Four-Dimensional Newtonian Relativity

Kennard Callender

ORCID ID: 0000-0002-7303-4848
Santiago de Veraguas, República de Panamá

Abstract

The constancy of the speed of light seems to imply that time and space are not absolute; however, we intend to show that that is not necessarily the case. In this article, we construct an alternative formulation to the theory of special relativity from the concepts of absolute time and absolute space defined by Newton and from the hypothesis that physical space is four-dimensional. We prove this formulation is mathematically equivalent to Einstein's theory by deriving the Lorentz transformation from the Galilean transformation for frames of reference in four-dimensional Euclidean space.

I. Introduction

According to Newton, time and space are absolute [1]. That means time and space exist independently from physical events and from each other. Furthermore, Newton argued that an object is at absolute rest if it is stationary with respect to absolute space or in absolute motion if it is moving with respect to absolute space [2]. For this reason, he contended that absolute space is a privileged frame of reference [3]. If Newton is correct, then the Galilean transformation should be the set of equations that accurately relate the time and space coordinates of two systems moving at a constant velocity relative to each other [4]. The constancy of the speed of light led Einstein to conclude that Newton's views are wrong; however, it can be shown that that conclusion is not necessarily true. In this article, we will construct an alternative formulation to the theory of special relativity from the postulates that time and space are absolute and from the hypothesis that physical space is four-dimensional. We will prove the mathematical validity of this formulation by deriving the Lorentz transformation from the Galilean transformation for frames of reference in four-dimensional Euclidean space.

II. Postulates

The alternative formulation to the theory of special relativity that we propose is based on the following postulates:

- Time and space are absolute.
- Space is four-dimensional.
- All objects move at the speed of light with respect to absolute space.

The first postulate refers to the same concepts defined by Newton in 1687. The second postulate states that physical space is a four-dimensional Euclidean space; that is our fundamental hypothesis. The third postulate posits that objects are never at rest with respect to absolute space and move only at one speed with respect to it: the speed of light. That proposition is similar to the one obtained from the theory of relativity, which asserts that all objects move through spacetime at the speed of light. These three postulates differ from Nordström's electromagnetic-gravitational theory and the Kaluza-Klein theory in that time and space are not absolute and space is not Euclidean in those formulations [5–9].

The mathematical formalism of these postulates is the following:

- The coordinates of two systems that move at a constant velocity relative to each other are related by the transformations of the Galilean group (any composition of uniform motions, translations and rotations in four-dimensional Euclidean space).
- Five equations are required to relate the coordinates between two inertial frames of reference (four equations for the spatial coordinates and one equation for the temporal coordinate).
- The speed between any inertial frame of reference and absolute space must be equal to the speed of light.

In addition to suggesting postulates about the nature of time and space, we need to take into account that the fundamental theories of modern physics are grounded on the assumption that space is three-dimensional. This remark can be stated as follows:

- The current scientific paradigm presupposes that space only has three dimensions, but if space actually has four dimensions, then that erroneous assumption would have affected the interpretation of experimental results and the formulation of the fundamental theories in modern physics.

We shall refer to this statement as the observer's principle (due to the role visual perception plays). This principle is inherently different from the aforementioned postulates because it does not describe the physical world. Instead, it points out that the wrong assumption made about the dimensionality of space during the process of observation would have led to a wrong interpretation of experimental results and, consequently, would have affected the mathematical formulation of fundamental theories such as quantum mechanics and relativity.

In the context of special relativity, the observer's principle highlights the fact that Hendrik Lorentz and Albert Einstein derived the Lorentz transformation under the assumption that space only has three dimensions. This means that they did not represent the fourth rectangular component of the position and velocity vectors in their formulations, implicitly assigning them a value of zero. Therefore, the assumption that space is three-dimensional affected Lorentz's and Einstein's formulations in the following way:

- They implicitly assigned a value of zero to the fourth spatial coordinates of physical events and to the fourth component of the velocity vector between inertial frames of reference.
- They assumed that only four equations are required to relate the coordinates between two inertial frames of reference (three equations for the spatial coordinates and one equation for the temporal coordinate).

In the next section, we will show how the Lorentz transformation emanates from these two statements.

