Title: A Chart that Shows Why the Thread Doesn't Break in Bell's Spaceship Paradox

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Abstract:

A chart is described that shows, according to inertial observers, the position versus time curves for an array of spaceships, all accelerating with the same constant acceleration (as confirmed by accelerometers). The initial slopes of the curves for the more distant spaceships are negative, which means that they are initially moving backwards (toward the trailing spaceship). And those slopes indicate that the speeds of those spaceships is greater than the speed of light. What is remarkable is that none of those results is problematical.

In the scenario that I am interested in, the spaceships all have attached accelerometers that all show the same constant acceleration "A" during the trip. Some interpretations of Bell's paradox are different, specifying that the accelerations are such that the initial inertial observers (who are stationary with respect to the spaceships immediately before the rockets are fired) will say that the distance between the spaceships is constant during the trip. THAT would require that the people on the trailing spaceship would say that the separation between the rockets is increasing, and that the rockets AREN'T producing the same thrust or the same acceleration. That is NOT the scenario I am interested in. I am interested in the scenario in which the people on the trailing spaceship say that the distance between the spaceships is constant during the trip, and that the accelerations are constant and identical.

Einstein, in his 1907 paper, https://einsteinpapers.press.princeton.edu/vol2-trans/319 , said that the distance "D" between two people who are accelerating with the same constant acceleration "A" ly/y/y (according to attached accelerometers) will be be constant. So he would have agreed that a thread connecting the two people would NOT break during their trip.

So, in my scenario, what do the initial inertial observers say about the separation of the rockets? The answer is simple, as required by the famous length contraction equation (LCE) of special relativity: an inertial observer will conclude that yardsticks moving at speed "v" relative to himself will be SHORTER than his own yardsticks, by the factor

gamma = 1 / {sqrt [1 - (v * v)] } ,

where "v" has units of ly/y, and

0 <= v < 1 .

So if the separation of the spaceships is constant at "D" ly, according to the people riding on the trailing spaceship, the initial inertial observers (of age "t") will say that the separation "S" between the trailing spaceship and the leading spaceship will decrease as "t" increases, as given by S(t) = D / gamma[v(t)] .

In order to plot the result, we first need to know what the trajectory of the trailing spaceship is. It is well-known that the speed of a rocket that starts accelerating at a constant “A” ly/y/y at t = 0, according to the initial inertial observers, is given by

$$v(t) = \tanh (A * t) .$$

The distance T(t) traveled by the trailing (“T”) spaceship is just the integral of v(t), which is

$$T(t) = (1 / A) * \ln [\cosh (A * t)] .$$

So in the chart I’m going to show, the curve which shows the trajectory of the trailing spaceship starts from the origin of the diagram, initially going almost horizontally to the right, and slowly increases its slope, until the slope approaches 1.0 ly/y (the speed of light).

Next, we need to determine the curve of the leading (“L”) spaceship. That’s easy: we’ve already determined above that the distance between the leading and trailing spaceships is

$$S(t) = D / \text{gamma}[v(t)] ,$$

so we just add that to the position of the trailing spaceship, to get

$$L(t) = T(t) + S(t) .$$

On the next page is the curve of the trailing spaceship (starting at the origin), and then four curves of the leading rocket, corresponding to four choices of the starting distance “D” between the leading and the trailing spaceship, for D = 0.5, 1.0, 2.0, and 3.0 ly.

~~Not correct.~~

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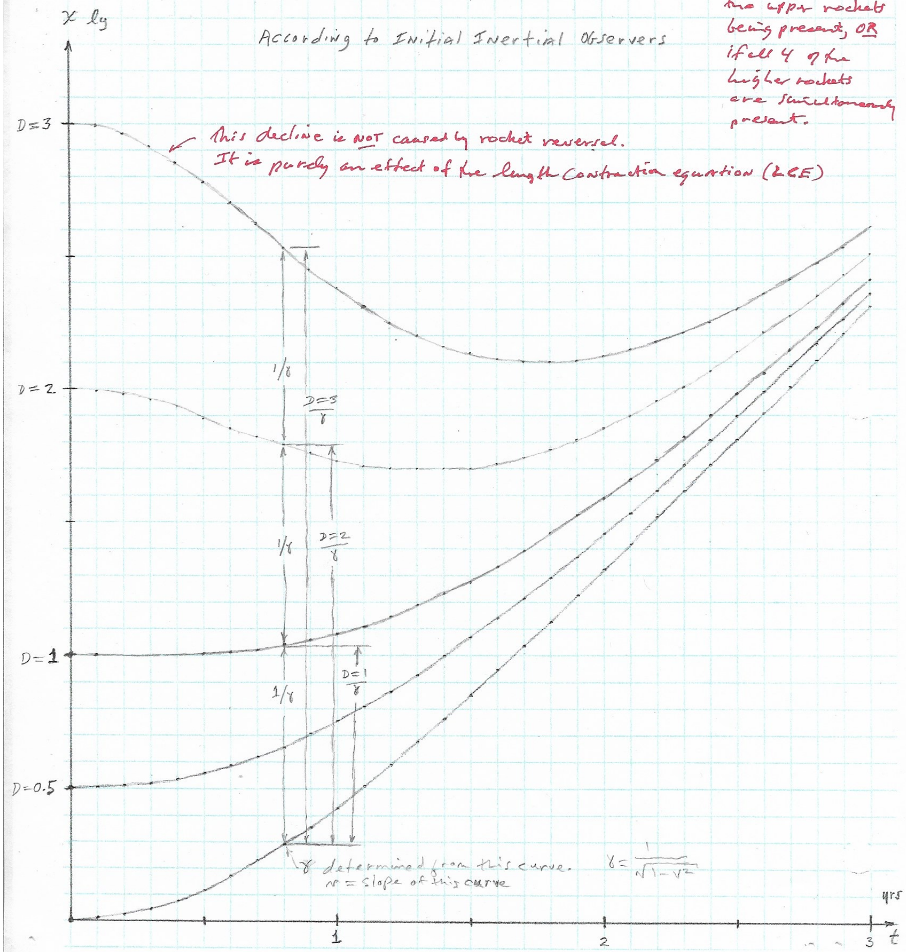
Two Simultaneously Accelerating Travelers
(Corrected Version)

I now* know it is correct. And the chart is correct both for only one of the upper rockets being present, OR if all 4 of the higher rockets are simultaneously present.

Acceleration = 1.0 ly/y/y (0.970 g)

~~Initial Seperation = 0.5 ly or 1.0~~

According to Initial Inertial Observers



You can think of the chart as one trailing spaceship and four different choices of the distance between the trailing spaceship and the leading spaceship.   OR, you can think of it as an array of FIVE separated spaceships that exist simultaneously.

All those vertical lines I've drawn show that for each choice of "t" (and I've chosen t = 0.8), the distance between each adjacent curve (excluding the D = 0.5 case) is equal to D / gamma.

The slope of each of the curves (at each value of "t") gives the speed of the spaceship at that instant. Look at the D = 3 curve. It curves downward at first, and the maximum slope in that region gives a velocity of more than 2 ly/y … more than twice the speed of light! But that isn't a problem. It is analogous to the situation where an inertial observer suddenly moves a yardstick (along the direction of its length), from zero speed to a speed near the speed of light. When the inertial observer does that, he will conclude that the ends of the yardstick approach each other at a speed MANY times larger than the speed of light. And that's not a problem! Neither is it a problem that the steep negative slope of the D = 3 curve is more than twice the speed of light … it is just an effect of the length contraction equation, nothing more.