Abstract

One of the most effective theories for dark matter is Milgrom’s Modified Newtonian Dynamics, where a modified law of gravity based on a fixed acceleration scale $a_0$ is postulated that provides a correct description of the gravitational fields in galaxies. However, the significance of $a_0$ is unknown, and the whole theory is generally viewed as a phenomenological description of the observations. Based on Newton’s gravitational law as applied to a uniform continuous mass we posit a non-homogeneous distribution of mass at cosmological scales that would give rise to a constant acceleration that agrees with MOND’s $a_0$. The implications for MOND as a viable theory of dark matter and for the problem of dark energy are discussed.

Modified Newtonian Dynamics (MOND) is a Newtonian-derived hypothetical model of gravity proposed 40 years ago by Mordehai Milgrom to explain the multiple gravitational anomalies observed in galaxies and galaxy clusters [1-3]. They are summarized and conventionally explained through the existence Dark Matter, an elusive new form of matter that interacts only gravitationally and is not included in the Standard Model of Particle Physics. While no such particles have yet been found, the search goes on and MOND usually plays a secondary role in the list of candidate explanations for dark matter. One of the reasons is that $a_0$, the distinctive feature of MOND, does not correspond to any physical entity, and –it is argued- was postulated solely as a means to obtain a gravitational law that fits the observations. It is sometimes called a phenomenological explanation.

While $a_0$ agrees to within one order of magnitude with the acceleration calculated at the border regions of the observable universe from the simple Newtonian gravitational formula and is also
found to relate to Hubble’s constant and to the square root of the cosmological constant $\Lambda$, in both cases scaled by the speed of light $c$, no physical representation of such an acceleration has been devised, and most physicists would agree that it represents another constant of nature, whose role would be to relate fundamental gravitational phenomena in the low-acceleration regime, implying probably some modification of the laws of gravity.

**The Newtonian ball model of gravity**

A generally accepted assumption of all current astrophysical models is the Cosmological Principle, the idea that the universe at large scales is both homogeneous and isotropic. While it may still be isotropic and strong constraints have been set on the range of variation in matter density, the homogeneity condition has little theoretical supporting evidence. Based on original ideas of Isaac Newton, we shall argue that the universe can be modelled as a nearly homogeneous continuous distribution of mass that obeys simple dynamics embodied in the Universal Law of Gravitation. As Newton amazingly found in the late 1600s [4], when a continuous distribution of mass with constant density is allowed to evolve according to such law, an acceleration appears that is null at the center and increases outwards in linear proportion to radial distance until it reaches, for a distance equal to the radius of the ball, the exact same value as predicted by conventional Newtonian gravity.

\[
F_B = \frac{G M m r}{R^3}
\]

as opposed to a point-mass gravitational field:

\[
F_N = \frac{G M m}{R^2}
\]

where $F_B$ (the force in the Newtonian ball model) and $F_N$ (Newton’s conventional point-mass gravitational force) are the force on a test particle with mass $m$ placed at a distance $r$ from the center of the $R$-ball, or at a distance $R$ from the central point-mass $M$, respectively. The acceleration for the ball with mass $M$ is then

\[
Acc_B = \frac{G M r}{R^3}
\]

and solving for $G$

\[
G = \frac{Acc_B}{R^3 / M r}
\]

We now define $G'$ as $4\pi G$ and substitute it for $G$ above. The resulting expression is mathematically equivalent, though it may facilitate the visualization of upcoming considerations.

\[
G' = \frac{(Acc_B 4\pi R^3)}{(M r)} \quad [G' := 4\pi G]
\]
And multiplying both parts of the right-hand quotient by a factor of three,

\[ G' = 3 \frac{Acc_B}{r} \cdot \frac{4/3 \pi R^3}{M} \]

and since \( \frac{4/3 \pi R^3}{M} \) is the inverse of the matter density for the spherical volume,

\[ G' = 3 \left( \frac{Acc_B}{r} \right) \cdot \left( \frac{1}{\rho} \right) \]

\[ G' = 3 \frac{Acc_B}{(r \cdot \rho)} \]  

(1)

where \( \rho \) is now the average, not necessarily constant matter density of the universe.

Looking at equation (1) we see that in such a ball model of the universe, if \( \rho \) is constant, then the quotient \( \left( \frac{Acc_B}{r} \right) \) must be constant, which agrees with the Newtonian view but does not help us understand the existence of a constant acceleration pervading the whole universe that at the same time agrees with the Newtonian acceleration at its border regions, as MOND postulates and available evidence strongly suggests.

