Negative Mass and Negative Refractive Index in Atom Nuclei

Nuclear Wave Equation

Gravitational and Inertial Control

Part 3

Author: Raúl Fattore

University Degree in Electronics Engineering
In This Paper

In this paper, the **nuclear response to external forces** is analyzed with the aim to observe any **changes in the nuclear mass** and study the **behavior of the refractive index** under such changes.

The analysis will be performed in the time domain as well as in the frequency domain by means of the Fast Fourier Transform (FFT) method. The external forces applied to the nucleus were classified into three types:

- The force originated from a polarized transverse electromagnetic wave (TEM)
- The force originated from a polarized TEM plus a static electric field (see Part-4)
- The force originated from a signal plus a static electric field (see Part-5)

Abstract

Some efforts have been made to prove negative mass behavior through some experiments performed in mechanics [1], and other disciplines [9], as well as some theories in electrostatics [2,3,4,5,6,7,8], but I haven’t found research about similar effects in atomic level, where the most elementary mass given by the atomic nucleus is to be found.

- Is the second Newton’s law still valid with negative mass?
- What could happen if we make the atom behave in a negative mass regime?
- Is the negative refractive index related to negative mass?
- Are we able to control the magnitude of mass?
- Are we able to control the sign of mass?

The answers to these questions are given through this series of papers, with results that are coincident with experimental data, except for the negative mass regime. Experiments must be done to confirm or invalidate the theory developed in these articles. Needless to say, if experiments validate this theory, then a significant change in mankind is going to happen. In that case, I strongly ask scientists to cooperate by making use of the derived technologies for good and refrain from doing it for evil.
Introduction
The theory presented in these papers, as described in Part 1, is based on three fundamental aspects that have proved to be extremely effective to describe physical phenomena and predicting results that agree with experimental data [10, 11, 12]:

- Spinning Ring Model of Elementary Particles (toroidal ring of continuous charge)
- New Atomic Model
- The Universal Electrodynamic Force

Based on the new atomic model, a shell arrangement of the nuclear particles has been assumed in Part 1, as shown in Fig. 1.

This sandwich configuration keeps the particles very tightly bound together. Note that at three shells in from the outermost shell, there are always two proton shells in a row for the larger nuclides.

This weak binding allows the outermost sandwich of shells to have liquid-like properties and forms the proper justification for a Liquid Drop Model of the nucleus.

As we already know, the torus ring model of the particles has an associated electric field as well as a magnetic field. However, due to the very tight packing configuration of the particles, we may safely assume that the distance among shells is extremely tiny and that the predominant force in the nucleus is of electrostatic origin, while the weaker magnetic forces will add some contribution to the equilibrium distance between each shell.

As demonstrated in Part 1, mass is an intrinsic property of the atomic nucleus. Under natural circumstances, it has a constant universal magnitude and is always positive. However, with some proper external agents, we might be able to manipulate the intrinsic mass by changing its magnitude and sign.
I. Nuclear Response to Force Caused by a Polarized Transverse Electromagnetic Wave (TEM)

Assume that a plane wave of amplitude $E_m$, frequency $\omega$, and propagation velocity $c$ in the y-direction strikes the outer shell of the "nuclear sphere".

We disregard some minor scattering caused by the few outer electrons in the atom, which are located at a very long distance from the nucleus. The incident wave energy may be totally absorbed, partially absorbed, or not absorbed at all by the nuclear shells.

The momentum density of any wave in vacuum is given by:

$$\frac{dp}{dV} = \frac{E^2}{\mu_0 c^2}.$$  

By replacing $B = \frac{E}{c}$, we get

$$\frac{dp}{dV} = \frac{\mu_0 E^2}{c^2}.$$  

By replacing $c = \frac{E}{c}$, we get

$$\frac{dp}{dV} = \frac{\mu_0 E^2}{c}.$$  

The momentum density as a function of only one of the EM fields is:

$$\frac{dp}{dV} = \frac{\varepsilon_0 E^2}{c}.$$  

Recall that we can write the force as the change in momentum, i.e.,

$$F = \frac{dp}{dt},$$  

so we can write the momentum density as the force exerted by the wave on the nucleus volume:

$$F \frac{dp}{dt} = \frac{\varepsilon_0 E^2}{c} dV \quad (1)$$

Where:

$dt$: time needed by the wave to travel with velocity $c$ (or $v$) across the nucleus diameter ($2r_n$) from $t = 0$ to $t = \frac{2r_n}{c}$ (or $t = \frac{2r_n}{v}$). Calculations will be made for a wave velocity of $c$ and $v$ in the nucleus.

$dV$: the volume element for the spherical nucleus ($r^2 \sin(\theta) dr d\theta d\phi$).

Note that the force given by (1) is calculated in vacuum and contains the vacuum permittivity $\varepsilon_0$.

We ignore the value of the nuclear permittivity of Aluminum. Therefore, the same permittivity will be used for further calculations to be consistent with the Coulomb Force and the Universal Force used in the study.

The polarized wave acting on the nucleus is given by Eq. (21) and (22) in Part-2, whose magnitude is repeated here:

$$E(r, t) = E_m \cos(Kr - \omega t) \quad (2)$$

We assume that the outer shell of the "nuclear sphere" is reached by a plane wavefront traveling in the $+y$-direction with velocity $c$ at $t = 0$. That is, the origin of coordinates is taken at the incident edge of the nucleus, which means the nucleus center is shifted ($r - r_n$).

