©2022 Leszek Mazurek
Independent Researcher
e-mail: mr.leszek.mazurek@gmail.com
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GENERAL RELATIVITY THEORY OF NUMBERS

LESZEK MAZUREK

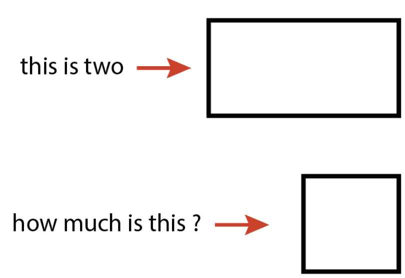
Abstract. In this paper we show that thorough understanding of numbers is possible only if we present them as value in relation to the certain reference measure. Commonly, we use number 1 as a reference measure, however, it does not have to always be 1, it can be any other number. To fully understand the meaning of numbers, we have to maintain their natural form which is a quotient of a value to a reference measure. Only by keeping this form we can do mathematics properly and appreciate its natural beauty.

This paper is still 'work in progress' version and will be updated and extended.

1. Test

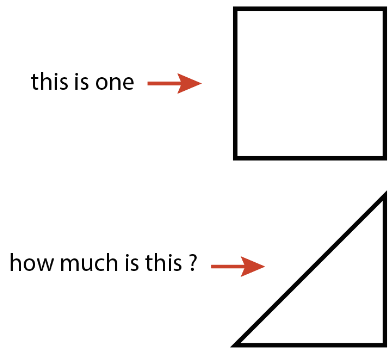
We start with a small test that will be explained later. Please answer questions in Figure 1 and Figure 2 and **write down** your responses.

Figure 1. Test 1



2020 *Mathematics Subject Classification.* Primary 11A99.

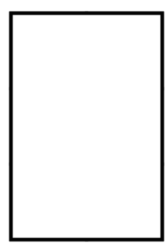
Figure 2. Test 2



2. Introduction

What is a number? After Wikipedia: "A number is a mathematical object used to count, measure, and label (...)". "To count" is obvious, as we are counting objects around us in our lives all the time. We know, very well, what "one car", "two apples" means. "To measure" - we want to measure this shape below and know how big is this object?

Figure 3. How big is this object?

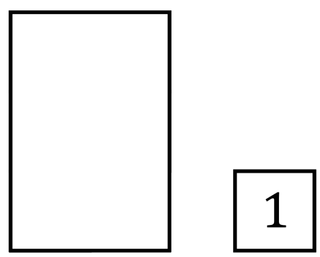


"To label" - we want to label this point on the axis in Figure 4. Which number does it represent? What is this number?

Figure 4. What is this number?

It looks like we are not able to do this. The reason for our confusion in both cases is very simple; we do not know what is a unit, what is one. If we do not have any "reference measure" that we can use to refer with our number to, we are not able to measure, to name a number, to label it. This is not the case when we count. When counting everyday things, we naturally know what "one thing" is, our unit is defined by an element - the things which we want to count a number of. We need to know what is one to name any number in reference to our unit - to compare it to the one. It is easy to say what the measure of an object is, if we know what is the one (unit). In Figure 5 we have no problem to say that the measure of the object, the number that this object represents is 6. It means that this object is 6 times as big as our unit and we know this by comparing (referencing with) our object to the unit.

FIGURE 5. Measure of an object in reference to 1.

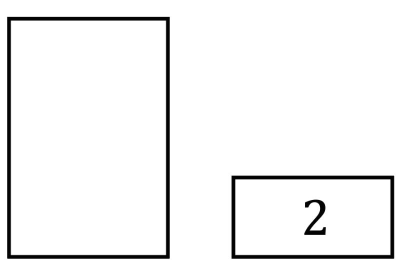


We have the same situation on the axis - once we define the unit, we can easily see that the point represents number 3.

FIGURE 6. The point on the axis with defined unit.

We can see that to understand a number, we need to know a unit, a reference measure, to which we can refer with our number to. By comparing to it we can understand a value of a number. But, do we always need to know 'one'? From example presented in Figure 7, we can easily understand which number represents the object even though the reference value in this case is 2. By comparing the bigger rectangle to the reference one, we can say it is (represents) 6.

