Theory of Original Gravity Revisited: Unified Field Theory, Starting out from the Quantum Formulation to the Manifold Geometry

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Abstract

This paper assumes an acquaintance with a new concept of time and gravity to provide the unified field theory. A new mathematical attempt intended to formalise the gravity current \( J^A \) and the speed of gravity signal \( V_g \) have been shown. The present author starts out from the “Theory of Original Gravity” [1] and presents to the unified field theory in a simple manner. This paper includes the formulations of quantum mechanics, pseudo-Riemannian manifold, four-gradient operator and Lorentz-invariant interval as a notable mathematical tools. All of the ideas and matters presented in this paper are mainly based on quantum theory and manifold geometry.

1. Introduction

The “Theory of Original Gravity” [1] is based on how energy operator, obtained from the heuristically assumed wave function \( \Psi_A \) of a tiny ambient space adjoining the particle in question acts on the hyperbolic version of the wave function \( \Psi_h \) of the Schrodinger’s particle to provide the gravitational energy. This interaction is cast in the language of quantum theory. Tensor field on a manifold (space - time) are maps which attach a tensor to each point of the manifold. A tensor field at each point of the manifold around a particle describes the geometry around that particle and determines the motion of the particle in that field. Understanding the above discussion, the present author throws a light on the interpretation of the wave function \( \Psi_A \) of a tiny ambient space adjoining the particle. It follows that \( \Psi_A = \frac{1}{\Omega} e^{-id\mu k^2/x} \). This means to saying that \( \Psi_A \) being a wave function of the suggested space is found an appropriate object to involve a tensor field on a manifold for describing the geometry around the particle and for determining the motion of the particle in that field.
It is important to note that the present author has used a defining term, the metric distance \( d_H \), contained in the wave function \( \Psi_A \) of a tiny ambient space geometry in the determination of space metric \( ds^2 \), the speed of the gravity signal \( v_g = \frac{\hbar}{m_{sdH}} \), gravitational energy etc..

In the General Theory of Relativity, matter-energy creates curvature in the space-time manifold and the curvature is the source of gravity. Newton’s law tells how mass generates gravitational force. In this paper, the present author demands from his theory that the gravity is the quantum interaction between \( \Psi_A \) and \( \Psi_h \), taking hyperbolic manifold associated with this interaction. Therefore, this theory is neither a geometry, nor does it depend on mass to tell the gravity.

There are three types of interactions, i.e., Quantum Chromodynamics( QCD), the electroweak interaction and gravity. The whole physics enterprise is now excited at the unification schemes of bringing the three fundamental interactions into one interaction.

Unlike, the other field theories, General Theory of Relativity is of the geometric nature. In 1967, Weinberg proposed his unified theory of weak and electromagnetic interactions, where electromagnetism was unified with the weak (nuclear) interaction, but not with the gravity. Now, the remarkable point is that the other interactions besides the gravity, cannot be described in geometric term, so they cannot be made to look like general relativity in four dimensions.

The most noticing progress is that this paper can describe this interaction in geometric term.

In this paper, \( \Psi_A \) is given by
\[
\Psi_A = \frac{1}{\Omega} e^{-idHk^2x}.
\]

Energy operator is calculated as follows:
\[
E_A = i\frac{\hbar^2}{2m_{sdH}} \frac{\partial}{\partial x}.
\]

Where, \( m_s \) is the mass of the particle whose wave function is \( \Psi_A \).

Hyperbolic version of Schrodinger’s particle wave function is given by
\[
\Psi_h = Ae^{-i(kx-wt)}.
\]

Gravitation energy is given by
\[
E_A = \frac{kA^2\hbar^2}{2\rho_{sdH}} = \frac{G^2}{v^2}\left(\frac{kA^2\hbar^2}{d_H}\right) = \hbar(G^2KA^2m_s v_g / v^2)
\]
Where, $\rho_s$ (space density) = $\frac{1}{m_s} \int_{vol} d^3 x$.

$G^A = \frac{v^2}{\rho_s}$, $v$ = frequency.

$v_g$ (speed of gravity signal) = $\frac{h}{m_s d\mu}$.