To calculate the exerted force on the atomic nucleus, we need to integrate Eq. (1). Considering that the force is constant on the time interval, the force integrated on the nucleus volume is:

$$F \int_0^t dt = \int_0^{2\pi} \int_0^\pi \int_0^{r_n} \frac{\varepsilon_0 E^2}{c} (r - r_n)^2 dr d\theta d\phi \quad (3)$$
Defining the time interval for the force integral

How to determine the time interval for integration? Three possibilities have been chosen that might give us a kind of “average force” (though is not an average) on the nuclear volume:

1. The wave’s travel time through the nucleus for wave velocity \( v_w \), that is \( t = 2 \frac{r_n}{v_w} \) (absorption)

2. The wave’s travel time through the nucleus for wave velocity \( c \), that is \( t = 2 \frac{r_n}{c} \) (transmission)

3. The wave period, that is \( t = T = \frac{2\pi}{\omega} \) (absorption/transmission, depends on wave vector’s choice)

1. Integrating the wave force for nuclear travel time, with wave velocity “\( v_w \)”

Replacing the wave Eq. (2) into the integral (3), and setting the time interval for this case,

\[
F \int_0^{2r_n} \frac{d r}{v_w} d t = \frac{\varepsilon_0}{c} \int_0^{2\pi} \int_0^{\pi} \int_0^{r_n} \left( E_m \cos(Kr - \omega t) \right)^2 (r - r_n)^2 \sin(\theta) d r d \theta d \varphi
\]

After integrating and doing some algebra, we get the expression of the wave force on the nucleus for this case:

\[
F_{ext} = \frac{\pi e_0 E_m^2}{6c^3 r_n} \left( 2K^3 r_n^3 + 6K^2 \cos(\omega t) \sin(\omega t) r_n^2 + 6K \cos^2(\omega t) \right) (r - r_n)^2 \sin(\theta) d r d \theta d \varphi
\]

2. Integrating the wave force for nuclear travel time, with wave velocity “\( c \)”

Replacing the wave Eq. (2) into the integral (3), and setting the time interval for this case,

\[
F \int_0^{2r_n} \frac{d r}{c} d t = \frac{\varepsilon_0}{c} \int_0^{2\pi} \int_0^{\pi} \int_0^{r_n} \left( E_m \cos(Kr - \omega t) \right)^2 (r - r_n)^2 \sin(\theta) d r d \theta d \varphi
\]

After integrating and doing some algebra, we get the expression of the wave force on the nucleus for this case:

\[
F_{ext} = \frac{\pi e_0 E_m^2}{6c^3 r_n} \left( 2K^3 r_n^3 + 6K^2 \cos(\omega t) \sin(\omega t) r_n^2 + 6K \cos^2(\omega t) \right) (r - r_n)^2 \sin(\theta) d r d \theta d \varphi
\]

3. Integrating the wave force for a wave period

Replacing the wave Eq. (2) into the integral (3), and setting the time interval for this case,

\[
F \int_0^{2\pi} \frac{d \omega}{\omega} d t = \frac{\varepsilon_0}{c} \int_0^{2\pi} \int_0^{r_n} \left( E_m \cos(Kr - \omega t) \right)^2 (r - r_n)^2 \sin(\theta) d r d \theta d \varphi
\]

After integrating and doing some algebra, we get the expression of the wave force on the nucleus for this case:
\[ F_{\text{ext}} = \frac{e_0 E_0 \omega}{6c K^3} \cdot (2K^3 r_n^3 + 6K^2 \cos(\omega t) \sin(\omega t) r_n^2 + 6K \cos^2(r_n K - \omega t) r_n + 6K \sin^2(r_n K - \omega t) r_n - 6K \sin^2(\omega t) r_n - 6 \cos^2(r_n K - \omega t) \omega t - 6 \sin^2(r_n K - \omega t) \omega t + 6 \cos^2(\omega t) \omega t + 6 \sin^2(\omega t) \omega t - 3r_n K - 3 \cos(r_n K - \omega t) \sin(r_n K - \omega t) - 3 \cos(\omega t) \sin(\omega t)) \]  

(6)

Now that we have the three versions of the external force exerted on the nucleus by a polarized TEM, it’s time to evaluate the nuclear response related to mass and refractive index behaviors.

\textbf{i.a Nuclear Mass Analysis due to Wave Force (4) – Partial or Total Energy Absorption}

The intrinsic net force in the nucleus was already defined with Eq. (23) in Part-1. Now we have the action of an external force acting on the nucleus that will interact with the internal force. By applying Newton’s second law, we have

\[ \Sigma F = m_n \cdot a_{\text{ep}} = F_{\text{net}} + F_{\text{ext}} \quad \Rightarrow \quad m_n \cdot a_{\text{ep}} = F_{\text{net}} + F_{\text{ext}} \]

(7)

By replacing the forces in (7), we obtain the expression of the nuclear mass for this case:

\[ m_n = \frac{1}{a_{\text{ep}}(t)} \cdot \left( \frac{3.410^{12} q^2 (1 - \frac{v^2}{c^2}) + \frac{v^2 p(t)}{c^2} + \frac{v^2 p(t)}{c^2} (2r_{\text{ep}}(t) v_{\text{ep}}(t))}{r_{\text{ep}}(t) v_{\text{ep}}(t)} \right) + \frac{2.0510^{13} q^2}{r_n^2} + \frac{\pi e_0 E_0^2 v_w}{6c K^3 r_n}. \]