FIGURE 7. Measure of an object in reference to 2.



From what was presented in all above examples we can draw the following conclusion.

A number is a mathematical object used to count, measure, and label which represents a relation between measured value and a certain reference measure that this value can be compared to.

$$number: \frac{value}{reference\ measure}$$

3. The meaning of numbers

Commonly, when we use numbers, we assume that the reference measure we refer to is 1. For example, by saying 5, what we actually mean is a ratio that this number has to 1 or that this number represents something 5 times as big as 1 (comparing to 1).

$$5 = \frac{5}{1} \text{ means a ratio of } 5 \text{ to } 1 \tag{3.1}$$

Usually we do not reflect this in our notation, yet everyone understands that we refer with our numbers to one. We instinctively know the meaning of numbers because we can compare them to the one and we all know what it is.

Is $\frac{1}{2}$ the same number as $\frac{2}{4}$?

We have no doubts to say that 5 is the same as 5, or x is the same as x. Saying that the thing is the same thing is one of fundamentals of logic. In mathematics we usually use equality symbol "=" to reflect this, therefore we have

$$5 = 5$$

and

$$x = x.$$

Should we really say the same comparing $\frac{1}{2}$ and $\frac{2}{4}$? To help with the answer to this silly question we check detailed comparison of these two quotients in the table below.

TABLE 1. Comparison of $\frac{1}{2}$ and $\frac{2}{4}$.

	$\mathbf{\frac{1}{2}}$	$\mathbf{\frac{2}{4}}$	is the same ?
numerator	1	2	no
denominator	2	4	no
division - operation	1 divided by 2	2 divided by 4	no
result of division	half	half	yes
fraction - operation	one half	two quarter	no
result of fraction	half	half	yes
ratio 'numerator' to 'denominator'	1 to 2	2 to 4	no
ratio result	half	half	yes

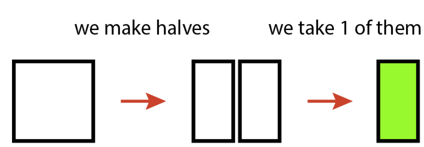
From Table 1 we see that by saying that $\frac{1}{2}$ is equal to $\frac{2}{4}$ we think only about the features which represent the results of the operations (half), however, we completely ignore operations that produces these results. We ignore what we divide by what, what we compare to what, what is the numerator and denominator we operate on. We oversimplify a lot. Please note that all results are represented by the word 'half' and its meaning is actually only understandable in reference to the 'whole', which we can present as

$$\frac{half}{whole}, \text{ which is } \frac{half}{1}.$$

As it was presented in previous section of this document, we are not able to understand what is 'half' without knowing what is the 'whole' in the first place.

Let us have a deeper look at what is $\frac{1}{2}$ and subsequently $\frac{2}{4}$.
Symbol $\frac{1}{2}$ can be interpreted as a fraction which is: we make halves and we take one of them.

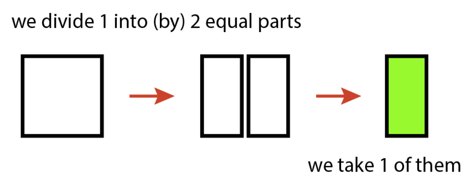
FIGURE 8. $\frac{1}{2}$ as a fraction - one half.



Notice that in this case we interpreted symbol $\frac{1}{2}$ "bottom up". We started by making halves $\frac{?}{2}$ and then took 1 of them $\frac{1}{?}$.

It can also be interpreted as a division which is: we divide 1 into 2 equal parts and we take one of them. Which is basically the same operation.

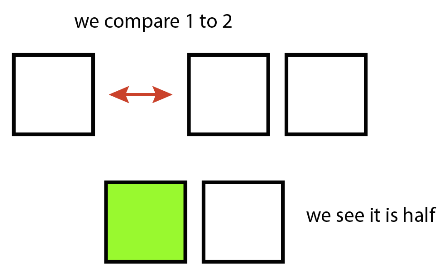
Figure 9. $\frac{1}{2}$ as a division.



In this case we interpreted symbol $\frac{1}{2}$ ”top down”. We started from taking one $\frac{1}{?}$ and we divided it into two equal parts $\frac{?}{2}$.