1. Connotational Ambiguity of Time

Let us now go onto the discussion of the connotational ambiguity of time. Time is said to be asymmetric, but there may be a phenomena with symmetry. In GTR, time only has physical meaning relative to a particular observer. Proper time ($\tau$) allows us to define four-vector which is considered as the most important mathematical structure for the handling of an event in space-time, and used to describe the evaluation of the system. But, in this case the system (four-vector, 4-acceleration etc.) should always depend on time to have it evaluated. Thus, the question related to the fundamental nature of time for macroscopic and microscopic system poses a serious challenge regarding the arrow of time and its reversal symmetry of the dynamics.$^{[4]}$

In this paper, the new concept of time has been proposed to build space-time metric and an attempt is made to reach the “Unified Field Theory” revisiting “Theory of Original Gravity”.$^{[1]}$

In the recent years the anti-de Sitter hyperboloid space-time maximally symmetric with a negative cosmological constant has become interesting in the field of string theory and M-theory. Kaluza-Klein theory provides a geometrical unified field theory of gravity and electromagnetism.$^{[5]}$

Klein-Gordon equation has used wave function $\varphi$ which is a Lorentz-invariant scalar function. Here $|\varphi|^2$ cannot represent a probability density because a density transforms a time-like component of four-vector, due to the Lorentz contraction of volume element. Klein Gordon for $\varphi$ and $\phi^*$ have used correct definition of density following the continuity equation:

$$\frac{\partial \rho}{\partial t} = - \nabla \cdot J$$

Where $J$ is the corresponding current vector, i.e., $\partial_{\mu} J^\mu = 0$.

In analogue with Klein-Gordon equations, the probability density ($\rho$) and the gravity current ($J^A$) has been calculated in this paper.
The product of the four-gradient by itself gives the d’Alembert operator in the scalar element of the resulting four-vector. The d’Alembert operator generates the wave equation when multiplied by same scalar field. \[^2\]

In analogue with d’Alembert operator, four-gradient operator has been determined to get four-force tensor equation in this paper. The present author has defined time as follows:

\[ T = \tilde{\gamma} + i t. \]

Where t is our clock time recorded by an observer and \( \tilde{\gamma} \) is described as real time, inherited by the particle itself owing to have its metric position in the said space-time. \( \tilde{\gamma} \) is taken as \( \tilde{\gamma} = \beta d_H. \)

Where \( \beta = \frac{v^2}{v_g^2} = \frac{v}{v_g} \theta \), and \( d_H = \int_{\lambda_0}^\lambda |ds_4| \), and \( x^\mu \) is supposed to be parameterised by \( \lambda \). The value of \( \beta \) has been calculated by the Lorentz transformation principle. In the equation (30), it has been shown that how \( d_H \) behaves as a real time and as a metric distance simultaneously. \( \tilde{\gamma} \) in this paper has some technical compatibility with proper time (\( \tau \)) in GTR. But \( = \tilde{\gamma} \) and \( T \) are conceptually different.

The gravity proposed in this paper finds its immediate compatibility with the Newtonian gravity, four-vector Maxwell’s equation and Planck energy equation.

2. **Suggested space-time and invariance symmetry:**

The hyperbolic n-space when isometrically embedded inside the (n+1)-dimensional Minkowski space becomes a Lorentzian manifold. A Lorentzian manifold is a special case of pseudo-Riemannian manifold in which the signature of the metric is (1, n-1). In this paper, the metric of type (1, n-1) is used to explain gravity.

The space of my preference is considered as a hyperboloid

\[ -v_g^2 \tilde{\gamma}^2 - v_g^2 t^2 + x^2 + y^2 + z^2 = -1 \] (1)

Which is isometrically embedded inside the flat 5-dimensional ambient space with metric

\[ ds_5^2 = -v_g^2 d\tilde{\gamma}^2 - v_g^2 dt^2 + dx^2 + dy^2 + dz^2 \] (2)
With $\eta = \text{diag.}(-1, -1, 1, 1, 1)$
This space is symmetric and the metric is invariant i.e., all observers in any frame of reference will agree on it. Eqn. (1) can be parameterised by
\[ v_g \tilde{Y} = r \]
\[ v_g t = \sqrt{1 - r^2} \cosh t \]
\[ x = \sqrt{1 - r^2} \sinh t \cos \phi \cos \theta \]
\[ y = \sqrt{1 - r^2} \sinh t \cos \phi \sin \theta \]
\[ z = \sqrt{1 - r^2} \sinh t \sin \phi \]
Eqn. (2) is reduced to
\[ ds_4^2 = -(\frac{1+\theta^2}{\theta^2}) v_g^2 t^2 + dx^2 + dy^2 + dz^2 \quad (3) \]

Where $T = \tilde{Y} + it$ and $\frac{t}{\tilde{Y}} = \theta$.

\[ |T|^2 = \tilde{Y}^2 + t^2 = (\frac{1+\theta^2}{\theta^2}) t^2 \]

The metric in Eqn. (3) is taken to define the real time $\tilde{Y}$.