(8)

\[ (2K^3 r_n^3 + 6 \cos(\omega t) \sin(\omega t) K^2 r_n^2 + 6K \cos^2(Kr_n - \omega t) r_n + 6 \sin^2(Kr_n - \omega t) Kr_n - 6K \sin^2(\omega t) r_n - 6 \cos^2(Kr_n - \omega t) \omega t - 6 \sin^2(Kr_n - \omega t) \omega t + 6 \sin^2(\omega t) \omega t + 6 \omega t \cos^2(\omega t) - 3Kr_n - 3 \sin(Kr_n - \omega t) \cos(Kr_n - \omega t) - 3 \cos(\omega t) \sin(\omega t)) \]

Recall that:

\[ r_{\text{ep}}(t) = (0.37 r_n + A_r \cos(\omega e t) - A_p \cos(\omega p t)) \]

\[ v_{\text{ep}}(t) = (-A_r \omega_e \sin(\omega e t) + A_p \sin(\omega p t) \omega_p) \]

\[ a_{\text{ep}}(t) = (-A_r \omega_e^2 \cos(\omega e t) + A_p \omega_p^2 \cos(\omega p t)) \]

Some graphs as examples are shown below to have a perception of what could be done to modify the nuclear mass magnitude and sign.

The main parameters used for the net force are:

\[ r_n = 3.5 \times 10^{-15} [m]; \quad A_e = 2 \times 10^{-16} [m]; \quad A_p = 10^{-3} A_e [m]; \quad N_0 = 378; \quad N_z = 312 \]

\[ \omega_e = 10^{15} [\frac{1}{s}]; \quad \omega_p = 10^{16} [\frac{1}{s}] \]

While for the wave: \[ v_w = v_{\text{ep}}(t); \quad K = \frac{\omega}{v_{\text{ep}}(t)} \]
Time Analysis of the Nuclear Mass

Figure 3
Mass magnitude vs. time for Wave param.: $\omega = 10^{10} \left(\frac{1}{s}\right)$ and $E_m = 10^5 \left(\frac{V}{m}\right)$

Figure 4
Mass magnitude vs. time for Wave param.: $\omega = 10^{10} \left(\frac{1}{s}\right)$ and $E_m = 2.5 \times 10^{26} \left(\frac{V}{m}\right)$

Figure 5
Mass magnitude vs. time for Wave param.: $\omega = 3 \times 10^{14} \left(\frac{1}{s}\right)$ and $E_m = 3 \times 10^{26} \left(\frac{V}{m}\right)$

From the period of the mass plot, we determine that the oscillation frequency is approximately:

$f = 1.6 \times 10^{14} \ [Hz]$
Frequency Analysis of the Nuclear Mass with FFT

Total number of samples $N = 2^{14}$, sampling frequency $f_s = 2^3 f_p$ (proton frequency), which gives a frequency resolution $\Delta f = \frac{f_s}{N} = 7.77 \times 10^{11} [Hz]$ and a total acquisition time of $T = \frac{N}{f_s} = 1.28 \times 10^{-12}[s]$. The frequency at the $i$-sample number on the plot is determined by $f = \frac{N(i)}{T} [Hz]$.

Figure 6
Frequency spectrum for Wave param.: $\omega = 10^{10} \frac{1}{s}$ and $E_m = 10^5 \left(\frac{V}{m}\right)$

Figure 7
Phase shift for Wave param.: $\omega = 10^{10} \frac{1}{s}$ and $E_m = 10^5 \left(\frac{V}{m}\right)$
Figure 8
Frequency spectrum for Wave param.: $\omega = 10^{10} \left[ \frac{1}{s} \right]$ and $E_m = 2.5 \times 10^{26} \left[ \frac{V}{m} \right]$

Figure 9
Phase shift for Wave param.: $\omega = 10^{10} \left[ \frac{1}{s} \right]$ and $E_m = 2.5 \times 10^{26} \left[ \frac{V}{m} \right]
From the Fourier frequency analysis, we see that the main frequency is:

\[ f_0 = 1.6 \times 10^{14} \text{ [Hz]} \]

In general, the main frequency and harmonics are given by the following formula:

\[ f_n = f_0 + nf_0, \quad n = 0, 1, 2, 3, \ldots \]
I.b Refractive Index Analysis due to Wave Force (4) – Partial or Total Energy Absorption

When the nucleus is under the action of external forces, and if it doesn’t break apart, then we can assume that a dynamic equilibrium state must exist. Under such circumstances, Newton’s second law requires that the sum of forces be equal to zero, \( \vec{F}_{\text{net}} + \vec{F}_{\text{ext}} = 0 \), that is,

\[
F_{\text{net}} = -F_{\text{ext}} \quad (9)
\]

Recall that the net nuclear force has already been written in terms of the index of refraction in Part-1, Eq. (23a):

\[
\vec{F}_{\text{net}} = -\frac{378k}{r_{\text{ep}}^2(t)} \left(1 - \frac{1}{n^2} + \frac{v_{\text{ep}}^2(t)r_{\text{ep}}(t)a_{\text{ep}}(t) + 1}{n^2c^2} + \frac{2r_{\text{ep}}(t)a_{\text{ep}}(t)}{c^2} \right) \hat{r} + \frac{2279.035793k}{r_n^2} q^2 \hat{r}
\]