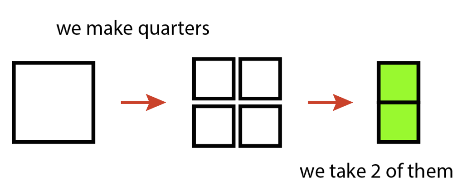
Finally, it can be interpreted as a ratio. We compare 1 to 2 and we see that 1 is a half of 2. Here our interpretation is from the middle, we compare numerator to denominator of our quotient $\frac{1}{?} \leftrightarrow \frac{?}{2}$.

Figure 10. $\frac{1}{2}$ as a ratio.



In case of $\frac{2}{4}$ when interpreted as a fraction: we make quarters and we take two of them.

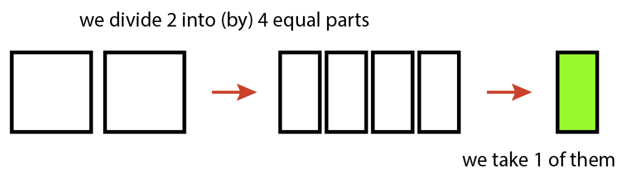
Figure 11. $\frac{2}{4}$ as a fraction - two quarters.



Here again we have interpretation ”bottom up”. We started from making quarters $\frac{?}{4}$ and then we take 2 of them $\frac{2}{?}$.

We can also interpret $\frac{2}{4}$ as division: we take two and divide it into (by) 4 equal parts, we take one of them.

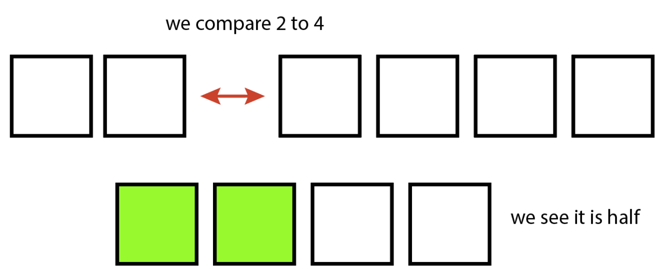
FIGURE 12. $\frac{2}{4}$ as a division - two divided by four.



In this division we interpreted symbol $\frac{2}{4}$ "top down". We started from taking two $\frac{2}{?}$ and we divided it into four equal parts $\frac{?}{4}$.

Finally, $\frac{2}{4}$ can be interpreted as a ratio.

FIGURE 13. $\frac{2}{4}$ as a ratio.



We compare 2 to 4 and we see that 2 is half of 4. Here our interpretation is from the middle; we compare numerator to denominator of our quotient $\frac{2}{?} \leftrightarrow \frac{?}{4}$.

As we can see from all the above examples, in each case we have the same result even though our operations that produced these results were different. It means that our quotients actually contain more information within than only results, and this information is completely ignored by our common understanding of numbers. Only by ignoring this additional information about operations, which initially exists, we can conclude that these two numbers $\frac{1}{2}$ and $\frac{2}{4}$ are equal.

To be able to say that $\frac{1}{2}$ and $\frac{2}{4}$ are equal we need to do the following transformations.

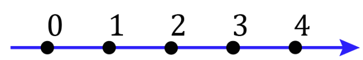
$$\frac{1}{2} \rightarrow \frac{half}{1} \leftarrow \frac{2}{4} \tag{3.2}$$

We need to transform our initial quotient, which represents certain relation between numerator and denominator, into a new quotient that represents a new value in relation to 1 that keeps the same ratio (numerator to denominator) as the initial one. During this transformation to denominator 1 some information that was initially present in our quotients is lost.

4. Graphical representation of numbers

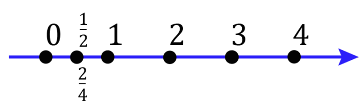
To represent numbers in graphical form, we often use an axis with a unit which defines distance from 0 to 1.

Figure 14. Number axis.



Using this kind of representation, we always refer with all numbers to 1 and both $\frac{1}{2}$ and $\frac{2}{4}$ can only be presented as one point which actually is $\frac{half}{1}$.

