vertex breeding ${}^{p}PF_{j_l}^{i_k} \uparrow P$; 6. Complement plafal $\overline{{}^{p}PF_{j_l}^{i_k}}$. The operations are illustrated in figure 9.

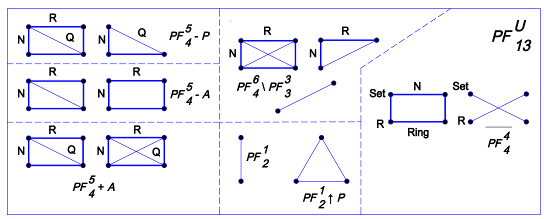


FIGURE 9. Elementary operations.

7.2. **Advanced operations.**

**Definition 7.1. Union of plafales.** Given plafales $PF_1, \dots, PF_n$, a union of plafales $PF_{S^n}^{un} = \bigcup_{i=1}^{n} PF_i$ is defined by the union of graphs[35] $G = \bigcup_{i=1}^{n} G(PF_i)$, if there exist: $P' \cup P''$ (at the common vertex) and $A' \cup A''$ (at the common edge), where $P', A' \in PF_{j'}$; $P'', A'' \in PF_{j''}$.

**Definition 7.2. Docking of plafales.** Given plafales $PF_1, \dots, PF_n$, a docking of plafales $PF_{S^n}^{doc} = \bigcup_{i=1}^{n} PF_i$ is defined by the union of graphs $G = \bigcup_{i=1}^{n} G(PF_i)$, for all common edges $a_1, \dots, a_k, \dots, a_m$ and vertices $p_1, \dots, p_l, \dots, p_q$, which are obtained by $G$, we have the following: $A' \leftrightarrow A_k$, $A'' \leftrightarrow A_k$, $P' \leftrightarrow P_l$, $P'' \leftrightarrow P_l$, where $P', A' \in PF_{j'}$; $P'', A'' \in PF_{j''}$.

**Definition 7.3. Docking• of plafales.** Given plafales $PF_1, \dots, PF_n$, a docking• of plafales $PF_{S^n}^{doc\bullet} = \bigcup_{i=1}^{n} PF_i$ is defined by the union of graphs $G = \bigcup_{i=1}^{n} G(PF_i)$, for all common edges $a_1, \dots, a_k, \dots, a_m$ and vertices $p_1, \dots, p_l, \dots, p_q$, which are obtained by $G$, we have the following: $A' \leftrightarrow A_k$, $A'' \leftrightarrow A_k$, $P' \leftrightarrow P_l$, $P'' \leftrightarrow P_l$, if there exist: $A_s = A''' \cup A''''$ and $P_t = P''' \cup P''''$; $P', P''', A', A''' \in PF_{j'}$; $P'', P''''$, $A'', A'''' \in PF_{j''}$.

**Definition 7.4. Intersection of plafales.** Given plafales $PF_1, \dots, PF_n$, an intersection of plafales $PF_{S^n}^{in} = \bigcap_{i=1}^{n} PF_i$ is defined by the intersection of graphs $G = \bigcap_{i=1}^{n} G(PF_i)$, if there exist: $P' \cap P''$ (at the common vertex) and $A' \cap A''$ (at the common edge), where $P', A' \in PF_{j'}$; $P'', A'' \in PF_{j''}$.

**Definition 7.5. Merger of plafales.** Given two plafales $PF_1, PF_2$, a merger of plafales $PF_{S^2}^{m} = \bigcap_{i=1}^{2} PF_i$ is defined by the intersection of graphs $G = \bigcap_{i=1}^{2} G(PF_i)$, for all edges $a_1, \dots, a_k, \dots, a_m$ and vertices $p_1, \dots, p_l, \dots, p_q$, which are obtained by $G$, we have the following: $A' \leftrightarrow A_k$, $A'' \leftrightarrow A_k$, $P' \leftrightarrow P_l$, $P'' \leftrightarrow P_l$, where $P', A' \in PF_1$; $P'', A'' \in PF_2$.

[35] No three of which do not have a common edge.

**Definition 7.6. Merger• of plafales.** Given two plafales $PF_1, PF_2$, a merger• of plafales $PF^{m\bullet}_{S^2} = \bigcap_{i=1}^{2} PF_i$ is defined by the intersection of graphs $G = \bigcap_{i=1}^{2} G(PF_i)$, for all edges $a_1, \ldots, a_k, \ldots, a_m$ and vertices $p_1, \ldots, p_l, \ldots, p_q$, which are obtained by $G$, we have the following: $A' \leftrightarrow A_k$, $A'' \leftrightarrow A_k$, $P' \leftrightarrow P_l$, $P'' \leftrightarrow P_l$, if there exist: $A_s = A''' \cap A''''$ and $P_t = P''' \cap P''''$; $P', P''', A', A''' \in PF_1$; $P'', P'''',$ $A'', A'''' \in PF_2$.

**Definition 7.7. Product• of plafales.** Given two plafales $PF_1$ and $PF_2$, a product of plafales $PF^{prod\bullet} = PF_1 \times PF_2$ is defined by the graph product [14] and existence of $P' \times P''$ (in accordance with the graph product), $P' \in PF_1$, $P'' \in PF_2$; and for edges $a_j \in (G(PF_1) \times G(PF_2))$ we have the following: the camoufleur[36] makes the correspondences.

**Definition 7.8. Decomposition of plafales.** Given plafales $PF_1, \ldots, PF_n$, a decomposition of plafales $PF^{d\bullet}_{S^n}$ is defined by the decomposition of graphs [16], [17], [18] and for all edges and vertices, which are obtained by the decomposition of graphs, we have the following: the camoufleur makes the correspondences.

**Remark 6.** $G$ (as defined in def. 7.1 – 7.7) is a simple graph and does not contain an isolated vertex. Decomposition of graphs are the simple graphs and do not contain the isolated vertices.

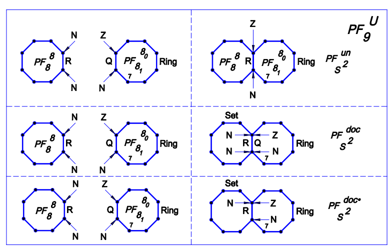


FIGURE 10. Union of plafales, docking of plafales, docking• of plafales.

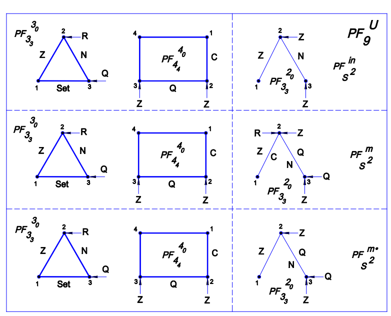


FIGURE 11. Intersection of plafales, merger of plafales, merger• of plafales.

[36]See section 10.

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