Now we can equate the forces according to Eq. (9), then solve for “n”,

\[
-\frac{3.4 \times 10^{12}q^2(1 - \frac{1}{n^2} + \frac{v_{\text{ep}}^2(t)r_{\text{ep}}(t)a_{\text{ep}}(t)}{n^2c^2})}{r_{\text{ep}}^2(t)} + \frac{2.0510^{13}q^2}{r_n^2} = -\frac{\pi \varepsilon_0 E_m v_w}{6c K^2 r_n} \cdot (2K^3 r_n^3 + 6 \cos(\omega t) \sin(\omega t) K^2 r_n^2 + 6K \cos^2(Kr_n - \omega t) r_n + 6 \sin^2(Kr_n - \omega t) Kr_n - 6K \sin^2(\omega t) r_n - 6 \cos^2(Kr_n - \omega t) \omega t - 6 \sin^2(Kr_n - \omega t) \omega t + 6 \sin^2(\omega t) \omega t + 6 \omega t \cos^2(\omega t) - 3Kr_n - 3 \sin(Kr_n - \omega t) \cos(Kr_n - \omega t) - 3 \cos(\omega t) \sin(\omega t))
\]

The refractive index “n” is a somewhat long-expression which is nonsense to copy here. Some plots as examples are shown below, where the main used parameters are:

\[
\begin{align*}
    r_n &= 3.5 \times 10^{-15} [m]; \\
    A_e &= 2 \times 10^{-16} [m]; \\
    A_p &= 10^{-3}A_e [m]; \\
    N_0 &= 378; \\
    N_s &= 312
\end{align*}
\]

\[
\begin{align*}
    \omega_e &= 10^{15} \left[ \frac{1}{s} \right]; \\
    \omega_p &= 10^{16} \left[ \frac{1}{s} \right]
\end{align*}
\]

While for the wave:

\[
\begin{align*}
    v_w &= v_{\text{ep}}(t); \\
    K &= \frac{\omega}{v_{\text{ep}}(t)}
\end{align*}
\]

\[\text{Figure 12} \]

Refractive Index vs. time for Wave param.: \( \omega = 10^{10} \left[ \frac{1}{s} \right] \) and \( E_m = 10^5 \left[ \frac{V}{m} \right] \)
Figure 13
Refractive Index vs. time for Wave param.: $\omega = 10^{10} \left[ \frac{1}{s} \right]$ and $E_m = 2.5 \times 10^{26} \left[ \frac{V}{m} \right]

Figure 14
Refractive Index vs. time for Wave param.: $\omega = 3 \times 10^{14} \left[ \frac{1}{s} \right]$ and $E_m = 3 \times 10^{26} \left[ \frac{V}{m} \right]$
### I.c  Comparison of Mass with Refractive Index Behavior due to Wave Force (4) – Partial or Total Energy Absorption

To analyze the changes in the refractive index “n” with respect to changes in nuclear mass, overlaid graphs of both quantities are shown below, which uncover interesting results.

**Figure 15**
Mass & Refractive Index vs. time for Wave param.: $\omega = 10^{10} \frac{1}{s}$ and $E_m = 10^5 \frac{V}{m}$

**Figure 16**
Mass & Refractive Index vs. time for Wave param.: $\omega = 10^{10} \frac{1}{s}$ and $E_m = 2.5 \times 10^{26} \frac{V}{m}$

**Figure 17**
Mass & Refractive Index vs. time for Wave param.: $\omega = 3 \times 10^{14} \frac{1}{s}$ and $E_m = 3 \times 10^{26} \frac{V}{m}$
The Refractive Index oscillates, and the period switches between \( n=\pm 1 \) at the crossing point of \( m=0 \), as well at some point with mass plot slope =0. It seems that the refractive index depends on the derivative of the mass, by changing the sign at both \( m=0 \) points, that is,

\[
n \propto \pm \frac{dm}{dt}
\]

This is an important result that tells us that the refractive index behavior is like a “beacon”, signaling the zones of the negative mass regime.

### II.a Nuclear Mass Analysis due to Wave Force (5) – Total Energy Transmission

The intrinsic net force in the nucleus was already defined with Eq. (23) in Part-1. Now we have the action of an external force acting on the nucleus that will interact with the internal force. By applying Newton’s second law, we have

\[
\sum F = m_n \cdot a_{ep} = F_{net} + F_{ext} \quad \Rightarrow \quad m_n \cdot a_{ep} = F_{net} + F_{ext}
\]

\[
m_n = \left\{ \frac{1}{a_{ep}} \left( F_{net} + F_{ext} \right) \right\}_{left} \right.
\]

By replacing the forces in (11), we obtain the expression of the nuclear mass for this case:

\[
m_n = \frac{1}{a_{ep(t)}} \left( -\frac{3.410^{12} q^2}{c^4} \left( \frac{v_{ep(t)}^2}{c^2} + \frac{v_{ep(t)}^2}{c^2} \right) \frac{a_{ep(t)}}{c^4} + \frac{2.0510^{13} q^2}{c^4} + \frac{\pi e_0 E_m^2}{6K^3} \right) + \frac{v_{ep(t)}}{c^4} - \frac{r_{ep(t)}}{c^4}
\]

\[
(2K^3 r_n^3 + 6K^2 \cos(\omega t) \sin(\omega t) r_n^2 + 6K \cos^2(r_n K - \omega t) r_n - 6K \sin^2(\omega t) r_n - 6 \cos^2(r_n K - \omega t) \omega t - 6 \sin^2(r_n K - \omega t) \omega t + 6 \cos^2(\omega t) \omega t + 6 \sin^2(\omega t) \omega t - 3r_n K -
\]

\[
3 \cos(r_n K - \omega t) \sin(r_n K - \omega t) - 3 \cos(\omega t) \sin(\omega t))
\]