We therefore let \( \rho \) vary with radial distance, however small the constant of proportionality may be, and assume that it is the product in the denominator of Equation (1) \( (r \cdot \rho) \) that is constant. In other words, we let density decay inversely with radial distance. We immediately see that since both \( G' \) and the product \( (r \cdot \rho) \) are constant, so must be \( Acc_B \), and this acceleration agrees with MOND's universal acceleration \( a_0 \) and with the calculated Newtonian acceleration at the border regions of the ball to within one order of magnitude, as can be easily checked. Indeed, feeding in the accepted values for the mass of the observable universe \( (10^{53} \text{ Kg}) \), radial distance \( (10^{26} \text{ m}) \) and \( G \), it turns out that the acceleration perceived at the border regions of the observable universe is about \( 3.4 \cdot 10^{-10} \text{ m.s}^{-2} \), quite close to the reported value for \( a_0 \) \( (1.2 \cdot 10^{-10}) \). And according to the Newtonian ball model, assuming \( r \cdot \rho \) constant, this same acceleration would be present as a background curvature in the whole universe, explaining its local influence in all galaxies, not just as a constant of nature, but as a real acceleration that would determine the observed accelerations through a geometrical averaging with the local, Newtonian-derived acceleration.

The range of variation in mass density that could be expected in this model depends on how far we are from the central region of the universe, and can be approximately estimated.

From Eq (1), taking \( Acc_B = a_0 = 1.2 \cdot 10^{-10} \text{ ms}^{-2} \), \( R_U = 4.4 \cdot 10^{26} \text{ m} \), \( G' = 8.38 \cdot 10^{-10} \text{ m}^3 \cdot \text{Kg}^{-1} \cdot \text{s}^{-2} \), we have

\[ r \cdot \rho = 0.4295 \text{ Kg.m}^{-2} \]

\[ \rho = 0.4295 / r \]
Assuming we are in a mid-radius region, $R_0 = 2.2 \cdot 10^{26} \text{ m}$ and making \( dr = 1 \text{ Mpc} = 3.1 \cdot 10^{22} \text{ m} \), it turns out that the expected decrease in density per Mpc at a radius half the universe’s radius would be:

\[
d\rho = -0.4295 \cdot R_0^{-2} \cdot dr
\]

\[
d\rho = 6.29 \cdot 10^{-30} \text{ Kg/m}^3/\text{Mpc}
\]

This is approximately 1% of the accepted baryonic mass density of the universe (4.6% of the critical density $10^{-26} \text{ Kg/m}^3$, or $4.6 \cdot 10^{-28} \text{ Kg/m}^3$). For regions closer to the center, the predicted relative variations are larger, in more external regions they would become much smaller and practically unmeasurable.

Observational evidence for the distribution of mass density in the universe is scant. The large-scale average density of the universe, known as the cosmic density parameter, $\Omega$, depends on its composition and, according to the $\Lambda\text{CDM}$ model, is very close to the critical mass density $\Omega_c$, the one required to make the universe flat. The density of matter, including dark matter would amount to about 28% of the global density ($\Omega_m = 0.28$), while the density of baryonic matter is though to comprise a bare 4.6% of the total density. Distribution of average density as a function of distance is generally assumed to follow the general trend of decreasing as the radius increases, reflecting the overall dilution of matter on larger scales, but observations are dominated by a complex hierarchical structure, the so-called cosmic web, that hinders any precise average estimates. Therefore, no reliable data are currently available.

Several authors [5 - 9] notably Peebles, Karachentsev, Nuza and others have probed into the mass distribution in the vicinity of our Milky Way and found that, on average, its density is significantly lower than the average for the whole universe. We would thus be in a local region of low density, the Local Void, which makes the observations not representative of the universe. The interpretation of the results is also compounded by the influence of dark matter and structure formation, two processes of which we know little.

In two important studies [5, 6] the authors examined the distribution of the mean density of matter in spheres of various radii in our Local Universe and found that matter density up to about 50 Mpc decays with distance. The authors conclude that density is on average lower than the global density for the universe ($\Omega_{m, \text{local}} = 0.08$ vs $\Omega_m = 0.28$) and tends to an asymptotic minimum value. However, looking at the data in the figures, we speculate that they might also be consistent with a $1/r$ decay in that range. However, as the authors point out, larger scale distances are needed to avoid local variations, probably 100 Mpc at least. In the papers, uncertainties in the range up to 90 Mpc seem too large to draw a conclusion. As we have seen, a reliable measurement of the average baryonic-mass density around the Milky Way could be used to gauge our proximity to the center of the universe.

