

# Graviton momentum – a natural source of dark energy

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## Abstract

The dark energy concept in the standard cosmological model can explain the expansion of the universe. However, the mysteries surrounding dark energy remain, such as its source, its unusual negative pressure, its long-range force, and its unchanged density as the universe expands. In this paper, we propose a graviton momentum hypothesis, develop a semiclassical model of gravitons, and explain the pervasive dark matter and accelerating universe. The graviton momentum hypothesis is incorporated into the standard model and explains well the mysteries related to dark energy.

**Key words:** cosmological constant, graviton momentum, dark matter, dark energy, standard cosmological model, accelerating universe

## 1. Introduction

The discoveries of an expanding universe by Slipher V., 1917 [1]<sup>i</sup> and Hubble E., 1929 [2]<sup>ii</sup> in 1920s, and of the accelerating expansion rate of the universe (Riess A.G., et al., 1998 [3]<sup>iii</sup>, Perlmutter S., et al, 1999 [4]<sup>iv</sup>) in the late 1990s, compelled physics theorists to attempt to explain the nature and causes of the accelerating universe. By adding an additional metric tensor, and assigning to it a very small cosmological constant, Einstein's theory of general relativity is able to explain the expansion of the universe mathematically<sup>1</sup>, but is unable to identify any physical cause for the expansion.

On the other hand, through the measurement of the radial velocities of galaxies, Fritz Zwicky Zwicky F., 1933 [5]<sup>v</sup> suspected that there was invisible dark matter associated with galaxies. The

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<sup>1</sup> Einstein's intention of introducing the cosmological term was to obtain a solution of static universe and he later considered the inclusion of cosmological term in his celebrated field equation was the biggest blunder he ever made in his life. Friedmann realized that Einstein's static universe solution was unstable. Lemaitre showed that Hubble's law followed from the cosmological term.

measurements of the velocity curve in the late 1970s (Rubin, V.C., Ford, W. K., 1970 [6]<sup>vi</sup>, Cram, T. R., Roberts M. S., Whitehurst R. N., 1980 [7]<sup>vii</sup>) showed that, of the total matter in universe, the luminous matter made up to only 2% while the other 98% was considered as dark matter. Later studies on mass density at the current era (Sadoulet, B., 1999 [8]<sup>viii</sup>; Griest K., Kamionkowski M., 2000, [9]<sup>ix</sup>) further confirmed the previous findings. The cosmic microwave background (CMB) survey results (Bennett C. et al., 2012 [10]<sup>x</sup>, Aurich R., Lustig S., 2015 [11]<sup>xi</sup>) suggested that dark energy consisted of about 70% of the energy density in the universe, which generated repulsive force responsible for the accelerating universe. While physicists are still search for exotic cold dark matter, they are unable to explain dark energy in any plausible way.

So far the best explanation of an accelerating universe and dark energy is based on the  $\Lambda$ CDM model – the currently standard cosmological model. The model includes a dark energy component and explain the existing observations well. However, the model takes dark energy as given and does not specify the source of dark energy. Given the success of the quantum mechanism for microworld, quantum vacuum energy was considered as a source of dark energy. However, the calculations showed that the natural size of quantum energy was  $10^{120}$  too large for the cosmological constant. This was labelled as the cosmological constant problem, which is unsolved till today (Weinberg, S., 1989 [12]<sup>xii</sup>, Peebles P.J.E., Ratra B., 2003, [13]<sup>xiii</sup>, Collin R.E. , 2006 [14]<sup>xiv</sup>, Frieman J.A., Turner M.S., Huterer D., 2008 [15]<sup>xv</sup>, Amendola L., Tsujikawa S. , 2010 [16]<sup>xvi</sup>, Arun K., 2017 [17]<sup>xvii</sup>).

Within the  $\Lambda$ CDM model framework, Farnes [18]<sup>xviii</sup> intend to explain why the dark energy generates negative pressure. It is assumed that, like charges, mass can be positive or negative: the same type of masses has attractive force while the different types of masses have repulsive force. Although this hypothesis can explain the negative pressure of dark energy, the assumption of negative mass is incomprehensible based on current knowledge in physics. More importantly, this hypothesis is unable to explain the long-range effect of the repulsive force of dark energy.