Let us consider two systems $S$ and $S_1$ which coincide at the time $t = t_1 = 0$, such that $S_1$ moves with constant velocity $u$ along the X axis of the system $S$. The Lorentz Transformation connects the space coordinates and time by the equations
\[ x_1 = \theta(x - ut), y_1 = y, z_1 = z, t_1 = \theta t + \beta x \]
and they satisfy the following equations
\[ x^2 + y^2 + z^2 = v_g^2 \tilde{Y}^2 + v_g^2 t^2 \quad (4) \]
\[ x_1^2 + y_1^2 + z_1^2 = v_g^2 \tilde{Y}^2 + v_g^2 t_1^2 \quad (5) \]

Therefore, we get
\[ \theta^2 (x - ut)^2 + y^2 + z^2 = v_g^2 \tilde{Y}^2 + v_g^2 (\theta t + \beta x)^2 \]
\[ \Rightarrow \theta^2 (x^2 + u^2 t^2 - 2xu t) + y^2 + z^2 = v_g^2 \tilde{Y}^2 + v_g^2 (\theta^2 t^2 + \beta^2 x^2 + 2x t \theta \beta) \]
\[ x^2(\theta^2 - \beta^2 \nu_g^2) - 2xt(\theta^2 u + \theta \beta \nu_g^2) + y^2 + z^2 = \nu_g^2 \ddot{Y}^2 + t^2(\theta^2 \nu_g^2 - \theta^2 u^2) \]

Comparing the coefficients of the different terms in the equation (6) with the corresponding terms in the eqn. (4)

\[ \theta^2 - \beta^2 \nu_g^2 = 1 \quad (a) \]
\[ \theta^2 u + \theta \beta \nu_g^2 = 0 \quad (b) \]
\[ \theta^2 \nu_g^2 - \theta^2 u^2 = \nu_g^2 \quad (c) \]

Eqn. (c) gives

\[ \theta = \frac{\nu_g^2}{\nu_g^2 - u^2} = \frac{1}{1 - \frac{u^2}{\nu_g^2}} \]  

\[ \Rightarrow \theta = \frac{1}{(1 - \frac{u^2}{\nu_g^2})^{\frac{1}{2}}} \quad (7) \]

From eqn. (b)

\[ \theta^2 u = - \theta \beta \nu_g^2 \]

\[ \Rightarrow \beta = - \frac{u}{\nu_g^2} \nu_g \quad (8) \]
3. Determination of four-gradient operator, gravity current (J^A), probability density (ρ), speed of gravity signal v_g :

Equation (3) is given by

\[ ds_4^2 = -\left( \frac{1+\theta^2}{\theta^2} \right) v_g^2 (dt)^2 + dx^2 + dy^2 + dz^2 \]

Let us define the interval four-vector using the above equation (3) as follows:\(^{[2]}\)

\[(ds_4)_\mu = (ds_4)^\mu = \left( i \sqrt{\frac{1+\theta^2}{\theta^2}} v_g dt, dx, dy, dz \right) \quad (9)\]

Where i is the imaginary unit and i = √−1.

\[ ds_4^2 = (ds_4)_\mu (ds_4)^\mu \] is a relativistic invariant, which appears from the product of the interval four-vector by itself. The advantage of four-vector is that there is no difference between the contravariant and covariant forms of four-vectors.

The total differential "\(ds_4^2\)" of the scalar field \(s_4\), which is a function of the coordinates and time, is written as follows:

\[ ds_4 = \frac{\partial s_4}{\partial x_0} dx_0 + \frac{\partial s_4}{\partial x} dx + \frac{\partial s_4}{\partial y} dy + \frac{\partial s_4}{\partial z} dz \]
Where, $\partial x_0 = i \sqrt{\frac{1+\theta^2}{\theta^2}} v_g \partial t$

$$d s_4 = \frac{\partial s_4}{i \sqrt{\frac{1+\theta^2}{\theta^2}} v_g} dx_0 + \frac{\partial s_4}{\partial x} dx + \frac{\partial s_4}{\partial y} dy + \frac{\partial s_4}{\partial z} dz$$  \hspace{0.5cm} (10)