The three postulates presented here, in conjunction with the observer's principle, constitute the four-dimensional Newtonian formulation of special relativity (or four-dimensional Newtonian relativity, for short). We want to emphasize that the postulates are proposed as a description of reality, whereas the observer's principle is offered as an explanation of how the Lorentz transformation emerges from the wrong assumption about the dimensionality of space. Thus, this formulation contains two sets of equations: the first one gives a mathematical description of the physical world as it actually is (the Galilean transformation for frames of reference in four-dimensional Euclidean space), while the second one arises from the assumption that space is three-dimensional (the Lorentz transformation). It is the latter set of equations that is mathematically equivalent to the theory of special relativity.

III. Derivation

In this section, we will prove that the four-dimensional Newtonian formulation of special relativity is mathematically equivalent to Einstein's theory. To do this, we will use four rectangular coordinate systems: S, A, A′ and S′. Each system contains four coordinates that specify the position of a physical event in four-dimensional Euclidean space and a time coordinate that specifies the instant in which that event takes place. Hence, the coordinates of an event E for each system are:

- (x_1, x_2, x_3, x_4, t) according to S
- (X_1, X_2, X_3, X_4, T) according to A
- $(X_1', X_2', X_3', X_4', T')$ according to A′
- $(x_1', x_2', x_3', x_4', t')$ according to S′

The instant in which an event takes place is independent of the reference frame it is measured from (time is absolute). Therefore, we have that

$$t = T = T' = t' \tag{1}$$

The specific form of the Lorentz transformation that we will derive is the Lorentz boost in the x_1 direction. We will derive it from a composition of three transformations of the Galilean group (two uniform motions and a rotation in four-dimensional Euclidean space). The configuration of that composition is the following: The four coordinate systems only move on the plane that contains the axes of the first and fourth dimensions, such that for any event E, we have

$$x_2 = X_2 = X_2' = x_2' \tag{2}$$

$$x_3 = X_3 = X_3' = x_3' \tag{3}$$

The coordinate systems A and A′ are fixed with respect to absolute space, and their axes are rotated according to

$$X_1' = X_1 \, cos\, \theta - X_4 \, sin\, \theta \tag{4}$$

$$X_4' = X_1 \, sin\, \theta + X_4 \, cos\, \theta \tag{5}$$

where θ is the angle of rotation ($-90° \leq \theta \leq 90°$). Solving for X_1 and X_4 in equations 4 and 5 gives

$$X_1 = X_1' \, cos\, \theta + X_4' \, sin\, \theta \tag{6}$$

$$X_4 = -X_1' \, sin\, \theta + X_4' \, cos\, \theta \tag{7}$$

The coordinate system S represents an inertial frame of reference. It moves along the common axis X_4–x_4. According to the postulates we propose, inertial frames of reference move at the speed of light with respect to absolute space. Thus, the Galilean transformation equations are

$$x_1 = X_1 \tag{8}$$

$$x_4 = X_4 - ct \tag{9}$$

where c is the speed of light. Likewise, the coordinate system S′, which also represents an inertial frame of reference, moves at the speed of light along the common axis X_4'–x_4'. Consequently, the Galilean transformation equations for this case are

$$x_1' = X_1' \tag{10}$$

$$x_4' = X_4' - ct' \tag{11}$$

Lastly, the velocity $\vec{v}$ of the frame of reference S′ with respect to the frame of reference S is

$$\vec{v} = v_1\,\hat{x}_1 + v_4\,\hat{x}_4 \tag{12}$$

where v_1 is the velocity component along the x_1 axis, v_4 is the velocity component along the x_4 axis, $\hat{x}_1$ is the unit vector along the x_1 axis and $\hat{x}_4$ is the unit vector along the x_4 axis. These equations (1–12) completely specify the configuration of the system.

The mathematical relation between the coordinates of S and S′ can be obtained by solving for X_1, X_4, X'_1 and X'_4 in equations 8, 9, 10 and 11, respectively, and substituting them into equations 4, 5, 6 and 7:

$$x'_1 = x_1\,cos\,\theta - (x_4 + ct)\,sin\,\theta \tag{13}$$

$$(x'_4 + ct') = x_1\,sin\,\theta + (x_4 + ct)\,cos\,\theta \tag{14}$$

$$x_1 = x'_1\,cos\,\theta + (x'_4 + ct')\,sin\,\theta \tag{15}$$

$$(x_4 + ct) = -x'_1\,sin\,\theta + (x'_4 + ct')\,cos\,\theta \tag{16}$$

Equations 13, 2, 3, 14 and 1 give us the Galilean transformation for the configuration described in this section. The corresponding inverse Galilean transformation is given by equations 15, 2, 3, 16 and 1. These transformations provide the complete relationship between the inertial frames of reference S and S′ when describing a single event occurring in four-dimensional Euclidean space. Using equation 1 to substitute t' for t and t for t' in equations 13, 14, 15 and 16 gives