Another interesting observation is the striking resemblance of equation (1) with the Friedmann equation. For flat space (\( k = 0 \)), the Friedman equation can be expressed as [10]

\[
a''/a = - 4/3 \cdot \pi \cdot G ( \rho + 3p/c^2) + \Lambda c^2/3
\]

And making a customary simplification [10] that consists of replacing
\[ \rho \rightarrow \rho - \frac{\Lambda c^2}{8\pi G} \]
\[ \rho \rightarrow \rho + \frac{\Lambda c^4}{8\pi G} \]

we have
\[ H^2 = \left( \frac{a'}{a} \right)^2 = \frac{8\pi G \rho - k c^2/a^2}{a^2} \]

and assuming flat space \((k=0)\) and substituting \(G'\) for \(G\) \((G' = 4\pi G)\) results in
\[ G' = 4\pi G = \frac{3(2)}{H^2 / \rho} \]

which certainly reminds us of Eq 1:
\[ G' = \frac{3}{2} \cdot \frac{\text{Accel}}{r} \]

In the last expression, since dimensions of \(\text{Accel} / r\) are \(1 / T^2\), we have
\[ G' = 3 \cdot \left( \frac{1}{t} \right)^2 \cdot \left( \frac{1}{r} \right) \]

If we now interpret \(1/t\) as the constant rate of expansion \(H\), it turns out that Eq (1) can be viewed as equivalent to
\[ G' = 3 \cdot \frac{H^2}{\rho} \]

which differs from the Friedman equation by a factor of 2. The reason for the discrepancy we ignore, but it has happened before in astrophysics that a classical, non-relativistic approach has been later superseded by the appropriate relativistic version that differs from it by a factor of two, e.g., in the old pre-Einstein estimation of the lensing of light from Newtonian gravity by Johann Soldner in 1804.

Thus, the hypothesis of a decreasing matter density that scales inversely with distance seems a reasonable one and, from Newtonian mechanics, this would lead to a constant background cosmic acceleration that agrees with MOND’s \(a_0\) and would account for the rotation curves in galaxies. The observed galactic accelerations below a certain threshold also agree with MOND and turn out to be the geometric average of the Newtonian and the background \(a_0\). This would now be understood as a real physical phenomenon related to the interaction of two competing accelerations, not only a mathematical construct.

We cannot discuss here the other predictions of MOND related to dark matter. We would rather refer the reader to the works of the original author [1-3]. We should point out however that the discrepancies of MOND with the observations in galaxy clusters might be addressed by an averaging of the gravitational fields due to the cluster itself and to nearby galaxies [11]. We are not aware of any consistent explanation for the accelerations observed in colliding clusters like the Bullet. As for the CMB, we have argued elsewhere [11] that our understanding of the CMB has some problems that limit its ability to be used as the gold standard to adjudicate prospective fundamental theories. We’d like to draw attention to one of these problems, namely the strong and regular anisotropy observed in the CMB, the so-called CMB dipole, that is generally disregared as originated from the movement of our galaxy with respect to the CMB, out of the Hubble flow. Such anisotropy has been measured as equivalent to 380 Km/s and is generally erased by subtracting the original images. More recent data indicate though that the velocity
might be about 600 Km/s [12], much higher than expected, which makes it difficult to explain and might call for a re-evaluation of the anisotropy postulate.

We therefore conclude that

1. In a modified Newtonian ball model of the universe, a continuously decreasing matter density that scales as 1/r, as opposed to the uniform distribution from the Cosmological Principle, would give rise to a constant universal physical acceleration that agrees with MOND’s $a_0$.

2. This would provide a physical basis for MOND and support it as a viable interpretation of the dark matter problem.

3. The resulting matter-density distribution may be hard to verify experimentally, for the densities involved, as well as the variations incurred are very low. A variation in mass density around 1% per Mpc is expected.
Cosmological acceleration as a basis for the universe’s expansion

We now turn our attention to the mysterious empirical relation observed between $a_0$ and the parameters that reflect the universe’s expansion, $H_0$ and $\Lambda$.