From this relation we extract the partial derivatives by separating them from the differential of interval, we get the four vector as follows

$$\partial \mu s_4 = \left\{ \frac{1}{i \sqrt{\frac{1+\theta^2}{\theta^2}} v_g} \frac{\partial s_4}{\partial t}, \frac{\partial s_4}{\partial x}, \frac{\partial s_4}{\partial y}, \frac{\partial s_4}{\partial z} \right\}$$

$$= \left\{ -i \sqrt{\frac{\theta^2}{1+\theta^2}} v_g \frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\}$$  \hspace{0.5cm} (11)

Now, we suppress the scalar field ($s_4$) and leave the rest as an empty operator to obtain the four-gradient

$$\partial \mu = \left\{ -i \sqrt{\frac{\theta^2}{1+\theta^2}} v_g \frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\}$$

$$= \left\{ -i \sqrt{\frac{\theta^2}{1+\theta^2}} v_g \frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\}$$  \hspace{0.5cm} (12)
The product of the four-gradient by itself gives the operator.

\[
\partial_\mu \partial^\mu = -\frac{\theta^2}{1+\theta^2} \times \frac{1}{v_g^2} \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}
\]

\[
\therefore \partial_\mu \partial^\mu = -\frac{\theta^2}{1+\theta^2} \times \frac{1}{v_g^2} \frac{\partial^2}{\partial t^2} + \nabla^2
\]

\[
=-\frac{\theta^2}{1+\theta^2} \times \frac{m_S^2 \mathcal{H}^2}{h^2} \frac{\partial^2}{\partial t^2} + \nabla^2
\]

\[(13)\]

Now, in order to determine gravity current \((J^A)\) and probability density \((\rho)\), we multiply the operator by \(A^2 = \psi_h \psi_h^*\)

We get, \(-\frac{\theta^2}{1+\theta^2} \times \frac{m_S^2 \mathcal{H}^2}{h^2} \frac{\partial^2}{\partial t^2} (\psi_h \psi_h^*) + \nabla^2 (\psi_h \psi_h^*) = 0\)

\[
\Rightarrow \frac{\theta^2}{1+\theta^2} \times \frac{m_S^2 \mathcal{H}^2}{h^2} \left[ \psi_h \frac{\partial^2}{\partial t^2} \psi_h^* - \psi_h^* \frac{\partial^2}{\partial t^2} \psi_h \right] = (\psi_h \nabla^2 \psi_h^* - \psi_h^* \nabla^2 \psi_h)
\]

\[(14)\]
Multiplying both side by \( \frac{i}{k} \), we get

\[
\Rightarrow i \sqrt{\frac{\theta^2}{1+\theta^2}} \frac{m_s^2 d_H^2}{h k} \partial^2 \left[ \psi_h \partial^2 \psi_h^* - \psi_h^* \partial^2 \psi_h \right] \\
= i \sqrt{\frac{\theta^2}{1+\theta^2}} \frac{h}{m_s k d_H} \nabla \left[ \psi_h \nabla \psi_h^* - \psi_h^* \nabla \psi_h \right] \tag{15}
\]

Eqn. (15) is compared to

\[
\frac{\partial \rho}{\partial t} = -\nabla J \tag{16}
\]

We get

\[
\rho = i \sqrt{\frac{\theta^2}{1+\theta^2}} \frac{m_s d_H}{h k} \left[ \psi_h \frac{\partial}{\partial t} \psi_h^* - \psi_h^* \frac{\partial}{\partial t} \psi_h \right] \\
= \text{probability density.} \tag{17}
\]

\[
J^A = -i \sqrt{\frac{\theta^2}{1+\theta^2}} \frac{h}{m_s k d_H} \left[ \psi_h \nabla \psi_h^* - \psi_h^* \nabla \psi_h \right] \tag{18}
\]
\[ J^A = -i \frac{\sqrt{1+\theta^2}}{\theta} \frac{h}{m_S k d_H} (ikA^2 + ikA^2) \]

\[ J^A = 2 \frac{\sqrt{1+\theta^2}}{\theta} \times A^2 \times \frac{h}{m_S d_H} \]

= Gravity current. \hspace{1cm} (19)

\[ J^A = 2 \frac{\sqrt{1+\theta^2}}{\theta} \times A^2 \times v_g \] \hspace{1cm} (19a)

Where, \( v_g = \frac{h}{m_S d_H} \) = speed of gravity signal. \hspace{1cm} (20)

4. Proposed time:

Time in physics is operationally defined as “what a clock reads.”