Recall that:

\[
r_{ep(t)} = (0.37r_n + A_e \cos(\omega_e t) - A_p \cos(\omega_p t))
\]

\[
v_{ep(t)} = (-A_e \omega_e \sin(\omega_e t) + A_p \sin(\omega_p t) \omega_p)
\]

\[
a_{ep(t)} = (-A_e \omega_e^2 \cos(\omega_e t) + A_p \omega_p^2 \cos(\omega_p t))
\]

Some graphs as examples are shown below to have a perception of what could be done to modify the nuclear mass magnitude and sign.

The main parameters used for the net force are:

\[
r_n = 3.5 \times 10^{-15} \text{[m]}; \quad A_e = 2 \times 10^{-16} \text{[m]}; \quad A_p = 10^{-3} A_e \text{[m]}; \quad N_0 = 378; \quad N_s = 312;
\]

\[
\omega_e = 10^{15} \text{[1/s]}; \quad \omega_p = 10^{16} \text{[1/s]}
\]

While for the wave:

\[
K = \frac{\omega}{c}
\]
**Time Analysis of the Nuclear Mass**

From the period of the mass plot, we determine that the oscillation frequency is approximately:

$$ f = 1.6 \times 10^{14} \ [Hz] $$
Frequency Analysis of the Nuclear Mass with FFT

Total number of samples $N = 2^{13}$, sampling frequency $f_s = 2^3 \times 3 f_p$ (proton frequency), which gives a frequency resolution $\Delta f = \frac{f_s}{N} = 4.66 \times 10^{12} [Hz]$ and a total acquisition time of $T = \frac{N}{f_s} = 2.14 \times 10^{-13} [s]$. The frequency at the $i$-sample number on the plot is determined by $f = \frac{N(i)}{T} [Hz]$.

Figure 21
Frequency spectrum for Wave param.: $\omega = 10^{15} \left[ \frac{1}{s} \right]$ and $E_m = 10^5 \left[ \frac{V}{m} \right]$

Figure 22
Phase shift for Wave param.: $\omega = 10^{15} \left[ \frac{1}{s} \right]$ and $E_m = 10^5 \left[ \frac{V}{m} \right]$
Figure 23
Frequency spectrum for Wave param.: $\omega = 10^{15} \frac{1}{s}$ and $E_m = 1.6 \times 10^{16} \frac{V}{m}$

Figure 24
Phase shift for Wave param.: $\omega = 10^{15} \frac{1}{s}$ and $E_m = 1.6 \times 10^{16} \frac{V}{m}$
From the Fourier frequency analysis, we see that the main frequency is:

\[ f_0 = 1.6 \times 10^{14} \text{ [Hz]} \]

In general, the main frequency and harmonics are given by the following formula:

\[ f_n = f_0 + nf_0, \quad n = 0, 1, 2, 3... \]
II.b Refractive Index Analysis due to Wave Force (5) – Total Energy Transmission

When the nucleus is under the action of external forces, and if it doesn’t break apart, then we can assume that a dynamic equilibrium state must exist. Under such circumstances, Newton’s second law requires that the sum of forces be equal to zero, \( \vec{F}_{\text{net}} + \vec{F}_{\text{ext}} = 0 \), that is,

\[
F_{\text{net}} = -F_{\text{ext}} \quad (13)
\]

Recall that the net nuclear force has already been written in terms of the index of refraction in Part-1, Eq. (23a):

\[
\vec{F}_{\text{net}} = -\frac{378k q^2}{r_{\text{ep}}^2(t)} \left( 1 - \frac{1}{n^2} + \frac{v_{\text{ep}}^2(t) a_{\text{ep}}(t)}{n^2 c^2} + \frac{1}{n^4} + \frac{2r_{\text{ep}}(t) a_{\text{ep}}(t)}{c^2} \right) \hat{r} + \frac{2279.035793k q^2 \dot{r}}{r_n^2}
\]

Now we can equate the forces according to Eq. (13), then solve for “n”,

\[
-3.4 \times 10^{12} q^2 \left( 1 - \frac{1}{n^2} + \frac{v_{\text{ep}}^2(t) a_{\text{ep}}(t)}{n^2 c^2} + \frac{1}{n^4} + \frac{2r_{\text{ep}}(t) a_{\text{ep}}(t)}{c^2} \right) \frac{\pi \varepsilon_0 E_m^2}{6K^3 n^3} = -\frac{\pi \varepsilon_0 E_m^2}{6K^3 n^3} \cdot (2K^3 n^3 + 6K^2 \cos(\omega t) \sin(\omega t) r_n^2 + 6K \cos(r_n K - \omega t) r_n - 6K \sin^2(r_n K - \omega t) r_n - 6K \cos^2(r_n K - \omega t) \omega t - 6 \sin^2(r_n K - \omega t) \omega t + 6 \cos^2(\omega t) \omega t + 6 \sin^2(\omega t) \omega t - 3r_n K - 3 \cos(r_n K - \omega t) \sin(r_n K - \omega t) - 3 \cos(\omega t) \sin(\omega t))
\]