There are a number of theories about dark energy, but they all involve some assumptions or outcomes what are not supported by current observations or the known physics knowledge. Milgrom (1983)[19]<sup>xix</sup>, Bekenstein (2004) [20]<sup>xx</sup>, and Dunkel (2004) [21]<sup>xxi</sup> required the

modification of Newtonian dynamics (MOND) and an ad hoc introduction of fundamental acceleration of  $10^{-10} \text{ m/s}^2$ . Dvali, Gabadadze and Porrati (2000) [22]<sup>xxii</sup> and Kunz and Sapone (2007) [23]<sup>xxiii</sup> required to modify the general relativity. The quintessence model of Peebles and Ratra (1988) [24]<sup>xxiv</sup> and Rozas-Fernandez (2012) [25]<sup>xxv</sup> assumed a fifth fundamental force. Caldwell, Kamionkowske and Weinberg (2003) [26]<sup>xxvi</sup> assumed a phantom energy, a negative kinetic energy that increases with the expansion of the universe. The quintom (combination of quintessence and phantom) dark energy of Upadhye, Ishak and Steinhardt (2005) [27]<sup>xxvii</sup> and Marciu (2016) [28]<sup>xxviii</sup> introduced a time varying parameter for equation of state but still could not explain why the universe entered the acceleration stage at the current epoch. Sivaram and Sinha (1975) [29]<sup>xxix</sup> and Sivaram and Arun (2013) [30]<sup>xxx</sup> assumed a time varying cosmological constant and a time dependent vacuum energy density. Gurzadyan and Xue (2003) [31]<sup>xxxi</sup> suggested the possibility of variation of the speed of light and the gravitational constant.

Given our limited knowledge about dark energy, it is understandable that various surprising assumptions are made in previous studies. However, it can be argued that if a theory employs more plausible assumptions, the theory should be preferred. The present paper is an attempt in this direction. Our assumption is that while graviton attraction causes gravitational force, graviton momentum leads to the repulsive force responsible for an accelerating universe. Although gravitons have not been detected yet (maybe due to the very weak strength of gravitational force), the concept of gravitons is generally accepted as a means of quantifying a gravity field. It is also accepted that gravitons travel at the same speed as photons. Since photons have momenta, it is reasonable to assume that the photon-like gravitons, or dark photons, also have momenta.

With these plausible assumptions, the paper intends to derive the cosmological constant. As will be seen, with a model of graviton momentum, the paper explains well the pervasive dark matter in the universe, the long-range nature of repulsive force from dark energy, the unusual assumptions about dark energy in the  $\Lambda$ CDM model frame work, and the unchanged density of dark energy in an expanding universe. Most importantly, the explanation is simple and elegant, supported by existing observations, and compatible with the general relativity and the standard cosmological model.

The remaining paper is structured as follows. Section 2 focuses on the development of a semiclassical model that can explain pervasive dark matter in the universe. Section 3 derives the cosmological constant and explains an accelerating universe. Section 4 discusses the compatibility between the graviton momentum hypothesis and the standard cosmological model. Section 5 concludes the paper.

## 2. A semiclassical model of gravitons

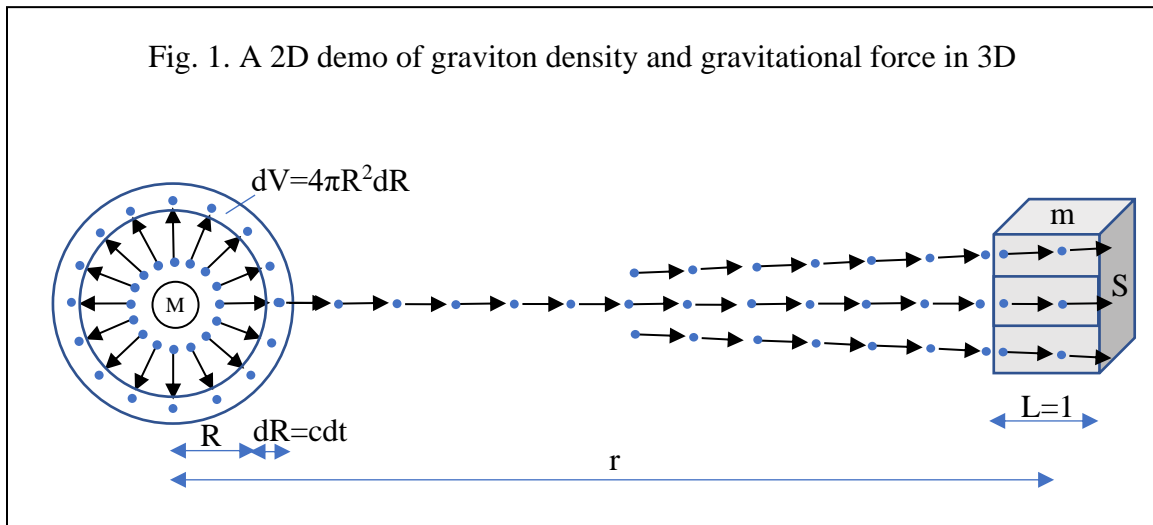
Graviton are a commonly accepted concept. However, it is generally interpreted as a virtue particle. This paper reviews gravitons as photon-like real particles or, loosely speaking, dark photons. Some may object to treating gravitons as dark photons because gravitons and photos are of spin-2 and spin-1, respectively. The spin-2 nature of gravitons is given by its rank-2 stress-energy tensor, which gives gravitons a fluidic feature. In terms of cosmology, this spin difference diminishes: the universe can be viewed as an idea fluid, so the stress-energy tensor for gravitons is diagonal and its rank reduces to 1.