Figure 15. Representation of numbers $\frac{1}{2}$ and $\frac{2}{4}$ as $\frac{half}{1}$.

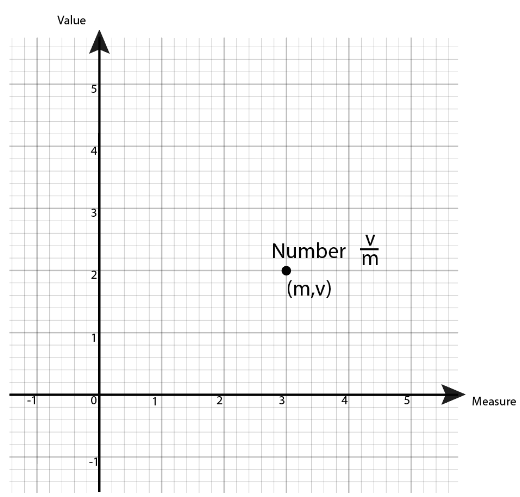


This representation of numbers using number axis is limited. As shown earlier, we do not always need to present numbers in reference to 1. Any other number can be used as the reference measure when presenting our value. To reflect this observation we propose the following representation.

$$number: \frac{value\ (v)}{reference\ measure\ (m)} \longmapsto (m, v) \tag{4.1}$$

By using this representation of numbers we can map every number into the point on the plain.

Figure 16. Representation of the number $\frac{v}{m}$ on the plain.



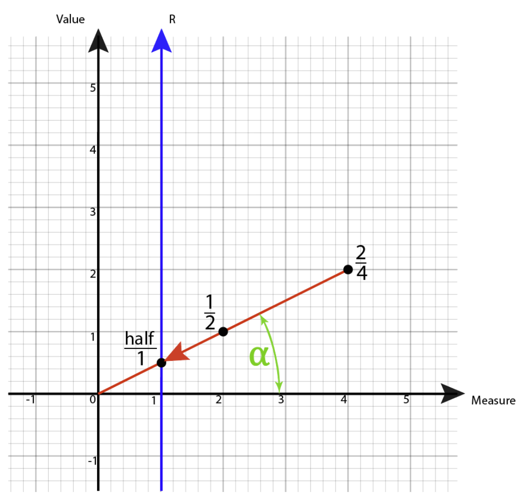
Using this method, we can see numbers in their full meaning with all the information that they have properly reflected. We can also easily understand the difference between $\frac{1}{2}$ and $\frac{2}{4}$ and why we think they are equal.

For clearer explanation we define two additional terms.

Real axis (R) - the axis on the Value/Measure plain connecting all points for which measure is equal 1. It is the representation of the normal number axis (from Figure 14 and Figure 15) on the Value/Measure plain. This will be highlighted in blue on all following figures.

Projection line - the line segment connecting point (m,v) - number's representation on the Value/Measure plain, the point (0,0) - the origin of the Value/Measure plain and Real axis. This will be highlighted in red on all following figures.

FIGURE 17. Representation of the numbers $\frac{1}{2}$ and $\frac{2}{4}$ on the plain.



In Figure 17, Real axis (in blue) represents numbers as we usually understand them, assuming that they are in reference to 1 (which is always the case when we count objects). We can also see how points (numbers) representing $\frac{1}{2}$ and $\frac{2}{4}$ are projected on the Real axis (from the perspective of point (0,0)) to the point $\frac{half}{1}$. All these points have the same value of ratio $\frac{v}{m}$ which we know as $tan(\alpha)$. Notice that by presenting numbers in their $\frac{value}{measure}$ form (as in Figure 17) we are reflecting all their features enumerated in Table 1. We also explain why we see some of these features as being the same - when we calculate the result of the ratio $\frac{v}{m}$ making it in relation to 1. While others are different - when we keep $\frac{v}{m}$ in quotient form accepting their true nature and only then we can express this richness of relations between different values and variant reference measures fully.

Numbers are always representing a comparison, a relation between a value and a certain reference measure. To understand numbers properly, and thus mathematics as well, we have to acknowledge that not only a ratio is important. Explicitly presented value and a reference measure, to which we refer our value to, are also meaningful.