The refractive index “n” is a somewhat long-expression which is nonsense to copy here. Some plots as examples are shown below, where the main used parameters are:

\[
r_n = 3.5 \times 10^{-15} [m]; \quad A_e = 2.1 \times 10^{-16} [m]; \quad A_p = 10^{-3} A_e [m]; \quad N_0 = 378; \quad N_s = 312
\]

\[
\omega_e = 10^{15} \left[ \frac{1}{s} \right]; \quad \omega_p = 10^{16} \left[ \frac{1}{s} \right]
\]

While for the wave:

\[
K = \frac{\omega}{c}
\]

Figure 27
Refractive Index vs. time for Wave param.: \( \omega = 10^{15} \left[ \frac{1}{s} \right] \) and \( E_m = 10^5 \left[ \frac{1}{m} \right] \)
Figure 28
Refractive Index vs. time for Wave param.: $\omega = 10^{15} \left( \frac{\hat{Z}}{s} \right)$ and $E_m = 1.6 \times 10^{16} \left( V_m \right)$

Figure 29
Refractive Index vs. time for Wave param.: $\omega = 10^{15} \left( \frac{\hat{Z}}{s} \right)$ and $E_m = 10^{20} \left( V_m \right)$
II.c Comparison of Mass with Refractive Index Behavior due to Wave Force (5) – Total Energy Transmission

To analyze the changes in the refractive index “n” with respect to changes in nuclear mass, overlaid graphs of both quantities are shown below, which uncover interesting results.

**Figure 30**
Mass & Refractive Index vs. time for Wave param.: $\omega = 10^{15} \frac{1}{s}$ and $E_m = 10^5 \frac{V}{m}$

**Figure 31**
Mass & Refractive Index vs. time for Wave param.: $\omega = 10^{15} \frac{1}{s}$ and $E_m = 1.6 \times 10^{16} \frac{V}{m}$

**Figure 32**
Mass & Refractive Index vs. time for Wave param.: $\omega = 10^{15} \frac{1}{s}$ and $E_m = 10^{20} \frac{V}{m}$
The Refractive Index depends on the slope of the mass plot. Here, it decreases with negative mass slope, and vice versa, until reaching the m=0 point, where abruptly switches between n=±1. It seems that the Refractive Index is proportional to the derivative of the mass, and the sign of the derivative changes when the mass sign changes.

\[ n \propto \pm \frac{dm}{dt} \]

This is an important result that tells us that the refractive index behavior is like a “beacon”, signaling the zones of the negative mass regime.

### III.a Nuclear Mass Analysis due to Wave Force (6) – Energy Absorption/Transmission

The intrinsic net force in the nucleus was already defined with Eq. (23) in Part-1. Now we have the action of an external force acting on the nucleus that will interact with the internal force. By applying Newton’s second law, we have

\[ \sum F = m_n \cdot a_{ep} = F_{net} + F_{ext} \quad \Rightarrow \quad m_n \cdot a_{ep} = F_{net} + F_{ext} \]

\[ m_n = \frac{1}{a_{ep}} \cdot (F_{net} + F_{ext}) \quad (14) \]

By replacing the forces in (14), we obtain the expression of the nuclear mass for this case:

\[ m_n = \left( \frac{3.410^{12} \cdot q^2 \cdot \left( 1 - \frac{v_{ep}^2(t)}{c^2} + \frac{v_{ep}^2(t) r_{ep}(t) a_{ep}(t)}{c^4} + \frac{2 r_{ep}(t) a_{ep}(t)}{c^2} \right)}{r_{ep}^2(t)} \right) + \frac{2.0510^{13} \cdot q^2}{r_n^2} + \frac{\varepsilon_0 \varepsilon_\lambda \omega}{6c K^3}. \]

\[ (2K^2 r_n^3 + 6K^2 \cos(\omega t) \sin(\omega t) r_n^2 + 6K \cos^2(r_n K - \omega t) r_n + 6K \sin^2(r_n K - \omega t) r_n - 6 \cos^2(r_n K - \omega t) \omega - 6 \sin^2(r_n K - \omega t) \omega + 6 \cos^2(\omega t) \omega t + 6 \sin^2(\omega t) \omega t - 3r_n K - \]

\[ 3 \cos(r_n K - \omega t) \sin(r_n K - \omega t) - 3 \cos(\omega t) \sin(\omega t) \right) \]

Recall that:

\[ r_{ep}(t) = (0.37 r_n + A_e \cos(\omega_e t) - A_p \cos(\omega_p t)) \]

\[ v_{ep}(t) = (-A_e \omega_e \sin(\omega_e t) + A_p \sin(\omega_p t) \omega_p) \]

\[ a_{ep}(t) = (-A_e \omega_e^2 \cos(\omega_e t) + A_p \omega_p^2 \cos(\omega_p t)) \]

Some graphs as examples are shown below to have a perception of what could be done to modify the nuclear mass magnitude and sign.

