Elementary proof of the Syracuse conjecture
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Abstract:
In application of the Collatz algorithm \((3* + 1)\) we have:
\[ x > 0 \quad , \quad x + V > 0 \quad (x + V \text{ successor of } x, \text{ } V \text{ variable of adjustment}) \]
and \( V < x \) (evaluation of \( V \) is in the main text, in first step \( V = 2 \) and \( x > 2 \)).
And as by hypothesis \( x \) gives a sequence of Syracuse \( S(x) = [x, \ldots, 1] \), \( x \rightarrow 1 \).
We deduce the rules:
\[ (x > 0) \land (V < x) \land (x + V > 0) \implies (0 < x + V < 2x) \]
\[ (0 < x + V < 2x) \implies [(x \rightarrow 1) \implies (x + V \rightarrow 1)] \]
The two sequences \( S(x) \) and \( S(x+V) \) converge to the only trivial cycle: \([4, 2, 1]\).
So by recurrence, every positive integer gives a sequence of Syracuse.

Syracuse conjecture (Collatz conjecture)
Algorithm of Collatz:
Let \( x \) a positive integer number.
1 - if \( x \) is even then \( x := x/2 \)
2 - if \( x \) is odd then \( x := x * 3 + 1 \)
We repeat 1 - 2 until obtain a cycle (is only cycle?) or \( x \) tends to infinity.
The symbol := means : assign value on right to variable on left.

Representation of numbers:
Let \( V \) a variable which, added to variable \( x \), gives the successor \( x + V \).
The variable \( V \) is a variable of adjustment.
Variables \( x \) and \( V \) are written in the form:
\[ x := a^\alpha \text{ with } a := 2^\alpha \text{ and } \alpha \text{ is integer } \geq 0, \text{ } y \text{ is an odd positive variable.} \]
\[ V := b^\beta \text{ with } b := 2^\beta \text{ and } \beta \text{ is integer } \geq 0, \text{ } z \text{ is an odd positive variable.} \]
\[ x + V := a^\alpha \text{ } y + b^\beta \text{ } z. \]
Application of the algorithm of Collatz:
The coefficient $a$ being power of 2, the algorithm is applied to the odd part $y$ of $x := a*(y)$ giving a sequence of Syracuse $S(x) = [x, ... ,1]$ and the odd part $z$ of $V := b*(z)$ is multiplied by 3 plus an adjustment.

In operation $3* + 1$, $x := a*(3*y + 1) = a''*(y')$, $x$ is increased by $(a - 1)$ to subtract from $V$ and we have for $V$ in $x + V$: $V := b*(3*z) - (a-1) = b'*(z')$.

So we have the equality

$$a*(3*y+1) + b*(3*z) - (a-1) = a*(3*y) + 1 + b*(3*z) = 3 * (a*(y) + b*(z)) + 1$$

giving $3*(x + V) + 1$, with $x$ and $V$ of before the operation $3* + 1$, according to the rule 2 of the algorithm.

$a'$ et $b'$ are power of 2 which can be equal to 1, $y'$ and $z'$ are odd numbers.

In the line $a''*(y') + b'*(z')$, $a'$ and $b'$ are divided by gcd($a',b'$) according to the rule 1 of the algorithm.

If gcd($a',b'$) = 1, the division by 2 is deferred.

Evaluation of the variable of adjustment $V$:
When $x$ is multiplied by 3 then $+ 1$, $V$ is multiplied by 3.
When $x$ is divided by 2, $V$ is divided by 2.
When $x = a(3*y+1)$, $x$ is increased by $(a - 1)$, $V$ is decreased by $(a - 1)$.

We deduce that $V$ is always less than $x$.

Conclusion:
In application of the Collatz' algorithm we have:

$x > 0$, $x + V > 0$ and $V < x$ (in first step $V = 2$ and $x > 2$).

And as by hypothesis $x$ gives a sequence of Syracuse $S(x) = [x, ... ,1]$, $x --> 1$.

We deduce the rules:

$$(x > 0) \land (V < x) \land (x + V > 0) \implies (0 < x + V < 2x)$$

$$(0 < x + V < 2x) \implies [(x --> 1) \implies (x + V --> 1)]$$

The two sequences $S(x)$ and $S(x+V)$ converge to the only trivial cycle: $[4, 2, 1]$.

So by recurrence, every positive integer gives a sequence of Syracuse.