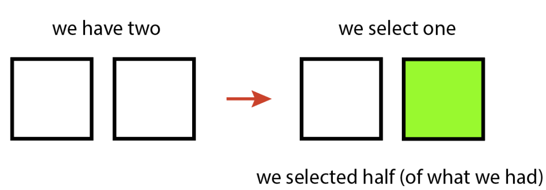
Now it is time to explain the test from the beginning of this paper. Of course the most obvious answers are: in Test 1 - one, and in Test 2 - half. Why probably no one answered 287/287 in Test 1? This, based on accepted mathematical understanding of numbers would be correct. Why probably no one answered 1001/2002 in Test 2? This, based on accepted understanding of numbers would be also correct. The reason is that we are guided by information provided in the tests and giving answer we are referring with our answer to the information that we know. The process looks like this. Test 1 - we know what is 2, we see that square is 2 times smaller so it has to be 1. Notice how we 'de-facto' referred with our answer to the number that we know. In Test 2 - we know what is 1, we see that triangle is half of this size (or 2 times smaller) so it has to be half (of one) or we can also say one half which is also half. Here, again, we refer with the answer to the number that we know. Now have a look on both tests together. Pictures are actually presenting exactly the same situation one big object and another two times smaller. We can clearly see how our answers are referenced to the 'reference measures' that were given. Exactly the same way we think about numbers always describing them in reference to the other one that we already know.

5. Operation of Selection

In Table 1 there were 3 different operations presented: division, fraction, ratio. Despite each of them describing different process of number manipulation, all produced the same result. In presented cases this was always *half*. Another such operation exists which produces the same result as the ones already presented, it is selection.

Interpreting symbol $\frac{1}{2}$ as selection we process "bottom up". We have two elements $\frac{?}{2}$ and we select one $\frac{1}{?}$ out of it.

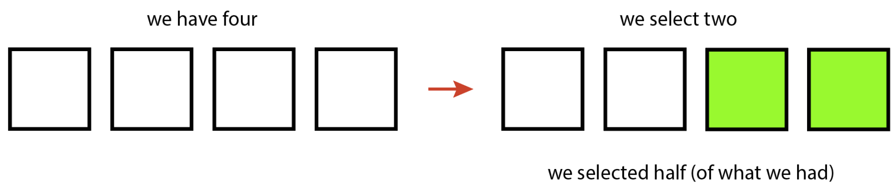
Figure 18. $\frac{1}{2}$ as a selection.



What we selected is half of what we had.

Interpreting symbol $\frac{2}{4}$ as selection; we have four elements $\frac{?}{4}$ and we select two $\frac{2}{?}$ out of it.

FIGURE 19. $\frac{2}{4}$ as a selection.



What we selected is half of what we had.

As we see this is another operation that produces the same results in both cases $\frac{1}{2}$ and $\frac{2}{4}$. We can extend Table 1 adding two additional rows.

TABLE 2. Comparison of $\frac{1}{2}$ and $\frac{2}{4}$.

	$\frac{1}{2}$	$\frac{2}{4}$	is the same ?
numerator	1	2	no
denominator	2	4	no
division - operation	1 divided by 2	2 divided by 4	no
result of division	half	half	yes
fraction - operation	one half	two quarter	no
result of fraction	half	half	yes
ratio 'numerator' to 'denominator'	1 to 2	2 to 4	no
ratio result	half	half	yes
selection 'numerator' from 'denominator'	1 from 2	2 from 4	no
result of selection	half	half	yes

Notice that all operations reflect certain 'tension' (relation) between one number and the other one, while all results are trying to reflect the same relation in reference to 1. The purpose of this paper is to propose paradigm shift and accept numbers in their natural quotient form where this 'tension' is properly reflected. To show why this is important we can analyse another example. Let us discuss two football matches. At the end, result in the first is "4:2" and in the second is "10:5". Would it be accepted by football fans to have these results listed in 'normalised form" as "2:1" in both matches? Probably not. This is why we should always reflect relations between numbers as they are. It seems to be important, and an extra information would be missed out.