The detailed assumptions for the graviton momentum model can be stated as follows. Extending from the photon emission phenomenon, we assume that all matter emits gravitons and that, after receiving a graviton, the matter reemits a graviton on the extended path of the original graviton. Gravitons are photon-like particles which have no mass and travel at the speed of light in a vacuum. However, different from the photons, gravitons have attractive interactions between them and are attracted by mass. Based on quantum theory, a photon has energy of  $hf$  and momentum of  $hf/c$ , where  $h$  is Planck's constant,  $f$  is the frequency of light and  $c$  is the speed of light in vacuum. Since gravitons are assumed to be photon-like and travel at the speed of light  $c$ , it is natural to assume that gravitons also have a momentum of  $hf/c$ , where  $f$  is frequency of a gravitational wave.

The direct outcome of these assumptions is that when a graviton contacts a mass particle, an attractive gravitational force (opposite to the direction of graviton movement) is generated due to attractive interaction with mass; meanwhile, a repulsive force (in the direction of graviton movement) also generated due to the momentum of the graviton. These two opposite forces come from the two compatible attributes of gravitons, so they are consistent. In considering gravitons as the carries of gravitational force, it is also reasonable to assume that the gravitational

force transferred by gravitons is proportional to the density of gravitons. With these assumptions, we can explain both gravitational force and dark energy.

We start with deriving Newtonian gravitational formula. Based on a simple setting shown in Fig. 1, mass  $M$  radiates gravitons in all directions and some of them are received by mass  $m$ . The density of gravitons can be measured by the number of gravitons in a small volume  $dV$ , enclosed by two spheres of radius  $R$  and  $R+dR$ . Since gravitons in a ray are attracted by each other and by mass, they establish the attractive force between  $M$  and  $m$ . The denser the graviton rays, the stronger gravitational force. As the gravitons in the ray from left to right attract each other, mass  $M$  can attract mass  $m$  through a great distance. However, as the distance increases, the graviton density decreases and the attraction between  $M$  and  $m$  will decrease significantly. Next, we examine these intuitive thoughts mathematically.



Let  $n$  be the number of gravitons per emission per unit mass,  $f$  the frequency of graviton emissions by  $M$ . The number of gravitons  $N$  emitted by mass  $M$  in one unit of time can be calculated as  $N = nfM$ . The total number of gravitons emitted in time period  $dt$  is:

$$dN = Ndt = nfMdt$$

Assume the gravitons in the first emission arrive at the sphere surface of radius  $R$ . Given that the gravitons travel at the speed of light  $c$ , the distance a graviton covers in time  $dt$  is  $cdt$ , so the

volume occupied by  $dN$  in time  $dt$  gravitons is the volume  $dV$  enclosed by the two spheres of radius  $R$  and  $R+cdt$  (i.e.  $dR=cdt$ ), which can be calculated as follows:

$$dV = \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} R^2 \sin\theta d\theta d\varphi dR = 4\pi R^2 cdt$$