$$x'_1 = x_1\,cos\,\theta - (x_4 + ct')\,sin\,\theta \tag{17}$$

$$(x'_4 + ct) = x_1\,sin\,\theta + (x_4 + ct')\,cos\,\theta \tag{18}$$

$$x_1 = x'_1\,cos\,\theta + (x'_4 + ct)\,sin\,\theta \tag{19}$$

$$(x_4 + ct') = -x'_1\,sin\,\theta + (x'_4 + ct)\,cos\,\theta \tag{20}$$

These equations (together with equations 1, 2 and 3) also provide a valid and accurate relationship between the inertial frames of reference S and S′ when describing a single event E.

As discussed in the previous section, Hendrik Lorentz and Albert Einstein derived the Lorentz transformation under the assumption that space is three-dimensional. This led them to implicitly assign a value of zero to the fourth spatial coordinates of physical events and to the fourth component of the velocity vector between inertial frames of reference (the observer's principle). This means that

$$x_4 = 0 \tag{21}$$

$$x'_4 = 0 \tag{22}$$

$$v_4 = 0 \tag{23}$$

By substituting eq. 23 into eq. 12, we arrive at the first way in which the assumption that space is three-dimensional affects the formulation of the Lorentz boost in the x_1 direction:

$$\vec{v} = v_1\,\hat{x}_1$$

$$v = v_1 \tag{24}$$

In special relativity, v is interpreted as being the velocity between the inertial frames of reference S and S′ along their common x_1–x'_1 axis. However, according to four-dimensional Newtonian relativity, v is only the component of the velocity of S′ with respect to S along the x_1 axis. That velocity component can be obtained from eq. 19 by taking the derivative of x_1 with respect to t:

$$v_1 = dx_1/dt = c\, sin\, \theta \tag{25}$$

Now we are ready to derive the Lorentz transformation and its inverse. First, we substitute eq. 25 into eq. 24 and solve for $sin\, \theta$:

$$sin\, \theta = \frac{v}{c} \tag{26}$$

Then we use the Pythagorean trigonometric identity to obtain the function of $cos\, \theta$, so that

$$cos\, \theta = \sqrt{1 - sin^2\, \theta} \tag{27}$$

Next, we substitute eq. 26 into eq. 27:

$$cos\, \theta = \sqrt{1 - \frac{v^2}{c^2}} \tag{28}$$

The Lorentz factor is a term that frequently appears in the equations of special relativity. It is given by

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{29}$$

Consequently, we have that

$$cos\, \theta = \frac{1}{\gamma} \tag{30}$$

The mathematical consequences of the observer's principle are represented by equations 21, 22, 26 and 30. For this reason, we substitute them into equations 17, 18, 19 and 20:

$$x_1' = \frac{x_1}{\gamma} - vt' \tag{31}$$

$$ct = \frac{vx_1}{c} + \frac{ct'}{\gamma} \tag{32}$$

$$x_1 = \frac{x_1'}{\gamma} + vt \tag{33}$$

$$ct' = -\frac{vx_1'}{c} + \frac{ct}{\gamma} \tag{34}$$

The last step of the derivation is to solve for the coordinates x_1, t', x_1' and t in equations 31, 32, 33 and 34, respectively. This gives us:

$$x_1 = \gamma(x_1' + vt') \tag{35}$$

$$t' = \gamma\left(t - \frac{vx_1}{c^2}\right) \tag{36}$$

$$x_1' = \gamma(x_1 - vt) \tag{37}$$

$$t = \gamma\left(t' + \frac{vx_1'}{c^2}\right) \tag{38}$$

Equations 37, 2, 3 and 36 form the Lorentz transformation for inertial frames of reference that move relative to each other at a constant velocity v along their common x_1–x'_1 axis (also known as the Lorentz boost in the x_1 direction). That transformation is given by

$$x'_1 = \gamma(x_1 - vt) \tag{39}$$

$$x'_2 = x_2 \tag{40}$$

$$x'_3 = x_3 \tag{41}$$

$$t' = \gamma\left(t - \frac{vx_1}{c^2}\right) \tag{42}$$

Likewise, equations 35, 2, 3 and 38 form the corresponding inverse transformation, which is

$$x_1 = \gamma(x'_1 + vt') \tag{43}$$

$$x_2 = x'_2 \tag{44}$$

$$x_3 = x'_3 \tag{45}$$

$$t = \gamma\left(t' + \frac{vx'_1}{c^2}\right) \tag{46}$$

Notice that only four equations (instead of five) relate the coordinates between the inertial frames of reference S and S′. This stems from the assumption that space is three-dimensional, which is what the second mathematical consequence of the observer's principle predicted. Also notice that the equation discarded is the one that represents that time is absolute (eq. 1). This remark completes the derivation. The more general form of the Lorentz transformation can be obtained by extending the procedure presented in this article.