The main parameters used for the net force are:

\[ r_n = 3.5 \times 10^{-15} [m]; \quad A_e = 2 \times 10^{-16} [m]; \quad A_p = 10^{-3} A_e [m]; \quad N_0 = 378; \quad N_3 = 312 \]

\[ \omega_e = 10^{15} \left[ \frac{1}{s} \right]; \quad \omega_p = 10^{16} \left[ \frac{1}{s} \right] \]

While for the wave:

\[ K = \frac{\omega}{c} \quad \text{and} \quad K = \frac{\omega}{v_{ep}(t)} \]
Time Analysis of the Nuclear Mass for Wave Velocity “c” and “$v_{ep}(t)$”

From the period of the mass plot, we determine that the oscillation frequency is approximately:

$$f = 1.6 \times 10^{14} \ [Hz]$$
Frequency Analysis of the Nuclear Mass with FFT for Wave Velocity “c”

Total number of samples $N = 2^{14}$, sampling frequency $f_s = 2^3 f_p$ (proton frequency), which gives a frequency resolution $\Delta f = \frac{f_s}{N} = 7.77 \times 10^{11} \text{Hz}$ and a total acquisition time of $T = \frac{N}{f_s} = 1.28 \times 10^{-12} \text{s}$.

The frequency at the $i$-sample number on the plot is determined by $f = \frac{N(i)}{T} \text{Hz}$.

From the Fourier frequency analysis, we see that the main frequency is:

$f_0 = 1.6 \times 10^{14} \text{Hz}$

In general, the main frequency and harmonics are given by the following formula:

$f_n = f_0 + nf_0, \quad n = 0, 1, 2, 3..$
**Frequency Analysis of the Nuclear Mass with FFT for Wave Velocity “v\text{ep}(t)”**

Total number of samples $N = 2^{14}$, sampling frequency $f_s = 2^3 f_p$ (proton frequency), which gives a frequency resolution $\Delta f = \frac{f_s}{N} = 7.77 \times 10^{11} \text{[Hz]}$ and a total acquisition time of $T = \frac{N}{f_s} = 1.28 \times 10^{-12} \text{[s]}$.

The frequency at the $i$-sample number on the plot is determined by $f = \frac{N(i)}{T} \text{[Hz]}$.

From the Fourier frequency analysis, we see that the main frequency is:

$f_0 = 1.6 \times 10^{14} \text{[Hz]}$

In general, the main frequency and harmonics are given by the following formula:

$f_n = f_0 + nf_0, \quad n = 0, 1, 2, 3..$
Refractive Index Analysis due to Wave Force (6) for Wave Velocity “c” and “v_{ep}(t)”

When the nucleus is under the action of external forces, and if it doesn’t break apart, then we can assume that a dynamic equilibrium state must exist. Under such circumstances, Newton’s second law requires that the sum of forces be equal to zero, \( \vec{F}_{net} + \vec{F}_{ext} = 0 \), that is,

\[
F_{\text{net}} = -F_{\text{ext}} \quad (16)
\]

Recall that the net nuclear force has already been written in terms of the index of refraction in Part-1, Eq. (23a):

\[
\vec{F}_{\text{net}} = -\frac{378k q^2}{r_{\text{ep}}^2(t)} \left( 1 - \frac{1}{n^2} + \frac{v_{\text{ep}}^2(t)r_{\text{ep}}(t)\alpha_{\text{ep}}(t)}{n^2 c^2} + \frac{1}{n^4} + \frac{2r_{\text{ep}}(t)\alpha_{\text{ep}}(t)}{c^2} \right) \vec{r} + \frac{2279.035793k q^2 \epsilon}{r_{\text{n}}^2}
\]

Now we can equate the forces according to Eq. (16), then solve for “n”,

\[
-3.4 \times 10^{12} q^2 \left( 1 - \frac{1}{n^2} + \frac{v_{\text{ep}}^2(t)r_{\text{ep}}(t)\alpha_{\text{ep}}(t)}{n^2 c^2} + \frac{1}{n^4} + \frac{2r_{\text{ep}}(t)\alpha_{\text{ep}}(t)}{c^2} \right) \epsilon = -\frac{\epsilon_0 E_{\text{m}}^2 \omega}{6c K^3} \cdot (2K^3 r_{\text{n}}^3 + 6K^2 \cos(\omega t) \sin(\omega t) r_{\text{n}}^2 + 6K \cos^2(r_{\text{n}}K - \omega t) r_{\text{n}} + 6K \sin^2(r_{\text{n}}K - \omega t) r_{\text{n}} - 6K \sin^2(\omega t) r_{\text{n}} - 6 \cos^2(r_{\text{n}}K - \omega t) \omega t - 6 \sin^2(r_{\text{n}}K - \omega t) \omega t + 6 \cos^2(\omega t) \omega t + 6 \sin^2(\omega t) \omega t - 3r_{\text{n}}K - 3 \cos(r_{\text{n}}K - \omega t) \sin(r_{\text{n}}K - \omega t) - 3 \cos(\omega t) \sin(\omega t))
\]

The refractive index “n” is a somewhat long-expression which is nonsense to copy here. Some plots as examples are shown below, where the main used parameters are:

\[
\begin{align*}
  r_{\text{n}} &= 3.5 \times 10^{-15} [\text{m}] ; & A_e &= 2.1 \times 10^{-16} [\text{m}] ; & A_p &= 10^{-3} A_e [\text{m}] ; & N_0 &= 378 ; & N_s &= 312 \\
  \omega_e &= 10^{15} [\text{s}^{-1}] ; & \omega_p &= 10^{16} [\text{s}^{-1}] \\
  \omega &= 10^{15} [\text{s}^{-1}] \quad \text{and} \quad K = \frac{\omega}{c} \quad \text{and} \quad K = \frac{\omega}{v_{\text{ep}}(t)}
\end{align*}
\]