The density of gravitons at radius  $R$  can be calculated as:

$$\rho_g = dN/dV = nfM/(4\pi cR^2)$$

Given the distance  $r$  between  $M$  and  $m$ , the density of gravitons from  $M$  at the testing object  $m$  is:

$$\rho_g = dN/dV = nfM/(4\pi cr^2) \quad (1)$$

Assuming that the gravity force  $F$  is proportional to graviton density  $\rho_g$ , and the mass  $m$  of the testing object, we have the gravitational force  $F$  enacted by object  $M$  on the testing object  $m$  at distance  $r$ :

$$F = C * \rho_g m = C * nfMm/(4\pi cr^2) \quad (2)$$

Here we have introduced a constant  $C$  – the amount of attraction force enacted by a graviton for a unit of mass. We can calibrate  $C$  to satisfy:

$$G = Cnf/(4\pi c) \quad \text{Or} \quad C = 4\pi cG/(nf)$$

Plugging above equation into eq. 2, we have Newton's gravity equation:

$$F = \frac{4\pi cG}{nf} m \rho_g = \frac{GMm}{r^2} \quad (3)$$

The derivation so far, however, ignores the impact of momentum from gravitons. Given the density of gravitons  $\rho_g$ , the cross-section  $S$  of the testing object  $m$ , and the speed of gravitons  $c$ , the flux of gravitons on  $m$  (the number of gravitons received by  $m$  in 1 unit of time) is  $\varphi = \rho_g * c * S$ . Since the momentum carried by a graviton is  $hf/c$ , the total momentum of gravitons transferred from  $M$  to  $m$  can be calculated as:

$$\Delta P = \varphi * \frac{hf}{c} = \rho_g c S * \frac{hf}{c} = \rho_g S h f$$

This result is analogical to that of solar sailing. However, a question could be raised regarding the consistency of momentum effect and gravitational effect: while it is understandable that the momentum is proportional to the number of gravitons received by the object, why is the gravitation effect proportional to the density of gravitons, rather than to the number of gravitons received? The answer lies in the assumption that when a graviton is absorbed by a mass particle, it re-emits a graviton on the extended path of the original graviton (i.e. shown as the gravitons travelling inside the object  $m$  in Fig.1). In this way, a graviton can act directly and indirectly on all mass particles in its pass, so the gravitational force is related to the density of gravitons, and is not blocked by any particles on its path.

Although the momentum is related to cross section, for more general application, it is useful to convert the cross section to the density of the testing object  $\rho_m$ . For convenience, we assume the length of testing object  $L=1$ , so the cross-section can be calculated as:  $S=V/L=V=m/\rho_m$  (in so doing, we imply that  $\rho_m$  is the column density of the testing object, or we simply view  $\rho_m=L\rho_m$  due to  $L=1$ . This is a way of eliminating parameters dependent on the shape and size of testing object, so the analysis can be simplified for more general cases). Since the transferred momentum  $\Delta P$  occurs in 1 unit of time, the force it generates is:

$$F' = \frac{\Delta P}{t} = \frac{\Delta P}{1} = \rho_g h f m / \rho_m \quad (4)$$

Since  $F'$  is opposite to gravitational force  $F$ , the total impact of gravitons from  $M$  on  $m$  can be expressed as the total force  $F_{\text{total}}$ :

$$F_{\text{total}} = F - F' = \frac{4\pi c G}{n f} m \rho_g - \frac{h f}{\rho_m} m \rho_g = m \rho_g \left( \frac{4\pi c G}{n f} - \frac{h f}{\rho_m} \right) \quad (5)$$

The first term in the bracket of eq. 5 is proportional to attractive gravity force, while the second term is related to the repulsive force caused by graviton momentum. It is clear that the density of testing object affects the relative size of the two terms. Equalizing these two terms, we have the break-free density for the testing object:

$$\rho_{m\_free} = \frac{n h f^2}{4\pi c G} \quad (6)$$

At this break-free density, the attractive force and repulsive force cancelled out each other, so the testing object  $m$  experiences zero net force from  $M$  and thus will not be attracted to  $M$ . Above this density, attractive gravity force dominates and vice versa. In short, eq. 5 shows the important role of the density of the testing object in the net force it receives.

