QED analysis of the two photons -- into -- a paraphoton inelastic scattering

V.A. Moiseeva and V.V. Filatov
Bauman Moscow State Technical University, Russia

*E-mail: vvfilatov@bmstu.ru

Abstract. Paraphoton is the axion-like chameleon (no fixed rest-mass) particle introduced to the quantum chromodynamics to solve the strong CP-problem. Nowadays, it is the best candidate to be the quantum of the dark matter. In the paper, we present the quantum analysis of the two-photons process resulting to the paraphoton formation.

1. Introduction
The recent experiments on the Large Hadron Collider [1] discover the elastic photon-photon scattering $\gamma + \gamma \rightarrow \gamma + \gamma$. This way, there is also possible the inelastic process (fig.1) combines two photons to the paraphoton [2] by the channel $\gamma + \gamma \rightarrow a$. The outcome $a$ is a massive boson with spin $s_a = 2$ and "chameleon" mass $m_a = 2h\nu/c^2$, which interacts neither the "strong" nor a "weak" way but potentially reveals itself in the neutrino experiments [3]. Because of its symmetry paraphoton is also the best candidate to be the gravitational gauge boson [4] which is probably form the dark matter. These all make the process $\gamma + \gamma \rightarrow a$ very important, and the paper performs its quantum analysis.

Figure 1. Feynman diagram for the process of inelastic interaction of two photons $\gamma + \gamma \rightarrow a$
2. Analysis

The analysis can be performed by the standard routine, described in [5]. Hereinafter the paper uses the nuclear system of units (Dirac constant is 1, speed of light is 1) and 4-vectors.

Let \( k_1 \) and \( k_2 \) be photon momenta (see fig.1), \( \lambda_1 \) and \( \lambda_2 \) its polarizations and \( q \) the momentum of paraphoton.

The initial state of the system (two photons) is

\[
|k_1\lambda_1, k_2\lambda_2\rangle = a^+_{\lambda_1}(k_1)a^+_{\lambda_2}(k_2)|0\rangle
\]

(1)

where \( |0\rangle \) is the ground state of a vacuum, \( a^+ \) is the photon "birth" operator.

The final state of the system (paraphoton) is \( |q\rangle \).

This way, the scattering matrix is following:

\[
\langle q|S|k_1\lambda_1, k_2\lambda_2\rangle = e^{i\lambda_1(k_1)\epsilon^\lambda_2(k_2)M_{\mu\nu}(k_1, k_2; q)(2\pi)^4\delta(k_1 + k_2 - q)}
\]

(2)

Here \( \epsilon^{\lambda_1}(k_1) \) and \( \epsilon^{\lambda_2}(k_2) \) are 4-polarizations of photons, \( e \) is the elementary charge, \( \delta \) is the Dirac \( \delta \)-function and \( M_{\mu\nu}(k_1, k_2; q) \) is unknown because of no good theory nowadays. Meanwhile, the analysis of the symmetry can get it with the single unknown constant.

To obtain \( M_{\mu\nu}(k_1, k_2; q) \), lets start from the evident commutativity

\[
[a^+_{\lambda_1}(k_1), a^+_{\lambda_2}(k_2)] = 0
\]

(3)

which gets

\[
\langle q|S|k_1\lambda_1, k_2\lambda_2\rangle = \langle q|S|k_2\lambda_2, k_1\lambda_1\rangle
\]

(4)

means that photons must be identical.

This way,

\[
M_{\mu\nu}(k_1, k_2; q) = M_{\nu\mu}(k_2, k_1; q)
\]

(5)

Next, Lorentz- and inversion-invariances makes \( M_{\mu\nu}(k_1, k_2; q) \) be a pseudotensor of rank 2. At fig.1 there are three 4-vectors \((k_1, k_2, q)\) bound by the energy and the momentum conservation

\[
k_1 + k_2 = q
\]

(6)

which makes only two \((k_1, k_2)\) of them independent ones. This two vectors can combine a rank 2 pseudotensor by the following way only:

\[
M_{\mu\nu}(k_1, k_2; q) = A\epsilon_{\mu\nu\rho\sigma}k_1^{\rho}k_2^{\sigma}
\]

(7)

where \( \epsilon \) is the Levi-Civita epsilon and \( A \) is a constant.

To construct scalar \( A \) from 4-vectors \( k_1 \) and \( k_2 \) there are two ways only:

1) \( k_1^2 = k_2^2 = 0 \) which is trivial, and
2) \( k_1 k_2 = \frac{1}{2} q^2 = -\frac{1}{2} m_a^2 \) where the paraphoton mass \( m_a \) is figured.

Therefore, \( A \) is determined by the rest-mass \( m_a \) of the paraphoton. In the local inertial system of paraphoton’s mass center the summation of (2) over \( \lambda_1 \) and \( \lambda_2 \) gives
\[ d\Gamma = \frac{1}{(2\pi)^2} \frac{1}{8m_a} e^4 \sum_{\lambda_1, \lambda_2 = 1, 2} (e_{\mu_1}^{\lambda_1}(k_1) e_{\nu}^{\lambda_2}(k_2) M_{\mu\nu}) \times \]
\[ \times (e_{\rho}^{\lambda_1}(k_1) e_{\sigma}^{\lambda_2}(k_2) M_{\rho\sigma})^* \delta(k_1 + k_2 - q) \frac{d k_1}{d k_{10}} \frac{d k_2}{d k_{20}} = \] (8)
\[ = \frac{1}{(2\pi)^2} \frac{1}{8m_a} e^4 \sum_{\mu, \nu} M_{\mu\nu} M_{\mu\nu}^* \eta_\mu \eta_\nu \delta(k_1 + k_2 - q) \frac{d k_1}{d k_{10}} \frac{d k_2}{d k_{20}} \]

with \( \eta_\lambda = (+1) \) for \( \lambda = 1, 2, 3 \), and \((-1)\) for \( \lambda = 4 \). This is the differential cross-section of \( \gamma + \gamma \to a \).

To simplify (8), one can use the following almost-evidences:
\[ \varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma} \eta_\mu \eta_\nu = - \varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma} \] (9)
and
\[ \varepsilon_{\mu\nu\rho\sigma} \varepsilon_{\mu\nu\rho\sigma'} = 2(\delta_{\rho\rho'}\delta_{\sigma\sigma'} - \delta_{\rho\sigma'}\delta_{\sigma\rho'}) \] (10)
which give
\[ \sum_{\mu, \nu} M_{\mu\nu} M_{\mu\nu}^* \eta_\mu \eta_\nu = \frac{1}{2} m_a^4 |A|^2 \] (11)

This way, we also can obtain the integral cross-section \( \Gamma \). To do it, note that \( d\Gamma (k_1, k_2) = d\Gamma (k_1, k_2) \). Therefore, to calculate \( \Gamma \), one should use the following trick: to integrate \( d\Gamma \) by all possible \( k_1 \) and \( k_2 \), and get a half of the result. During the integration one should also use the feature of the Dirac \( \delta \):
\[ \int \delta(k_1 + k_2 - q) \frac{d k_1}{d k_{10}} \frac{d k_2}{d k_{20}} = 2\pi \] (12)
which after all gives the final result:
\[ \Gamma = \frac{1}{4} \pi \alpha^2 m_a^3 |A|^2 \] (13)

where \( \alpha = e^2/4\pi = 1/137 \) is the fine structure constant, \( m_a \) is the (chameleonic) rest-mass of the paraphoton, \( A \) is a constant to be measured during the experiment.

3. Conclusion
The performed QED analysis gives the differential (8) and the integral (13) cross-sections of the photon-photon process resulting to the birth of the paraphoton. Because of T-symmetry, these also are the cross-sections of the invert process of paraphoton decay onto two photons. Therefore, we predict the two photons -- into --the paraphoton oscillations, which can be detected by the known distribution of the dark matter in the Universe [6,7]. These oscillations makes the Universe be transparent to observe ultra-deep field at the Hubble Space Telescope (HST anomaly, [8]). The \( \gamma + \gamma \to a \) channel also influence to the registered star energy radiation resulting to the anomalous flux of the neutrino (solar neutrino anomaly [9]) and unexpectedly high positron ratio in the cosmic rays (PAMELA anomaly [10]). At last, one should notice the most recent experiments on the electromagnetically induced over-transparency of the crystalline ruby [11] and the Primakoff ("Light shining through a wall") effect in the photonic crystals [12].

References