The Hexagon

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Abstract

We provide coordinates of a regular hexagon

There are some general facts about a regular n-gon, see [1] or [2], but we have found only one website about a *regular 6-gon* or *hexagon*.

There are approximate coordinates of a hexagon in the German wikipedia, see [3]. The exact values of a hexagon appear as a mystery. To our great surprise, we have found only one more information. It seems that the first who calculated the coordinates of a hexagon was the Indian Gopal Menon. See [4]. He yields formulas for a hexagon depending on a number a. Here are exact values if $a = 2$. It holds that

$$(-2, 0); \left(-1, +\sqrt{3}\right); \left(+1, +\sqrt{3}\right); (+2, 0); \left(+1, -\sqrt{3}\right); \left(-1, -\sqrt{3}\right)$$

are coordinates for a horizontal hexagon, and

$$(0, +2); \left(+\sqrt{3}, +1\right); \left(+\sqrt{3}, -1\right); (0, -2); \left(-\sqrt{3}, -1\right); \left(-\sqrt{3}, +1\right)$$

are coordinates of an upright hexagon.

Proof. An easy calculation shows that in the horizontal 6-gon the six interior angles have 120 degrees. All edgelengths are 2. The proof is done. $\square$

We have made the coodinates as simple as possible. The calculation of the above coordinates without [4] would be easy, if one knows that a hexagon consists of six equilateral triangles.

Now we make a conjecture about regular polygons.

Definition
A regular n-gon is a polygon with n vertices such that the edges have the same length, and all inner angles are equal.

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Conjecture
Let $n > 2$ be a natural number, where n is not 4. It holds:
An n-gon is regular if and only if all inner angles are equal.

References

[1] https://math.stackexchange.com/q/1982828

[2] https://math.stackexchange.com/q/117164

[3] https://de.wikipedia.org/wiki/Sechseck

[4] https://www.quora.com/How-can-you-find-the-coordinates-in-a-hexagon