### 3. Gravity momentum leading to elusive dark matter and an accelerating universe

Given the pervasive and dominant dark matter in the universe, we can ignore the high-density luminary matter when we consider issues at the cosmological scale. After the big bang (i.e. the inflation epoch), the matter density was relatively high, so  $\rho_m$  is small and the second term in the brackets of eq.5 was dominated by the first term. As such, the attractive force dominated and the expansion of the universe decelerated. As the universe expands, the density of dark matter decreases. At the current epoch, the density of dark matter is so low that the second term in the brackets of eq.5 dominates and the net force between dark matter particles are repulsive. This repulsive force keep dark matter particles from clumping together, so dark matter cannot form pockets of dense particles to cause disturbance to electromagnetic wave. Consequently, dark matter is very hard to detect.

The repulsive force between pervasive dark matter particles also causes the accelerated expansion of the universe. Next, we explain how this force is related to the acceleration rate of the universe expansion and to the cosmological constant.

Assuming the density of the dark matter is  $\rho_D$ , The amount of dark matter  $M_D$  within a sphere of radius  $r$  can be calculated as:

$$M_D = \frac{4}{3} \pi r^3 \rho_D$$

Applying eq. 1, we can calculate the graviton density at distance  $r$  due to dark matter:

$$\rho_{gD} = \frac{nfM_D}{4\pi cr^2} = \frac{\rho_D nf}{3c} r$$



Using eq. 5, we have the total force due to dark matter:

$$F_{total} = m\rho_{gD} \left( \frac{4\pi cG}{nf} - \frac{hf}{\rho_m} \right) = \frac{m\rho_D nf}{3c} r \left( \frac{4\pi cG}{nf} - \frac{hf}{\rho_m} \right) = \frac{GM_D m}{r^2} - \frac{hmf^2 \rho_D}{3c\rho_m} mr \quad (7)$$

The first term on the righthand side of eq. 7 is the gravitational force, which decrease at the pace of  $1/r^2$ . The second term is the repulsive force due to graviton momentum, which is proportional to distance  $r$  and thus a long-range force. Considering this total force at the cosmological scale (extremely large distance and ultralow density), we can ignore the first term and also equalize the density of the testing matter  $\rho_m$  with the overall dark matter density  $\rho_D$ , so the acceleration rate of the universe due to graviton momentum can be expressed as:

$$g_\Lambda = \frac{hmf^2}{3c} r = \frac{hmf^2}{3c^3} c^2 r \quad (8)$$

This acceleration rate should be equivalent to that calculated from the general relativity. From Einstein's field equation we can have:

$$\nabla^2 \Phi = 4\pi G \left( \rho + \frac{3p}{c^2} \right)$$

Considering the density of matter  $\rho_M$  and dark energy  $\rho_\Lambda$ , we have  $\rho = \rho_M + \rho_\Lambda$ , so the above equation becomes:

$$\nabla^2 \Phi = 4\pi G \left( \rho_M + \frac{3p_M}{c^2} + \rho_\Lambda + \frac{3p_\Lambda}{c^2} \right)$$

Setting  $p_M = 0$  (the pressure from matter is negligible) and using the popular assumption for dark energy:  $p_\Lambda = -\rho_\Lambda c^2$  (the negative sign coming from the equation-of-state parameter value of -1 for dark energy), we have

$$\nabla^2 \Phi = 4\pi G \left( \rho_M + \frac{3p_M}{c^2} + \rho_\Lambda - 3\rho_\Lambda \right) = 4\pi G \rho_M - 8\pi G \rho_\Lambda$$

Following the tradition, we can define:

$$\Lambda = \frac{8\pi G \rho_\Lambda}{c^2} \quad (9)$$

In considering that, in a dark energy dominated universe,  $4\pi G\rho_M \ll \Lambda c^2$ , we have:

$$\nabla^2\Phi = 4\pi G\rho_M - \Lambda c^2 \approx -\Lambda c^2$$

The solution to the above Poisson equation is:  $\Phi_\Lambda = -\frac{\Lambda c^2}{6}r^2$ , so we have:

$$\mathbf{g}_\Lambda = -\nabla\Phi_\Lambda = \frac{\Lambda c^2}{3}r \quad (10)$$

Comparing above equation with eq. 8, we have:

$$\Lambda = \frac{hmf^2}{c^3} \quad (11)$$

Is the size of this cosmological constant consistent with existing observations? We do not know the answer because the frequency of graviton emissions  $f$  and the number of gravitons per emission per unit mass  $n$  are unknown. With the improvement of technology in detecting gravitational waves, the amplitude and frequency of gravitons may be measured in the future, so the cosmological constant may be calculated from eq. 11 and to be compared with observations. At present, we can use the measured density of dark energy to perform a plausibility check by calculating the cosmological constant from astronomical measurements and then calibrating the graviton emission frequency or amplitude.