As a final note, we want to emphasize that the Galilean transformation derived in this section (given by equations 17, 2, 3, 18 and 1) and its corresponding inverse (equations 19, 2, 3, 20 and 1) describe a single event. However, when eq. 1 is discarded and the mathematical conditions of the observer's principle are imposed (equations 21, 22 and 23), then the resulting equations describe two events that occur at the same place but at different times. That would be the interpretation of this result from a mathematical perspective. From a physical perspective, this result tells us that the effects from the Lorentz transformation (such as time dilation, length contraction and the constancy of the speed of light) are actually depth perception effects that are being interpreted as real effects because the fourth spatial dimension is not being taken into account. Other interesting remarks can be made about this derivation, but we will address them more profoundly in a future paper.

IV. Conclusion

An alternative formulation to the theory of special relativity was constructed from the concepts of absolute time and absolute space defined by Newton and from the hypothesis that physical space is four-dimensional. The formulation contains two sets of equations: the first one describes the physical world as it actually is, while the second one emerges from the wrong assumption about the dimensionality of space. The second set of equations (the Lorentz transformation) is mathematically equivalent to Einstein's theory; however, the interpretation of those equations is significantly different. The four-dimensional Newtonian formulation of special relativity interprets the effects predicted by the Lorentz transformation as depth perception effects, whereas special relativity interprets them as being real. Therefore, four-dimensional Newtonian relativity proves that the constancy of the speed of light is not necessarily incompatible with the concepts of absolute time and absolute space. Furthermore, and perhaps more remarkable, the result presented here could be seen as mathematical evidence that physical space is four-dimensional.

Dedication

This article is dedicated to the memory of my father, Dr. Lorenzo León Callender López, who always supported me and was there for me. Without him, this work would not have been possible.

References

[1] Newton, I. *The Mathematical Principles of Natural Philosophy*. New-York, D. Adee, 1848, p. 77. https://www.loc.gov/item/04014428/

[2] Dainton, B. *Time and Space*. 2nd ed., McGill-Queen's University Press, 2010, p. 182. https://archive.org/details/timespace0000dain/page/182

[3] Dainton, B. *Time and Space*. 2nd ed., McGill-Queen's University Press, 2010, pp. 165–166. https://archive.org/details/timespace0000dain/page/165

[4] Arnold, V. *Mathematical Methods of Classical Mechanics*. 2nd ed., Springer-Verlag, 1989, pp. 3–6. https://archive.org/details/mathematicalmeth0000arno/page/3

[5] Nordström, G. "On the Possibility of a Unification of the Electromagnetic and Gravitation Fields". *Modern Kaluza-Klein Theories*, eds. Appelquist et al., Addison-Wesley, 1987, pp. 50–56. https://archive.org/details/modernkaluzaklei0000unse/page/50

[6] Kaluza, T. "On the Unity Problem of Physics". *Modern Kaluza-Klein Theories*, eds. Appelquist et al., Addison-Wesley, 1987, pp. 61–68. https://archive.org/details/modernkaluzaklei0000unse/page/61

[7] Klein, O. "Quantum Theory and Five Dimensional Theory of Relativity". *Modern Kaluza-Klein Theories*, eds. Appelquist et al., Addison-Wesley, 1987, pp. 76–87. https://archive.org/details/modernkaluzaklei0000unse/page/76

[8] Klein, O. "The Atomicity of Electricity as a Quantum Theory Law". *Modern Kaluza-Klein Theories*, eds. Appelquist et al., Addison-Wesley, 1987, p. 88. https://archive.org/details/modernkaluzaklei0000unse/page/88

[9] Einstein, A., and P. Bergmann. "On a Generalization of Kaluza's Theory of Electricity". *Modern Kaluza-Klein Theories*, eds. Appelquist et al., Addison-Wesley, 1987, pp. 89–107. https://archive.org/details/modernkaluzaklei0000unse/page/89