Using Euler’s Identity to Prove the Existence of Natural Logarithms of Numbers Approaching 0\(^+\) on the Complex Plane

Shikhar Sehgal

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Abstract
This paper provides an overview of using Euler's identity to prove that natural logarithms of numbers approaching zero exist on the complex plane.

\[ e^{ix} = -1 \] (by Euler’s identity)

Hence, \( \ln (-1) = i\pi \),

Which means: \( \ln (0-1) = i\pi \)

We know that \( \ln(a-b) = \ln(a(1-b/a)) = \ln a + \ln (1-b/a) \)

Hence,
\[ \ln(x(1-1/x)) = \ln x + \ln (1-1/x) = i\pi \]

When \( x \to 0 \)

Hence,
\[ \ln x = i\pi - \ln (1-1/x) \text{ as } x \to 0^+ \] \( \ldots 1 \)