Now, the problem raises the question of what time really is in the physical sense and whether it is truly real, distinct phenomenon.

In classical mechanics, time is treated as background parameter, external to the system itself. Following the classical kinematics of Newton’s law of motion, the kinematics of quantum mechanics is built in pre-supposing nothing about the time reversal symmetry of the dynamics. Special theory of relativity has defined time as space-time continuum. But from a fixed Lorentz observers’ viewpoint time remains absolute and external parameter.

In GTR, time only has meaning relative to a particular observer. It does not address why even can happen forward and backward in space, whereas events only happen in the forward progress of time.
The problem now involves the related question of why time seems to flow in a single direction, despite the fact that no known physical laws at the microscopic level seem to require a single direction.\footnote{3}

Time reversal symmetry laws under the transformation of time reversal is given by,

\[ T: t \rightarrow -t \]

Though in special equilibrium states the second law of thermodynamics predicts the time symmetry to hold, the macroscopic universe does not show the symmetry under time reversal.

In classical mechanics, a velocity \( v \) reverses under the operation of time, but acceleration does not.\footnote{6}

A closer look assumes that the motion of classical charged particle in electromagnetic field is also time reversal invariant.

In this paper, time is given by \( T = \bar{\gamma} + it \), where \( t = \theta \bar{\gamma}, \bar{\gamma} = \beta d_H \).

Where \( \beta = -\frac{v}{\sqrt{v^2}} = -\frac{v}{\sqrt{v^2}} \theta \), and \( d_H = \lambda^\lambda_{\lambda_0} \left| ds_4 \right| \).

It has been shown that how \( d_H \) behaves as a real time and as a metric distance simultaneously.

We note that \( ds_4^2 \) is invariant under Lorentz transformations and \( \bar{\gamma} \) can hold time reversal symmetry. The reason for \( \bar{\gamma} \) to hold time reversal symmetry is that \( \bar{\gamma} \) has been defined by \( \bar{\gamma} = \beta d_H \) it is inherited by the events owing to have their metric position in space-time and always stays with events whether the events happen backward or forward in space.
5. **Determination of four-force tensor equation** :

From eqn. (13), the four-gradient operator is

\[- \frac{\theta^2}{1+\theta^2} \times \frac{1}{v_{g}^2} \frac{\partial^2}{\partial t^2} + \nabla^2 = \partial_{\mu} \partial^{\mu}\]

Multiplying the equation by

\[E_A = G^A K A^2 h^2 \frac{\mu^2}{\nu^2 d_H},\] we get

\[- \frac{\theta^2}{1+\theta^2} \times \frac{1}{v_{g}^2} \frac{\partial^2}{\partial t^2} \left( \frac{G^A K A^2 h^2}{\nu^2 d_H} \right) + \nabla^2 \left( \frac{G^A K A^2 h^2}{\nu^2 d_H} \right) = 0\]

\[\Rightarrow \frac{\theta^2}{1+\theta^2} \times \frac{1}{v_{g}^2} \frac{\partial^2}{\partial t^2} \left( \frac{G^A K A^2 h^2}{\nu^2 d_H} \right) = \nabla^2 \left( \frac{G^A K A^2 h^2}{\nu^2 d_H} \right)\]

In L.H.S., putting \(d_H = \frac{t}{\theta \beta}\)

\[\Rightarrow \frac{\theta^2}{1+\theta^2} \times \frac{1}{v_{g}^2} \frac{\partial^2}{\partial t^2} \left( \frac{G^A K A^2 h^2}{\nu^2} \times \frac{\theta \beta}{t} \right) = \nabla^2 \left( \frac{G^A K A^2 h^2}{\nu^2 d_H} \right)\] (21)
\[ \Rightarrow - \frac{\theta^2}{1+\theta^2} \times \frac{1}{u_g} \frac{\partial}{\partial t} \left( \frac{\theta \beta G^A K A^2 h^2}{v^2 t^2} \right) = \nabla^2 (E_A) \]

In R.H.S. \( \nabla^2 (E_A) \) can be written as \( \nabla(\nabla E_A) \), since \( E_A \) is a scalar function.