Indeed, the numerical value of MOND’s $a_0$ has been found to be approximately

$$a_0 \sim \left(\frac{c}{2\pi}\right) \cdot H_0 \sim \left(\frac{c^2}{2\pi}\right) \cdot \sqrt{\Lambda/3}$$

Why is that? What is the intimate relation of $a_0$ to the accelerated expansion of the universe?

Fig 1. A non-homogeneous universe with matter density that decays as $1/r$ would generate a constant background acceleration $a_0$ that pervades the whole universe. This would give rise to a redshift corresponding to velocities that increase more than linearly with distance. No expansion is needed. Varying intervals of elementary space layers representing acceleration are greatly exaggerated.
Current models of the universe postulate a constant acceleration for all unbound stellar bodies that agrees with $a_0$, $H$, and $\Lambda$. Recessional velocities and redshift that increase with distance would suggest that an expansion is taking place.

Let’s take a look at the modified Newtonian ball model of gravity as applied to the whole universe. The postulated real universe (Fig 1) consists of a spacetime network with a constant acceleration ($a_0$) where the separation between neighbouring layers of the network decays linearly with radial distance, i.e., there is a constant gradient of the deformation in the radial direction that corresponds to the constant acceleration $a_0$. In our current cosmological models in contrast, flat space is assumed (Fig 2) and the intervals between neighboring nodes and layers of the network are constant, there is no definite acceleration in space. But we then observe that faraway galaxies seem to recede with a constant acceleration that turns out to be the same or very close to $a_0$ (Fig 2). From Hubble’s Law and previous observations on the redshift of moving stellar bodies, recessional velocities that increased linearly with distance was interpreted as the cumulative effect of a universe that was expanding. More precise observations in 1998 showed that the speeds were actually accelerating, as if a constant energy pervading all the space was driving them apart.

From general Relativity’s Equivalence Principle, the scenario of a flat universe and a cosmic accelerated expansion (Fig 2) is completely indistinguishable from a static universe with a constant gravitational field pervading it (Fig 1). All gravity-related phenomena –including redshift- must be present equally in both situations. A decision as to which is happening must come from external observations or reasoning. For any observer bound to a galaxy and placed a large distance away from us, it would be impossible to tell which one of these is happening:
1. The galaxy is accelerating away from the center of the universe by $a_0$ in flat space. (Fig 2). And since all neighboring galaxies are reporting the same, with velocities that increase with distance at a constant rate, the universe must be expanding all around them at an accelerated rate. Or else,

2. The galaxy is rotationally bound but static, immersed in a field with constant acceleration $a_0$ (Fig 1). And since all other galaxies are experiencing the same and the field does not decay as the inverse square of distance but remains constant, the conclusion would be that a constant acceleration due to a new form of gravity is pervading the whole universe.

Redshift alone would not allow them to tell which one is true. Of course, if there is evidence suggesting a non-homogeneous distribution of matter, or if we lack any ideas about the origin of the energy that drives the expansion, this would favor the conclusion that the accelerated outward motion is apparent and the real thing is a constant gravitational field that generates a redshift proportional to recessional velocity. Moreover, any interpretation that relies on option 1 must remain silent about the relation of $a_0$ with the Hubble parameter and the cosmological constant $\Lambda$. According to option 2, these are but two equivalent ways to represent the same phenomenon.

An objection could be raised from the fact that the equivalence principle guarantees the local equivalence of accelerations in flat space to gravitational fields in curved space, but the equivalence is not guaranteed globally. However, given that acceleration in this case is postulated to be constant everywhere and so would be the background curvature, we argue that an equivalence that is valid locally at all locations in the universe should be viewed as globally valid.

Notice that this does not invalidate redshift as an accurate indicator of velocity and distance for luminous stellar bodies. That redshift increases linearly with recessional velocity has been confirmed in multiple studies. Galaxies are either accelerating away from us with a very small acceleration $a_0$ (option 1, Fig 2) or they stay separated by a spacetime with constant curvature (option 2, Fig 1) but in both cases the amount of redshift is proportionally to their velocity along line of sight. When this velocity includes an additional local component due to a real movement with respect to neighbouring galaxies, redshift is a correct measure of that velocity provided the component due to expansion is subtracted and the ‘proper’ velocity obtained. The component due to expansion -that can also be viewed as due to curvature of intermediate space- is always proportional to distance from us, both according to options (1) and (2). Since $a_0$ is so small, this relation is approximately linear, and therefore redshift is a reliable indicator of distance at all but the largest scales. It is only with very precise measurements at cosmic scales that a non-linear, accelerated relation of redshift with position is noticeable. This is what the groups of Riess, Perlmutter and Schmidt accomplished in 1998, establishing a solid relation of redshift to velocities that are slowly increasing at a rate that scales with $\Lambda$ and $a_0$.

