Abstract

Investigation is tried about approach to Collatz conjecture and its integer space.

1. Introduction

Collatz conjecture operation is repeating following calculation regarding to positive odd integer (POI) $n_i$ until $n_{i+1} = 1$.

$$n_{i+1} = \frac{3n_i + 1}{2^{m_i}}, \quad i = 1, 2, 3, \ldots$$

(1)

Here dividing by $2^{m_i}$ means dividing by possible max. power of 2 to make POI.

Meaning of this formula is:

Based on binary floating-point representation, calculation $3n_i + 1$ is done using integer mantissa $n_i$. Dividing by $2^{m_i}$ is to normalize $n_i$ and to get new integer mantissa $n_{i+1}$. Integer mantissa: It is mantissa whose part is integer.

2. Characteristic of Collatz Conjecture operation

Regarding to arithmetic procedure, operation generally might be done regard to variables and how to operate is represented using formulas. But actually, sometimes operation could be different procedure depending to the value in the variables.

For example, $m_i$ of (1) is repeating times of dividing by 2 and this procedure depends on the value of $n_i$. In such case, operation is different depending on each $n_i$ value.

Therefore, the difficulty to mention summarized result of Collatz Conjecture in general may be on such point.

Regarding to Collatz Conjecture, main theme to be resolved is following two issues.

1) All series of the POI is terminated at 1.
2) All series of the POI have no looping in it.

Each of two themes may be able to be approached independently. For the approach, there could be such methods as arithmetical, statistical, logical, topological and etc.

For example,

Report1 is arithmetically for 2). *1
Report2 is statistically for 1). *2
Report3 is logically for 2). *3
This report is statistically for 1).
3. Collatz Conjecture POI Space

Above (1) can be rewritten.

\[ n' = \frac{3n+1}{2^m} \quad (2) \]

\( m \): possible max. power number of 2, \( n \): current POI of Collatz Conjecture POI series

About (2), relation between \( m \) and \( n \) is represented as follow table1 (\( m \leq 6 \)).

<table>
<thead>
<tr>
<th>( m )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 4l + 3 )</td>
</tr>
<tr>
<td>2</td>
<td>( 8l + 1 )</td>
</tr>
<tr>
<td>3</td>
<td>( 16l + 13 )</td>
</tr>
<tr>
<td>4</td>
<td>( 32l + 5 )</td>
</tr>
<tr>
<td>5</td>
<td>( 64l + 53 )</td>
</tr>
<tr>
<td>6</td>
<td>( 128l + 21 )</td>
</tr>
</tbody>
</table>

Table1

\( l \) is positive integer.

For example,

in the case \( m = 1 \), \( n \) is 7 (\( 4 \times 1 + 3 \)) when \( l = 1 \),

in the case \( m = 6 \), \( n \) is 21 (\( 128 \times 0 + 21 \)) when \( l = 0 \).

In general, a value \( m \) and POI \( n \)'s have following relation (3).

\[ n = 2^{m+1}l + c \quad (3) \]

\( l \): positive integer

\( c \) is odd integer constant calculated on (4) using minimum POI \( k \) which satisfies (4) without remain.

\[ c = \frac{2^{m}k-1}{3} \quad (4) \]

Based on (3), occurrence rate of \( n \) in the case of \( m \) is following.

Occurrence rate to positive integers: \( \frac{1}{2^{m+1}} \quad (5) \)

Occurrence rate to positive odd integers: \( \frac{1}{2^m} \quad (6) \)

Therefore, expected value for \( m \) is

\[ \frac{m}{2^m} \quad (7) \]

Total expected value for whole POI space is

\[ e = \sum_{m=1}^{\infty} \frac{m}{2^m} \quad (8) \]

\( e \) from 1 to \( j \) is

\[ \sum_{m=1}^{j} \frac{m}{2^m} = 2 \left( \frac{1}{2^0} - \frac{j+1}{2^j} + \frac{j}{2^{j+1}} \right) \quad (9) \]
When \( j = \infty \), \( e \) is
\[
e = \sum_{m=1}^{\infty} \frac{m}{2^m} = 2. \quad (10)
\]

4. Collatz Conjecture operation

Collatz Conjecture operation is doing (11) (*1, 6. (4) page6.)
\[
n_i = \frac{3^n}{2^{m_1+m_2+\cdots+m_l}} + \frac{3^{l-1}}{2^{m_1+m_2+\cdots+m_l}} + \frac{3^{l-2}}{2^{m_2+m_3+\cdots+m_l}} + \cdots + \frac{3^1}{2^{m_{l-1}+m_l}} + \frac{3^0}{2^{m_l}} \quad (11)
\]

n: initial value POI

One of the space characteristics is that m is distributed periodically or equalized in the space based on (4).

(a)
Selection of \( n_1 \) and \( n_i \) is arbitrary or at random.

(b)
Based on (a)(b), according to increasing of \( i \), \( 2^{m_1+m_2+\cdots+m_l} \) is reaching to \( 2^{e_i} \). Therefore, in order to calculate targeted value, \( m_i \) is replaced by expected value e. and \( \alpha \) is defined.
\[
\alpha = \frac{3}{2^e} \quad (12)
\]

Then (11) is
\[
n_i = \alpha^i n + \frac{1}{2^e} (\alpha^{i-1} + \alpha^{i-2} + \cdots + \alpha^1 + \alpha^0). \quad (13)
\]

Based on (10), according to increasing of \( i \), \( \alpha^i \) is reaching to 0, then \( n_i \) is reaching to 1. This is estimated target value and Collatz Conjecture operation is reaching toward it, or never stop until reaching to it.

5. Conclusion

Investigation for the characteristic of Collatz Conjecture POI space shows its every POI series is reaching toward 1 on the assumption that sufficient times unique trial (=no looping) can be done.

References

*1 https://vixra.org/abs/2302.0015
*2 https://vixra.org/abs/2204.0151
*3 https://vixra.org/abs/2304.0070