\[ \Rightarrow - \frac{\theta^2 \beta}{1+\theta^2} \times \frac{1}{u_g} \frac{\partial}{\partial t} \left( \frac{G^A K A^2 h^2}{v^2 t^2} \right) = \nabla (\nabla E_A) \]

\[ \Rightarrow \frac{2 \theta^3 \beta}{1+\theta^2} \times \frac{1}{u_g} \left( \frac{G^A K A^2 h^2}{v^2 t^3} \right) = \nabla F \quad (22) \]

Where \( F = \nabla E_A \)

From eqn. (19a), we get

\[ J^A = \frac{2\sqrt{1+\theta^2}}{\theta} A^2 \times \Omega_g \quad (17) \]

\[ \therefore A^2 = \frac{\theta}{2\sqrt{1+\theta^2}} \times \frac{1}{u_g} \times J^A \]
Replacing $A^2$ in eqn. (22), we get

$$\frac{\theta^4 \beta}{(1+\theta^2)^{3/2}} \times \frac{1}{v^3} \times \frac{G^A k h^2}{\sqrt{2} t^3} J^A = \nabla F^\gamma$$

Eqn. (23) can be written as

$$\frac{\theta^4 \beta}{(1+\theta^2)^{3/2}} \times \frac{1}{v^3} \times \frac{G^A k h^2}{\sqrt{2} t^3} J^A = \nabla F^\gamma$$

(24)

This is the four-force tensor equation.

It is interesting to note that in the left hand side of the equation (21), $d_H$ is replaced by the relation

It is interesting to note that in the left hand side of the equation (21), $d_H = \frac{t}{\theta \beta}$. Thus, $d_H$ plays the role of time. Whereas, in the right hand side of the eqn.(21), $d_H$ plays the role of metryic distance and $d_H$ is given by

$$d_H = \int_{\lambda_0}^{\lambda} |ds_4|.$$ 

6.a) Compatibility with the proper time ($\tau$) of the particle in the special theory of Relativity:

Let the coordinates of the two events in different reference frames $S_1$ and $S_2$ are connected by a Lorentz transformation.

The coordinate differences are connected by
\[ \Delta t = \gamma \left( \nabla t_1 + \frac{v}{c^2} \Delta x_1 \right), \]
\[ \Delta x = \gamma (\Delta x_1 + \nu \nabla t_1), \]
\[ \Delta y = \Delta y_1, \Delta z = \Delta z_1 \] (25)\text{c}

Since \((\Delta y)^2 + (\Delta x)^2\) is invariant under a rotation about the x-axis, \(-c^2\Delta t^2 + \nabla x^2 + \nabla y^2 + \nabla z^2\) is also invariant under Lorentz Transformations, i.e.,

\[ \Delta s^2 = - c^2 \Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 \]

\[ = - c^2 \Delta t_1^2 + \Delta x_1^2 + \Delta y_1^2 + \Delta z_1^2 \] (26)

If the velocity of the particle is \(u\) along x-axis, then \(u = \frac{\Delta x}{\Delta t}\)

Eqn. (26) gives

\[ \Delta s^2 = - c^2 \Delta t^2 + u^2 \Delta t^2 \]

\[ = - \left( 1 - \frac{u^2}{c^2} \right) c^2 \Delta t^2 \] (27)

In the rest frame \(S_2\) of the particle, \(\Delta x_1 = 0\), giving

\[ \Delta s^2 = - c^2 \Delta t_1^2 \] (28)
The time $t_1$ in the rest-frame of the particle is the same as the time measured on a clock carried by the particle. This time is called proper time ($\tau$) of the particle.

From eqn. (27) and eqn. (28), we get

$$-c^2\Delta\tau^2 = -(1 - \frac{u^2}{c^2})c^2\Delta t^2$$

$$\Rightarrow \frac{\Delta t}{\Delta\tau} = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} = \gamma \quad (29)$$