Refractive Index vs. time for Wave param.: 
\[
\omega = 10^{15} [\text{s}^{-1}] \quad \text{and} \quad E_{\text{m}} = 10^5 [\text{V} / \text{m}]
\]

Refractive Index vs. time for Wave param.: 
\[
\omega = 10^{10} [\text{s}^{-1}] \quad \text{and} \quad E_{\text{m}} = 10^5 [\text{V} / \text{m}]
\]

Figure 42

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\[ K = \frac{\omega}{c} \]

Refractive Index vs. time for Wave param.: \[ \omega = 10^{15} \left( \frac{1}{s} \right) \text{ and } E_m = 3 \times 10^{20} \left( \frac{V}{m} \right) \]

\[ K = \frac{\omega}{V_{ep}(\xi)} \]

Refractive Index vs. time for Wave param.: \[ \omega = 10^{16} \left( \frac{1}{s} \right) \text{ and } E_m = 10^{26} \left( \frac{V}{m} \right) \]

\[ K = \frac{\omega}{c} \]

Refractive Index vs. time for Wave param.: \[ \omega = 10^{15} \left( \frac{1}{s} \right) \text{ and } E_m = 10^{23} \left( \frac{V}{m} \right) \]

\[ K = \frac{\omega}{V_{ep}(\xi)} \]

Refractive Index vs. time for Wave param.: \[ \omega = 3 \times 10^{14} \left( \frac{1}{s} \right) \text{ and } E_m = 3 \times 10^{26} \left( \frac{V}{m} \right) \]
III.c Comparison of Mass with Refractive Index Behavior due to Wave Force (6) for Wave Velocity “c” and “vep(t)”

To analyze the changes in the refractive index “n” with respect to changes in nuclear mass, overlaid graphs of both quantities are shown below, which uncover interesting results.

$$K = \frac{\omega}{c}$$

Mass & Refractive Index vs. time for Wave param.: 
$$\omega = 10^{15} \left[ \frac{1}{s} \right] \text{ and } E_m = 10^5 \left[ \frac{V}{m} \right]$$

$$K = \frac{\omega}{vep(t)}$$

Mass & Refractive Index vs. time for Wave param.: 
$$\omega = 10^{10} \left[ \frac{1}{s} \right] \text{ and } E_m = 10^5 \left[ \frac{V}{m} \right]$$

$$K = \frac{\omega}{c}$$

Mass & Refractive Index vs. time for Wave param.: 
$$\omega = 10^{15} \left[ \frac{1}{s} \right] \text{ and } E_m = 3 \times 10^{20} \left[ \frac{V}{m} \right]$$

$$K = \frac{\omega}{vep(t)}$$

Mass & Refractive Index vs. time for Wave param.: 
$$\omega = 10^{16} \left[ \frac{1}{s} \right] \text{ and } E_m = 10^{26} \left[ \frac{V}{m} \right]$$

$$K = \frac{\omega}{c}$$

Mass & Refractive Index vs. time for Wave param.: 
$$\omega = 10^{15} \left[ \frac{1}{s} \right] \text{ and } E_m = 10^{23} \left[ \frac{V}{m} \right]$$

$$K = \frac{\omega}{vep(t)}$$

Mass & Refractive Index vs. time for Wave param.: 
$$\omega = 3 \times 10^{14} \left[ \frac{1}{s} \right] \text{ and } E_m = 3 \times 10^{26} \left[ \frac{V}{m} \right]$$
The Refractive Index depends on the slope of the mass plot. For wave velocity “c”, it decreases with negative mass slope, and vice versa, until reaching m=0 point, where abruptly switches between n=±1. It seems that the Refractive Index is proportional to the derivative of the mass, and the sign of the derivative changes when the mass sign changes.

For wave velocity “\(v_{ep}(t)\)”, the Refractive Index oscillates and switches between n=±1 at m=0 and also when the slope of the mass =0. It seems that the refractive index depends on the derivative of the mass.

\[ n \propto \pm \frac{dm}{dt} \]

These are important results that tell us that the refractive index behavior is like a “beacon”, signaling the zones of the negative mass regime.

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**Conclusions**

It has been demonstrated that the application of the Universal Electrodynamics Force to the new Atomic Model predicts important changes in nuclear mass when an external force caused by a TEM is acting on the atomic nucleus.

As we have demonstrated in previous papers, mass is an electrodynamic quantity and as such, it can be manipulated at will. It was demonstrated in this paper that one means to achieve mass changes is by striking the nucleus with a TEM.

It was clearly seen from the results and corresponding graphs, that the magnitude and the sign of the mass can be modified by changing the amplitude and/or frequency of the external wave, within a certain range.

The somewhat abrupt sign change of the refractive index values during mass sign change was also clearly demonstrated. There is clear evidence that the refractive index is proportional to the mass change with time, i.e., to the derivative of mass with respect to time.

The refractive index can be used as a tool to discover the negative mass region of the nucleus, as well as in any piece of “macro” material. The refractive index is a “beacon” that signals the exact point of mass sign change and its range.

Fourier's analysis show in the phase shift graphs many swings of phase between ±\(\pi\), which clearly indicate resonance states in the nucleus at those frequencies, as well interferences with the external agent.

In previous papers, we described that the main force that keeps the nuclear shell in a very tight packing structure in such a tiny space is the electrostatic force. This is really an enormous force. It means that we also need huge external electromagnetic fields to achieve some nuclear interaction, and this may represent a technical limitation in the present time.
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