The CMB survey data (Bennett C. et al., 2012<sup>xxxii</sup>, Aurich R., Lustig S., 2015<sup>xxxiii</sup>) show that the ratio of density of dark energy  $\rho_\Lambda$  to critical density  $\rho_c$  is 0.7181. Using the critical density  $\rho_c = 9.47 \times 10^{-27} \text{ kg/m}^3$ , we can obtain the value for cosmological constant:

$$\Lambda = \frac{8\pi G\rho_\Lambda}{c^2} = 1.267 * 10^{-52}$$

Use eq.11 we can find:

$$nf^2 = \frac{\Lambda c^3}{h} = 3.257 * 10^7$$

Since the range of gravitational waves is  $10^{-16} - 10^4 \text{ Hz}$ , we assume an average frequency of  $f=1$  for a back-of-envelope calculation. We obtain the number of gravitons per emission per kg mass

$n=3.257 \cdot 10^7$ . This is a plausible number. Although we cannot estimate the cosmological constant directly from graviton momentum due to lack of information on graviton emissions, this back of envelope calculation shows that graviton momentum is a plausible physical cause for cosmological constant.

Moreover, the graviton momentum model can also explain the mystery of constant dark energy density. The conventionally derived cosmological constant in eq. 9 depends on the density of dark energy. As the universe expands, the density of dark energy should decrease. This would lead to a decreasing cosmological constant. The current common wisdom is that the density of dark energy is *somehow* constant as the universe expands so that the cosmological constant does not change. With our derived eq. 11, it is easy to see that the cosmological constant is related only to the frequency of graviton emissions and the number of gravitons per emission, so it does not change with expansion of universe nor dark energy density. The mystery is revealed by eq.7: As the universe expands, a decrease in density of dark matter ( $\rho_D$ ) is cancel out by the same degree decrease in density of testing object ( $\rho_m$ ), leaving the repelling force unchanged. In other words, the density of dark energy decreases as universe expands, but this decrease is balanced out by the same degree of increase in the volume of the testing object of given amount of mass  $t$ . Consequently, the amount of repulsive force or the acceleration rate is unaffected, so is the cosmological constant.

#### **4. Incorporating the graviton momentum hypothesis into the $\Lambda$ CDM model**

We can replace the traditional cosmological constant term (i.e. eq. 9) derived from Einstein field equation with the that derived from graviton momentum (i.e. eq. 11), so the field equation is intact. The Friedman-Lemaitre-Robertson-Walker (FLRW) model and the  $\Lambda$ CDM model are developed directly from the general relativity, so they should also be compatible with graviton momentum hypothesis. Below we show how to incorporate the graviton momentum term into a  $\Lambda$ CDM model.

As the backbone of the  $\Lambda$ CDM model, the Friedmann equations including dark matter and dark energy<sup>2</sup> are as follows:

$$\frac{\dot{a}^2 + kc^2/R_0^2}{a^2} = \frac{8\pi G}{3} (\rho_M + \rho_\Lambda) \quad (12)$$

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{c^2} [(p_M + p_\Lambda) + (\rho_M + \rho_\Lambda)c^2/3] \quad (13)$$

From equation 1 and 2, we can have:

$$\dot{\rho}c^2 = -3(\rho c^2 + p)\frac{\dot{a}}{a}, \text{ where } p = p_M + p_\Lambda, \rho = \rho_M + \rho_\Lambda \quad (14)$$

It is assumed that the pressure due to matter is negligible ( $p_M=0$ ). Dark energy has negative pressure  $p_\Lambda = w\rho_\Lambda c^2 = -\rho_\Lambda c^2$ . Including these assumptions, we can obtain the equation for the evolution of Hubble constant:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} [\rho_M a^{-3} + \rho_\Lambda a^{-3(1+w)}], \text{ with } w = -1 \text{ for dark energy} \quad (15)$$

In considering graviton momentum as the source of dark energy, we do not need the unusual assumption of negative pressure  $p_\Lambda = -\rho_\Lambda c^2$ . Instead, using eq. 9 and 11, we can obtain a constant density of dark energy:

$$\rho_\Lambda = \frac{nhf^2}{8\pi Gc} \quad (16)$$

We have:

$$p_\Lambda = -\rho_\Lambda c^2 = -\frac{nhf^2 c}{8\pi G} \quad (17)$$

The above equation shows that the new specification of  $p_\Lambda$  and  $\rho_\Lambda$  based on graviton momentum naturally leads to constant density of dark energy. If we plug the new specification of  $p_\Lambda$  and  $\rho_\Lambda$  (i.e., eqs. 16, 17) into eqs. 12, 13 and 14, we can obtain the same equation as eq.15.