6. CONSEQUENCES

Mathematics is based on numbers. When accepted, proposed above understanding of numbers, will have many significant consequences. We can summarise this change as a transformation of mathematics from one-dimensional (where all numbers were always projected on the Real axis and always referenced to 1, even in case of quotients), to two-dimensional, where numbers are located on the Value/Measure plain and can be fully interpreted as they really are. This change requires to rethink and redefine mathematical operations, number relations and verification of everything that was build on these foundations. It also explains and solves many problems that were undefined, unsolved or beyond our reach until now. In the next few pages this process will only be outlined, and some most important consequences will be presented, but to fully complete this task and understand all consequences of this change, cooperation of many mathematicians and probably many years of work is needed.

6.1. Relations between numbers: $\frac{1}{2}$ and $\frac{2}{4}$, $\frac{-1}{1}$ and $\frac{1}{-1}$.

As presented in previous sections of this paper, $\frac{1}{2}$, when understood correctly **is not** $\frac{2}{4}$. They only share one common property, which is when projected on Real axis and converted to denominator 1, they have the same numerator - *half* (compare (3.2) and Figure 17).

Depending on the context, $\frac{1}{2}$ can represent:
- one divided by two,
- one half,
- ratio between one and two,
- one selected out of two,

but in fact it is simply a number; **one in relation to two**.

Depending on the context $\frac{2}{4}$ can represent:
- two divided by four,
- two quarter,
- ratio between two and four,
- two selected out of four,

but it is also a number; **two in relation to four**.

If we accept that numbers should be presented in their natural form - as quotients, we see that these two numbers $\frac{1}{2}$ and $\frac{2}{4}$ are different.

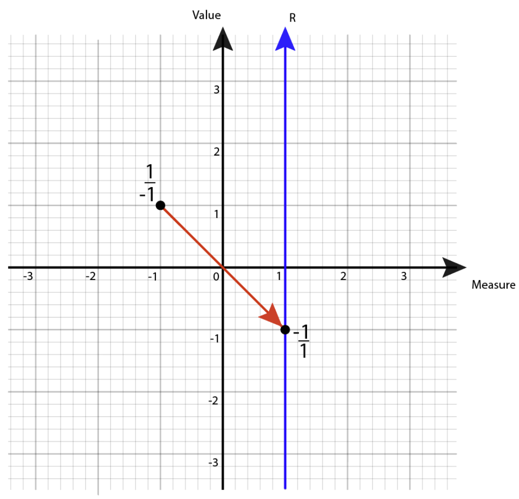
$$\frac{1}{2} \neq \frac{2}{4} \tag{6.1}$$

If they are different, can we say which one is bigger? All relations "greater", "smaller" or "equal", as we use them, are only reflecting the relation between projections of numbers to the Real axis. When we represent numbers on the Value/Measure plain, relations between these numbers are more complex. Each number can have greater Value or smaller Value, greater Measure or smaller Measure than the other number. It is also possible that two numbers will have the same Value but different Measure or vice versa. These relations together with their meanings still have to be properly defined.

Remark 6.1. *This document is in very initial version and therefore proposition of definitions of relations between numbers in their quotient form* $\frac{Value}{Measure}$ *will be described in next revision.*

Another interesting consequence of this new approach to numbers is the fact, it is now important whether the minus sign is located in a numerator or a denominator. Therefore $\frac{-1}{1}$ and $\frac{1}{-1}$ are also two different numbers with contrasting meanings.

Figure 20. Representation of the numbers $\frac{-1}{1}$ and $\frac{1}{-1}$ on the plain.



In Figure 20 we see that both $\frac{-1}{1}$ and $\frac{1}{-1}$ are projected to the same point on Real axis, but their locations on the plain are different. All pairs which have minus sign either in numerator or denominator, will have the same characteristics. Each pair will be projected to the same point on Real axis.