Let us now consider two systems $s$ and $s_1$ which coincide at the time $t = 0$, such that $s_1$ moves with constant velocity $u$ along the x axis of the system $s$. Then the velocity $u$ of a particle referred to the system $s$ is $u = \frac{dx}{dt}$. The Lorentz Transformation connects the space coordinates and time by the equations

$$dx_1 = 0(dx - udt), dy_1 = dy, dz_1 = dz, dt_1 = 0dt + \beta dx$$, where

$$\theta = \frac{1}{\frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}}$$ and $$\beta = -\frac{v}{v_\theta}$$

$$ds^2_5 = -v_\theta^2d\bar{Y}^2 - v_\theta^2 dt^2 + dx^2 + dy^2 + dz^2 = -v_\theta^2d\bar{Y}^2 - v_\theta^2 dt^2 + u^2dt^2$$

$$= -v_\theta^2d\bar{Y}^2 - v_\theta^2 dt_1^2 + dx_1^2 + dy_1^2 + dz_1^2 \quad (30)$$

The invariance of this equation with respect to the Lorentz Transformations suggests that the space in this paper is defined by the metric

$$ds^2_5 = -v_\theta^2d\bar{Y}^2 - v_\theta^2 dt^2 + dx^2 + dy^2 + dz^2$$ is appropriate for the geometrical interpretation of the research goal in this paper. The present author wants to clarify as to why $d\bar{Y}$ remains the same in both of the systems $s$ and $s_1$, extending a support that $\bar{Y}$ is inherited by the events owing to have their respective metric position in space-time and always stays with events whether the events happen backward or forward in space. The real time $\bar{Y}$ and the metric distance $d_H$ go hand in hand by the relation $\bar{Y} = \beta d_H$ where

$$\beta = -\frac{v}{v_\theta}$$

Again in the rest $s_1$ of the particle, $dx_1 = 0$, it follows from the equation (30) that

$$ds^2_5 = -v_\theta^2d\bar{Y}^2 - v_\theta^2 dt_1^2 \quad (31)$$
This time $t_1$ in the rest frame $s_1$ must be the same as the real time $\dot{Y}$ which all observers agree upon. It follows from the eqn. (31) that

$$ds^2 = -v_g^2 d\dot{Y}^2 - v_g^2 dY^2 \tag{32}$$

From eqn. (30) and eqn. (32), we get

$$-v_g^2 d\dot{Y}^2 - v_g^2 dt^2 + u^2 dt^2 = -v_g^2 d\dot{Y}^2$$

$$\Rightarrow -v_g^2 dt^2 + u^2 dt^2 = -v_g^2 d\dot{Y}^2$$

$$\Rightarrow - (1 - \frac{u^2}{v_g^2}) v_g^2 dt^2 = -v_g^2 d\dot{Y}^2$$

$$\Rightarrow (\frac{dt}{d\dot{Y}})^2 = \frac{v_g^2}{(1-\frac{u^2}{v_g^2})v_g^2}$$

$$\Rightarrow \frac{dt}{d\dot{Y}} = \frac{1}{\sqrt{1-\frac{u^2}{v_g^2}}} = \theta \tag{33}$$

Thus, from eqn. (29) and eqn. (33), we find a similar form of $\frac{\Delta t}{\Delta \tau} = \frac{1}{\sqrt{(1-\frac{u^2}{c^2})}} = \gamma$ in the Special Theory of Relativity with the form of $\frac{dt}{dY} = \theta$ in this paper.

It is evident in this paper that the proper time $\tau$ in the GTR and the real time $\dot{Y}$ in this paper have different physical interpretation.

The finest trick has been adopted by the present author in the choice of real time $\dot{Y}$. This is the time which every particle inherits due to have its position in space-time. The real time of the particle ($\dot{Y}$) can easily give the relationship between an old fashioned Newtonian 3-velocity $u(t)$ and our clock-time $t$ by the relation $\frac{dt}{d\dot{Y}} = \theta(u)$.
6. (b) **Compatibility with Newtonian gravity and Planck energy equation:**

According to the Newtonian gravitationa energy, the field I and potential energy are given by

\[ I = -\frac{GM}{r^2} r^A \quad \text{and} \quad U = -\frac{GMm}{r} \quad (34) \]

The field depend on \( r \) but not on \( t \). Such a field is incompatible with special relativity. This is not Lorentz invariant field. Had these been a four-vector rather than a three vector and would have depended on \( t \) as well as on \( r \), so that the equation of gravity looked the same in all frame of reference by Lorentz transformations.

In this paper, gravitational energy is given by:[1]

\[ E_A = \frac{k A^2 h^2}{2 \rho_S d_H} = \frac{G^A}{v^2} \left( \frac{k A^2 h^2}{d_H} \right). \quad (35) \]

Where, \( \rho_S \) (space density) = \( \frac{1}{m_s} \int_{vol} d^3x \).