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<sup>2</sup> For simplicity, we omit the density of radiation energy, the curvature parameter and other parameters in a typical  $\Lambda$ CDM model.

As a result, not only does the new specification keep the same setting of a  $\Lambda$ CDM model, but also avoids the unusual assumptions about dark energy.

Another way to verify the consistency between the graviton momentum hypothesis and the standard cosmological model is to rewrite eq. 7 in the form of the acceleration rate:

$$\mathbf{g}_{total} = \rho_{gD} \left( \frac{4\pi cG}{nf} - \frac{hf}{\rho_m} \right) = \frac{\rho_D nf}{3c} \left( \frac{4\pi cG}{nf} - \frac{hf}{\rho_m} \right) r = \left( \frac{4\pi G \rho_D}{3} - \frac{nhf^2}{3c} \right) r = \frac{4\pi G}{3} \left( \rho_D - \frac{nhf^2}{4\pi Gc} \right) r \quad (18)$$

In the above equation, we equalized  $\rho_D$  and  $\rho_m$  because the density of the universe is almost constant at the cosmological scale. The  $\mathbf{g}_{total}$  is the acceleration rate in the direction of attractive force, so it is the acceleration rate of contraction, corresponding to the negative acceleration rate of expansion in the second Friedmann equation,  $-\ddot{a}$ . The distance  $r$  is corresponding to the scale factor  $a$ . Once we replace  $\mathbf{g}_{total}$  and  $r$  in equation 18 with  $-\ddot{a}$  and  $a$ , respectively, we obtain the second Friedmann equation for matter and dark energy.

The graviton momentum hypothesis can also be incorporated into the Einstein Field equation. Replacing the  $p_\Lambda$  and  $\rho_\Lambda$  in the tensor for cosmological constant, we have:

$$T_{\mu\nu}^\Lambda = \frac{\Lambda g_{\mu\nu}}{\kappa} = -\frac{c^4}{8\pi G} \Lambda g_{\mu\nu} = \begin{pmatrix} \rho_\Lambda c^2 & 0 \\ 0 & p_\Lambda g_{ij} \end{pmatrix} = \begin{pmatrix} \frac{nhf^2 c}{8\pi G} & 0 \\ 0 & -\frac{nhf^2 c}{8\pi G} g_{ij} \end{pmatrix}$$

Adding this tensor to Einstein's field equation  $G_{\mu\nu} = \kappa T_{\mu\nu}$ , we have a new field equation including graviton momentum or dark energy:  $G_{\mu\nu} = \kappa(T_{\mu\nu} + T_{\mu\nu}^\Lambda)$ . The new field can explain the source and nature of dark matter and dark energy.

## 5. Conclusions

Based on the generally accepted concept of graviton, the paper presented a model of graviton momentum, which explains the gravitational force, the pervasive dark matter, the accelerating universe, as well as mysteries surrounding dark energy, including the long-range nature of repulsive force from dark energy, the negative pressure from the dark energy in the  $\Lambda$ CDM model framework, and the unchanged density of dark energy in an expanding universe.

The repulsive force from graviton momentum can be transformed to the cosmological constant and thus be included in the Einstein field equation. Similarly, the repulsive force can be linked to the density and pressure from the dark energy. By replacing the dark energy density with the term derived from graviton momentum, the paper upgraded the  $\Lambda$ CDM model and naturally explains the seemingly unusual assumptions about dark energy. Since the graviton momentum hypothesis does not affect the inflation epoch nor the epochs in big bang cosmology, both inflationary cosmology and big bang theory are not affected by the results presented in this paper. As a result, the graviton momentum hypothesis can be incorporated into the standard cosmological model.

*Data availability statement: No new data were generated or analysed in support of this research.*

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