6.2. **Operations on numbers.**

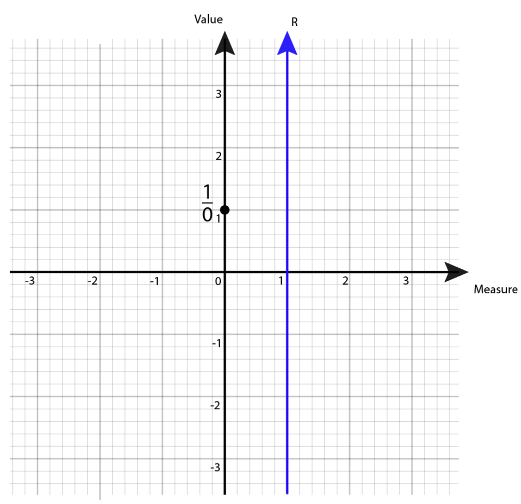
At the beginning, mathematics was very closely connected to our physical world. When people started counting, they did it on physical objects, they measured physical objects and their calculations were describing real things. Unfortunately, later mathematics was almost completely disconnected from the real world becoming an abstract science about abstract ideas. No one cares anymore if latest discoveries in mathematics are in any way related to the real world. Mathematicians even discovered irrational numbers which by the name are ir-rational quoting dictionary "not logical". When mathematics was still strictly connected to the real world, people naturally discovered mathematical operations: addition, subtraction later multiplication and division. All of those operations have one and the same problem; they look only on one side of operation. For example: we have two apples and we add another two apples, now we have four, but where did we get those two new apples from? Another example: we have one stick and we divide this stick into two equal parts, now we mathematically think we have half of the stick, but what we actually have are two such halves, not one. We can easily propose similar examples for subtraction and multiplication. To be precise and mathematically reflect what is really going on, we should always describe the entirety of the situation in mathematical operation not only part of it. Things do not come out of nowhere, do not disappear to nowhere and are not multiplied out of nowhere, all operations are just certain transformations from one state to another. We always start with a certain numerical setup that we then change by doing mathematical operations to a new numerical setup that we end with.

Remark 6.2. *This document is in very initial version and therefore more detailed description of many different operations based on the representation of numbers in their quotient form* $\frac{Value}{Measure}$ *perceived as transformations will be described in next revision.*

6.3. **Division by zero $\frac{1}{0}$.** [1]

One of the important benefits of this new way of understanding numbers is explanation for division by zero. Symbol that represents division by zero is $\frac{1}{0}$. According to proposed interpretation of numbers it is a normal point on the plain $\frac{Value}{Measure}$.

FIGURE 21. Representation of the numbers $\frac{1}{0}$ on the plain.



Please notice that it is impossible to draw a projection line for this number. There is no such line that goes through the point (0,0), number $\frac{1}{0}$ and touches the Real axis. This means that we can not do such transformation (find value of x) that satisfies

$$\frac{1}{0} \rightarrow \frac{x}{1} \tag{6.2}$$

whilst keeping the same ratio for $\frac{x}{1}$ as $\frac{1}{0}$ has. Impossibility to project $\frac{1}{0}$ on the Real axis is the key to understand true meaning of division by zero, but for this we need correct understanding of numbers in the first place.

What is a 'division'?

Division is nothing else but projection of any number represented by $\frac{value}{measure}$ onto the Real axis. We are just trying to project numbers from their natural form to denominator 1.

$$\frac{v}{m} \rightarrow \frac{?}{1} \tag{6.3}$$

However, in case of number $\frac{1}{0}$, this projection is not possible due to the fact that the line that goes through origin of Value/Measure plain, and the number $\frac{1}{0}$ is parallel to the Real axis and never touches it.
We conclude that we can not divide one by zero, but we can understand what the symbol $\frac{1}{0}$ means. It is just a number in its natural form reflecting the relation of 1 in reference to 0. What we can also easily understand is that $\frac{2}{0}$ is just another number, different than $\frac{1}{0}$.

6.4. Division of zero by zero $\frac{0}{0}$.

In case of $\frac{0}{0}$, it is another point on the Value/Measure plain. It can not be projected on the Real axis, but it is a special point because it is present in all projection lines from all existing points. It has a crucial meaning in connection between mathematics, as we know it, based on numbers, as we know it, that are always referred to 1, and new mathematics as we do not know it yet, that is based on a new understanding of numbers as proposed in this paper.

Remark 6.3. More about the $\frac{0}{0}$ number in the next revision of this document.

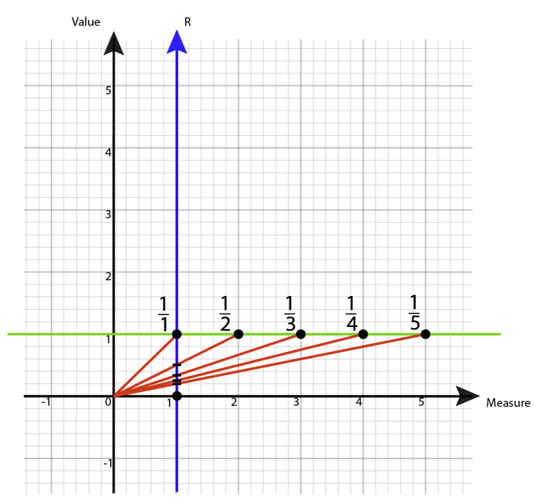
6.5. Understanding of limits.

We consider a sequence a_n defined as

$$a_n = \frac{1}{n}, \; for \; n \in \mathbb{N}. \tag{6.4}$$

We know from school that the limit of such sequence is 0. We see representation of this sequence on the $\frac{value}{measure}$ plain in Figure 22

FIGURE 22. Representation of the numbers from the sequence a_n on the plain.



When we progress with this sequence, as $n \to \infty$, we see that projections of the numbers on the Real axis get closer and closer to the point $(1,0)$ which represents number $\frac{0}{1}$. We are projecting all numbers $\frac{1}{n}$ to its corresponding representations (on Real axis) $\frac{r_n}{1}$ in such way that the ratio remains the same.

$$\frac{1}{n} \to \frac{r_n}{1} \tag{6.5}$$

Please notice that all subsequent r_n are decreasing which is also reflected by decreasing angle between subsequent projection lines (red) and measure axis. Therefore we say that

$$\lim_{n \to \infty} \frac{1}{n} = \frac{0}{1} = 0 \tag{6.6}$$

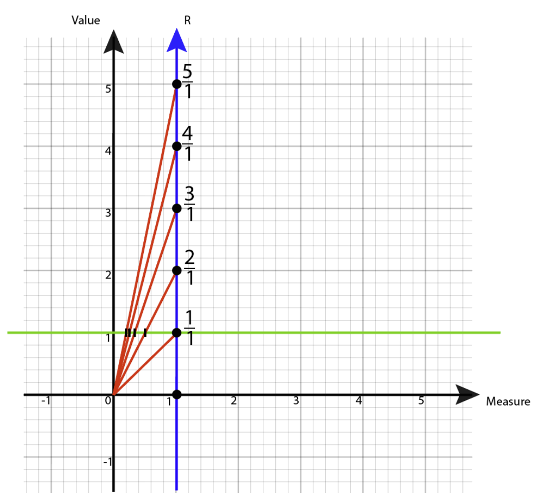
When we present numbers in this sequence on the plain using proposed representation, we see that actually all numbers in this sequence are located on one line (green line in Figure 22) and they have nothing to do with 0, and are also not limited by any limit. Only their projections on the Real axis (representations in reference to 1) get closer and closer to the $\frac{0}{1} = 0$.

For comparison, we consider another sequence b_n defined as

$$b_n = n = \frac{n}{1}, \ for \ n \in \mathbb{N}. \tag{6.7}$$

We know from school that the limit of such sequence does not exist because terms approach ∞.

FIGURE 23. Representation of the numbers from the sequence b_n on the plain.



In Figure 23 we see that all subsequent terms are located on the Real axis (all points $\frac{n}{1}$ are represented in reference to 1) and as $n \to \infty$ the points corresponding with the terms of the sequence become more and more distant from the Measure axis. Therefore, the angles between their projection lines and the Measure axis become bigger. Comparing these two examples we see that they are not to different from each other. In comparison to the previous example we can say that

$$\lim_{n \to \infty} \frac{n}{1} = \frac{1}{0} \tag{6.8}$$

In fact, in this example terms of the sequence are also not limited by any limit, they all are just located on the Real axis.

References

[1] Leszek Mazurek. Division by zero. URL: https://www.researchgate.net/publication/339350024_Division_by_zero.

Email address: mr.leszek.mazurek@gmail.com ,