\( G^A = \frac{v^2}{\rho_s} \) and \( v \) is a frequency

\[ v_g (\text{speed of gravity signal}) = \frac{h}{m_s d_H}, \quad \int_{k_0}^A \left| ds_4 \right| = d_H \]

\( d_H \) is thus behaving as a metric distance. This provides an immediate test of compatibility with the Newtonian gravity.
Again, $d_H$ is a significator of real time as $\bar{Y} = \beta \int_{s_0}^{1} |d s_4| = \beta d_H$. Thus, the trick is made in extremely easy way to show that how $d_H$ works as time and distance as well. The advantage of $\bar{Y}$ is that it is invariant and all observers, no matter which their reference frame, look the same.

In order to compare with Planck energy equation, $E_A$ is written as follows:

$$E_A = h \left( \frac{G^A}{v^2} kA^2 m_s v_g \right).$$

Where, dimension of $\left( \frac{G^A}{v^2} kA^2 m_s v_g \right)$ is the dimension of frequency. $E_A$ is found parallel to the Planck's equation $E = h \nu$.

6. (c) Compatibility with the four-vector Maxwell's equation:

Maxwell's four-vector equation is given by

$$m \frac{d u^\mu}{d \tau} = F^\mu_\gamma q u^\gamma \quad (36)$$

where, $F^\mu_\gamma$ is the electro-magnetic field tensor components.

Four-force equation has been determined in the eqn. (24) in this paper is

$$\frac{\theta^4 \beta}{(1+\theta^2)^{\frac{3}{2}}} \times \frac{1}{v_3^2} \times \frac{G^A}{v^2 t^3} k h^2 \times J^A_\mu = \nabla_\gamma F^\gamma_\mu \quad (37)$$

We note that the eqn. (37) mimics the eqn. (36)

Discussion:
In the beginning of the discussion, the present author would like to give the clarification as to how $v_g$ has been used in framing the space-time metric in eqn. (2) and eqn. (3). An arbitrary parameter is a variable whose value is specified, well, arbitrarily. An example is the calculation of ballistic motion using $g$ with value of 9.8 m/s$^2$. We note that in fundamental physics, the meson theory of Dyson when encountered by Fermi turned out to be pretty much wrong. In his theory, arbitrary parameters include a very sensible ratio i.e., the ratio of the photon mass to the electron mass.

In this paper, it has been understood that $ds_5^2$ in eqn. (2) and $ds_4^2$ in eqn. (3) depend on $v_g = \left(\frac{\hbar}{ms_dH}\right)$ but when evaluating $ds_5^2$ or $ds_4^2$, $v_g$ is held constant. So, it can be considered to be a parameter. Again, when we are interested in the value of $ds_5^2$ or $ds_4^2$ for different values of $v_g$, we then consider $v_g$ to be variable. This is the reason why $v_g$ as arbitrary parameter best fits in the eqn. (2) and the eqn. (3).

As for $\theta$, the argument of the function is invariably called parameter. So, in this paper $\theta$ is used as a parameter.

The interval four-vector and four-gradient operator equation, in the form proposed by the present author, emerge in this paper as the correct mathematical tools that have been used to determine the gravity current and four-force tensor equation.

Many differences have been proposed in this paper with respect to the existing models, with an aim to answer the research question. This is now left to the enthusiastic physicists, mathematicians and the readers to consider the reality of this theory.

We note that the Einstein’s manifold is a pseudo-Riemannian manifold whose Ricci tensor is proportional to the metric and this metric is the solution of the vacuum EFE. Likewise, in this paper pseudo-Riemannian manifold has been used to understand the geometric aspect of this theory.

The underlying difference of this paper from the General Theory of Relativity involves with the signification of the metric. In this paper, the metric also signifies the real time $\tilde{\gamma}$. The present paper holds the position that the constitution of time is complex in nature, and the real time $\tilde{\gamma}$ is associated with the metric distance $d_H$ by the relation $\tilde{\gamma} = \beta \int_{\lambda_0}^{\lambda_1} |ds_4| = \beta d_H$.

In GTR, gravity refers to the curvature of space-time which involves the dynamic interaction between matter and space-time. In this paper, the gravity is the quantum interaction between $\Psi_A$ and $\Psi_h$, taking hyperbolic manifold associated with this interaction. Therefore, this theory is neither a geometry, nor does it depend on mass to tell the gravity.

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References:
