An Elementary Solution of the Four-color Conjecture

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Abstract

The four-color conjecture (Guthrie's problem after F. Guthrie) is solved simply. In summary, if we choose four colors—red (Ar), blue (Bb), yellow (Ay), and green (Bg) (which can be any colors)—and divide them into Group A (Ar, Ay) and Group B (Bb, Bg); if we choose any color from Group A to color the first non-peripheral country, then to color the infinitely many countries adjacent to it, we can alternately use the two colors from the other group to color any number of countries. If the last two countries encounter the same color from the different group, then one of them must be replaced with a different color from the same group (of course, not the same color). Such a replacement only needs to occur once. The same logic applies to coloring any newly colored countries of any of the four colors, and so on, up to infinity.

Key words: four-color conjecture, Guthrie’s problem

The four-color conjecture (Guthrie's problem after F. Guthrie) states that any map in a plane can be colored using four-colors in such a way that regions sharing a common boundary (other than a single point) do not share the same color.

At the now time, the solutions of mathematical problems are too mathematical and make problems more complex, if we divide a complex problem into a few of the simpler problems and solve them one by one, the solutions will be much simpler and clearer.

Let's choose four colors: red (Ar), blue (Bb), yellow (Ay), and green (Bg), and divide them into Group A (Ar, Ay) and Group B (Bb, Bg).

1. First, we color yellow (Ay) as the color of the first non-peripheral country, and red (Ar) as the color of its first adjacent country. If it has fewer than three adjacent countries, the conclusion is self-evident: we simply color the other two countries with blue (Bb) and green (Bg), with no further analysis needed. If it has three or more adjacent countries, up to infinitely many, we can alternately color each country with blue (Bb) and green (Bg) indefinitely. If the last two countries end up being adjacent and of the same color (Bb and Bb, or Bg and Bg), one of them must be replaced with red (Ar). This completes the coloring of the first layer of adjacent countries. The number of these countries ranges from 3 to infinitely many. Among them, the second layer introduces at most 2 newly

colored red (Ar = 1 or 2) countries, 0 yellow (Ay = 0) countries, while blue (Bb = 1, 2, … or ∞) and green (Bg = 1, 2, … or ∞) can be infinitely many.

2. To color the countries adjacent to the red (Ar) in the second layer, we only need to consider those with three or more adjacent countries, up to infinitely many. We can alternately color each country with blue (Bb) and green (Bg). If the last two countries end up being adjacent and of the same color (Bb and Bb, or Bg and Bg), one of them must be replaced with yellow (Ay). This completes the coloring of the countries adjacent to the red (Ar) in the second layer. The number of adjacent countries ranges from 3 to infinitely many. Among them, at most 1 newly colored yellow (Ay = 0 or 1) country is introduced, 0 red (Ar = 0) countries, while blue (Bb = 1, 2, … or ∞) and green (Bg = 1, 2, … or ∞) can be infinitely many.

3. To color the countries adjacent to the blue (Bb) in the second layer, we only need to consider those with three or more adjacent countries, up to infinitely many. We can alternately color each country with red (Ar) and yellow (Ay). If the last two countries end up being adjacent and of the same color (Ar and Ar, or Ay and Ay), one of them must be replaced with green (Bg). This completes the coloring of the countries adjacent to the blue (Bb) in the second layer. The number of adjacent countries ranges from 3 to infinitely many. Among them, at most 1 newly colored green (Bg = 0 or 1) country is introduced, 0 blue (Bb = 0) countries, while red (Ar = 1, 2, … or ∞) and yellow (Ay = 1, 2, … or ∞) can be infinitely many.

4. To color the countries adjacent to the green (Bg) in the second layer, we only need to consider those with three or more adjacent countries, up to infinitely many. We can alternately color each country with red (Ar) and yellow (Ay). If the last two countries end up being adjacent and of the same color (Ar and Ar, or Ay and Ay), one of them must be replaced with blue (Bb). This completes the coloring of the countries adjacent to the green (Bg) in the second layer. The number of adjacent countries ranges from 3 to infinitely many. Among them, at most 1 newly colored blue (Bb = 0 or 1) country is introduced, 0 green (Bg = 0) countries, while red (Ar = 1, 2, … or ∞) and yellow (Ay = 1, 2, … or ∞) can be infinitely many.

Then, by continuously repeating steps 2, 3, and 4, we color any uncolored countries up to infinitely many. In these three steps, there is no example of a yellow (Ay) country. If a yellow (Ay) country is encountered, then step 5 is performed (which is essentially the same as step 1):

5. To color any countries adjacent to yellow (Ay), we only need to consider those with three or more adjacent countries, up to infinitely many. We can alternately color each country with blue (Bb) and green (Bg). If the last two countries end up being adjacent and of the same color (Bb and Bb, or Bg and Bg), one of them must be replaced with red (Ar). This completes the coloring of the countries adjacent to yellow (Ay). The number of adjacent countries ranges from 3 to infinitely many. Among them, at most 1 newly colored red (Ar = 0 or 1) country is introduced, 0 yellow (Ay = 0) countries, while blue (Bb = 1, 2, … or ∞) and green (Bg = 1, 2, … or ∞) can be infinitely many.

In summary, if we choose four colors—red (Ar), blue (Bb), yellow (Ay), and green (Bg) (which can be any colors)—and divide them into Group A (Ar, Ay) and Group B (Bb, Bg); if we choose any color from Group A to color the first non-peripheral country, then to color the infinitely many countries adjacent to it, we can alternately use the two colors from the other group to color any number of countries. If the last two countries encounter the same color from the different group, then one of them must be replaced with a different color from the same group (of course, not the same color). Such a replacement only needs to occur once. The same logic applies to coloring any newly colored countries of any of the four colors, and so on, up to infinity.