Discussion and Q&A
The first comment that comes to mind is how plausible a non-uniform distribution of matter is, given the fundamental character of homogeneity and isotropy in modern cosmology. Even assuming that inhomogeneities in the mass distribution are constrained by the CMB and the wide field observations of distant galaxies, small inhomogeneities like the ones predicted here cannot be currently ruled out. As for the theoretical arguments, given that we can only record with certainty a minute fraction of the matter assumed as present in the universe, this highlights our limitations to determine theoretical estimates for the mass density distribution.

On the other hand, a central concentration of mass density would intuitively make sense when considering masses governed only by gravity. As we have seen, densities that decrease with distance have been reported in the local universe, although they cannot be taken as representative of the whole universe.

To our knowledge, at least two authors, Lombriser in 2023, and Buchert, Roukema et al, 2008-2013 [13-15] have proposed that the universe expansion might be an apparent phenomenon caused by disturbances in the gravitational fields resulting in abnormal curvatures at cosmic scales. The origin of those curvatures is difficult to ascertain though, and neither proposal included a constant cosmological acceleration among its features.

Moreover, an agreement with observations without the need for dark matter or dark energy might count as partial supporting evidence. But a definitive answer must come from direct observations. Measuring large scale mass densities across the universe seems the most promising, if challenging path. The expected rate of decrease in mass density (about 1% per Mpc) might be within reach of our present technology.

Q1. -Assuming that mass density varies with radial distance in a model that was meant to describe uniformly distributed masses, does it not disprove the argument?

A. The Newtonian model for gravity in solid spheres is valid not only for spheres with uniform density, but for any sphere in which density depends only on radial distance, i.e, for any spherically symmetric distribution of matter.

Q2. -How well does the model support MOND as an effective theory for dark matter?

A. MOND has been considered by most authors either as a modification of the laws of gravity awaiting proper justification, or as a mere mathematical description based on the introduction of a new free variable $a_0$, to fit the observations. We would claim that it is neither. The main drawback of MOND has been so far its speculative nature and the arbitrary splitting of the gravitational law in two domains, corresponding to accelerations higher and lower than $a_0$. The view that $a_0$ is a real acceleration based on a plausible distribution of matter and on the Newtonian laws of gravity makes it more plausible. It would explain not only the value of $a_0$, but also the fact that MOND kicks in at a definite threshold acceleration. MOND asserts that the observed accelerations in MOND regime are the geometric average of Newtonian acceleration and $a_0$, and this can now be understood as the influence of a constant, background acceleration playing its role only.
when the Newtonian gravitational field is comparable or weaker than $a_0$.

Q3. -How does this model affect other parts of the current CDM cosmological framework such as dark energy and the Big Bang?

A. It is currently difficult to foresee the impact that a confirmation of the physical nature of $a_0$ and a non-homogeneous distribution of matter would bring about, but a revision of dark energy and the concept of expansion would certainly be among the first consequences. The idea that the observed expansion of the universe is an apparent phenomenon due to a constant cosmological acceleration seems a disturbing hypothesis that is almost guaranteed to earn strong rejection. And yet, most of our observations would remain valid. Our fundamental theories, in particular General Relativity would not only be unaffected but would be further confirmed, driving away recent suspicions that it might need modification. It would mean just a change in perspective, an indeed a more effective one in terms of explanatory power vs complexity. Perhaps it would also make Einstein happy, for he could finally do away with the cosmological constant, that dreaded parameter that he called the worst blunder of his life.

Q4. –MOND postulates a fundamental acceleration that affects local dynamics. Trying to obtain the same number from a large scale cosmological model, how can this give us the required local behavior?

A. MOND's acceleration scale $a_0$ has been also called the 'cosmological acceleration' ($a_L$) and has a very clear global meaning: it is a new scale for very low energy astrophysics that goes far beyond describing the rotations curves of galaxies. The local behavior of all galaxies observed so far with reliable data is determined by $a_0$. This was Milgrom's key idea, with no exception found so far. The whole point of this work is to examine whether this 'local' parameter reflects a real physical phenomenon, namely a physical acceleration that would operate at a cosmological level. The numerical coincidence of $a_0$ with the acceleration that results from applying Newtonian mechanics to the data for the whole universe is just another clue suggesting a cosmological significance for $a_0$, but it is not the only nor the main one. The other two are (1) the fact that all galaxies at all distances probed so far behave as being influenced by $a_0$, with dimensions of acceleration, and (2) the fact that in pure, simple Newtonian dynamics and assuming a matter density that decays inversely with radius, a constant acceleration appears necessarily, and must therefore agree with both accelerations, the one calculated at the border regions and the one operating locally at each galaxy. The numerical coincidence is predicted by the Newtonian equation and agrees with available data to a high degree.

