

On Goldbach Conjecture and Twin Prime Conjecture

Part one: history, development and doubt

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Abstract

In this paper, we introduce Goldbach Conjecture and Twin Prime Conjecture: history, development, public dissemination in China, and propose doubt about the effectiveness of Analytical Number Theory

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AMS subject classifications. 11P32, 11A41

1 Introduction

Goldbach Conjecture and Twin Prime Conjecture are the most famous, long-standing open problems of number theory. In 1742, Germany mathematician Christian Goldbach (1690-1764) wrote a letter to his friend Leonhard Euler (1707-1783) and proposed that:

- ① Any even number greater than 6 can be expressed as the sum of two odd primes.
- ② Any odd number greater than 9 can be expressed as the sum of three odd primes.

Goldbach could not prove it. So, he asked for Euler's help. After careful consideration, Euler wrote in reply and admit that this was affirmative theorem, though I could not prove it [1].

The Twin Prime Conjecture arose from an open question about the "distribution of prime number". The conjecture states that: "there exist infinitely many primes P such that $P+2$ is a prime". For example, the twin prime 29 and 31 have a gap of $31 - 29 = 2$ " [2].

In 1900, German mathematician D. Hilbert raised 23 problems at the international Mathematical Conference held in Paris. The Goldbach Conjecture and Twin Prime Conjecture were listed as the 8th problems. Since then, tens of thousands of mathematicians all over the world have devoted to solve these problems. However, neither the Goldbach Conjecture nor the Twin Prime Conjecture, despite being absolutely true, have been proved.

2 National enthusiasm for Goldbach Conjecture in China

An article was published on *People's Daily of China* on Feb.17, 1978, and the beginning of the editor's note was "we are excited to recommend readers with the reportage titled 'Goldbach Conjecture' witted by Xu Chi. This article was firstly published on the first number 1978 of *People's Literature*. It faithfully introduces stirring deeds of our famous mathematician Chen Jingrun who is not afraid of difficulties to get on a new step in a lively style, which is warmly received by wide readers." This information was spread over the country to initiate quite a storm in China. Every

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barber knew that. This article presents the details in 2-3 pages, with three impression highlights as follows:

First, when Chen Jingrun in the middle school, his math teacher who came from other place, had a wide range of knowledge and enthusiasm in teaching. He told students about many interesting math knowledge in his class, those students who showed no interest in math were attracted, not to mention those math fans. Someday, the teacher told them Goldbach Conjecture that “every even number not less than 6 was the sum of two odd primes”, this difficult problem fails to be proved for more than two hundred years. The story made the classroom like a boiling water, students were crazy in chattering about this. The teacher said that “math is the queen of nature science; number theory is the crown of math; Goldbach Conjecture is a pearl on the crown.” All students opened their eyes wide in surprise! This miraculous praise gave them lesson with a deep impression including Chen Jingrun, also gave Chinese readers with miraculous impression, never forget. Since then, the term of “Goldbach Conjecture” was cased a mysterious veil on it!

Second, Xiong Qinlqi who was mathematician and educationalist of the older generation had the good sense to recognize Hua Luogeng, and Hua Luogeng found Chen Jingrun with the same sense. In 1957, Hua Luogeng arranged a strategic development in Chinese Academy of Sciences Math Institute that push forward Goldbach Conjecture, a bright pearl on the crown while paying attention to applied mathematical!

Third is the process to the Conjecture by international mathematicians in the number theory aiming at Goldbach Conjecture, call (1+1). In 1920, Norway mathematician Brun proved (9+9); in 1924, Rademacher proved (7+7); in 1932, Estermann proved (6+6); in 1938, A. A. Buchstab proved (5+5), and then he proved (4+4) in 1940; in 1956, I. M. Vinogradov proved (3+3); in 1958, Chinese mathematician Wang Yuan proved (2+3); in 1948, Hungry mathematician Renyi proved (1+6); in 1962, Chinese mathematician Pan Chengdong (1+5), Wang Yuan and Pan Chengdong proved (1+4); in 1965, A. A. Buchstab and I. M. Vinogradov proved (1+3); in 1966, Cheng Jingrun proved (1+2) with hard efforts and challenging spirit, honored by foreigners such as “distinguished achievement”, “splendid theorem”, “glorious top of Sieve Method” etc. to win honor for China. “Now, it is only step to the pearl of the crown, but it is the most difficult step, let’s wait to see who can get it!” Since then, Goldbach Conjecture, a name with mystery is connecting with Chinese, never forget! [3].

3 On Analytic Number Theory: a doubt

Goldbach Conjecture had been open for 280 years. It is a core to develop analytic number theory throughout the 20th century. The analytic number theory includes the sieve method as a foundational method, density theory, circle method, ζ -function, L-function theory, triangle of estimation of linear prime variables, complex analysis and integral transform etc. As the integrated application of these method developments, the main achievements were as follows:

- ① Russia mathematician I. M. Vinogradov proved that a greater odd number can be the sum of three odd primes (called “three primes theorem”);
- ② Chinese mathematician Cheng Jingrun proved that a greater even number can be the sum of a prime and the product of at most two primes (1+2).

However, these methods prove ineffective in (1+1). In 1979, Chinese mathematician Pan Chengdong pointed out that “it seems more difficult to finish this last step than we met before, and we are also not sure that this conjecture could be proved with the present methods. We even fail to give an assumptive proof on (1+1).” The method of analytical number theory is applicable to “greater numbers” with the theoretical formula, but the “greater numbers” are unimaginable. For example, “a greater number” in three-primes-theorem is greater than $e^{e^{16038}}$ (the number with 40 million digits), but to those odd numbers less than it, the theorem is not tenable. In this case,

experts admit that it is solved [4].

As for the Twin Prime Conjecture, Y. Zhang and J. Maynard made the spectacular breakthroughs. Y. Zhang obtained a great attention with his proof that there are infinitely many consecutive primes with a distance of 70,000.000 at most[5] . With the project "PolyMath 8" this bound could be lessened down to 246 respectively to 12 assuming validity of the Elliott-Halberstam Conjecture [6]. Since Twin Prime Conjecture and Goldbach Conjecture have common characters, similar methods are used for the proof of Twin Prime Conjecture[7]. However, they seems ineffective for the final proof.

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