Q5. -If the universe has been found to be approximately flat from multiple observational evidences, how can it contain a constant acceleration that behaves as a positive curvature?

A. The curvature implied in the Newtonian ball model is subsumed and interpreted in the FLRW model by the idea of expansion. It was argued that both concepts are equivalent and interchangeable, according to the Equivalence Principle. The only thing needed to
change the framework is to put away the assumption of homogeneity and to show that a
physical constant acceleration exists. A cosmological acceleration $a_0$ is then substituted
for the cosmological constant, and an equivalent universe without dark energy nor
expansion results. In $\Lambda$CDM as it stands, this is not possible, for both $\Lambda$CDM and the
Friedmann equations depend on a uniform and isotropic universe as their fundamental
starting point.

Q6. -Why don’t astrophysical observations support the view that there is a center of the
universe? Why is redshift approximately the same in all directions?

A. Unless we are very close to the actual center of the universe—which seems unlikely
though not impossible- there should be ways to tell where the center of the universe is,
or at least in what direction it lies. Masses and distances for faraway galaxies are hard to
assess by means other than redshift. Uncertainties grow at larger distances, and current
technology has limited power to resolve them at cosmic scales. On the other hand, it
seems that redshift should provide useful clues. Unfortunately, light reaching us from the
center regions of the universe is also expected to be redshifted, just as when it comes
from the periphery. The same reasoning applies as above. When looking towards the
center of the universe, a constant gravitational field is present, with us in the high-
potential side of it. The situation is equivalent to us moving away from the center, which
in turn is equivalent to the observed stars in the center moving away from us. Thus, light
received from stationary galaxies in the central regions of the universe would be also
redshifted. (Incidently, this is probably the main reason why the universe was viewed as
isotropic from the very start by cosmologists). However, when looking in the centripetal
direction, the number of galaxies and the mass density as a function of redshift should
increase as we approach the center, followed by a steep decrease when looking through
into the other side. Conversely, when looking in the other direction away from the center,
the number of galaxies and matter density should always decrease, as does the average
density in the theoretical model. The precision needed would still be a challenge, the
expected variations in matter density being of the order of $10^{-30}$ Kg/m$^3$/Mpc, or around
1% per Mpc for the mid-radius regions of the universe. As for the CMB signal itself, it
was pointed out the striking anisotropy in the CMB signal that is usually attributed to
relative motion of our local universe but, as can be imagined from the previous
considerations, the possibility cannot be ruled out that such a dipole is genuine and
arises from real differences when looking into the central or peripheral regions of the
universe.

Lastly, it can be argued that this discussion is limited to the non-relativistic case, where time is
absolute and space is treated quite naively. The observation would be pertinent, but we are not
all that sure that absolute space is not a real feature of the universe. Special and General
Relativity are the only correct descriptions of the universe we live in, but this might be only as
long as we do not take actual deformations of spacetime into account. Einstein himself warned
us in his famous Leiden address of 1920 that spacetime might in the end turn out to be a real
entity, albeit possibly an unmeasurable one. And perhaps by real, in this context, he meant
absolute. At any rate, the present discussion seeks to address very real problems in the
astrophysical realm that for the most part take place at sub-relativistic speeds. The approach is classical, intuitive and Newtonian because this is the only way our imagination can be put to work, but we expect that a formulation that respects Lorentz invariance and the postulates of Relativity should be eventually worked out.

Two developments are expected for the future that should help in solving these issues: On the one hand, accurate measurements of mass densities at large scales. High expectations are placed in this regard on the James Web Space telescope and on the recently launched Euclid Space Telescope. A high amount of data on large-distance galaxies up to 10 billion-light-years away are already being collected and analyzed with the specific goal of studying the behavior and nature of dark energy. On the other hand, an understanding of the deep mechanisms of gravity based on the fundamental structure of spacetime, a goal that both Newton and Einstein longed for unsuccessfully, would help in solving these and other standing problems in astrophysics and cosmology.
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