Calculation of the energy density parameter \( \rho \) in the cosmological model \( \Lambda \)CDM and through experimental data

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Abstract

We have calculated the energy density parameter, which appears in the cosmological model \( \Lambda \)CDM in the Friedman equation, using cosmological data values from recent experimental results reported in the literature. A statistical calculation has been performed to determine its uncertainty. Finally, the result is commented linking it with the possible existence of dark energy.

Keywords

Dark energy, \( \Lambda \)CDM model, general relativity.

1.- Equation to solve

According to the Friedman equation, the Hubble parameter \( H \), the Gaussian curvature of the space-time \( K \), and the energy density parameter \( \rho_m \), are related in the \( \Lambda \)CDM cosmological model as follows [1].

\[
H^2 = \left( \frac{a'}{a} \right)^2 = \frac{8\pi G \rho_m + \Lambda c^2}{3} - Kc^2 \tag{1}
\]

being “a” the scale factor of the universe, \( G \) the universal gravitational constant, \( \Lambda \) the cosmological constant. The energy density parameter \( \rho_m \) is commonly called \( \rho \) in the literature.

The critical density \( \rho_c \) is called the energy density that results when in the Friedman equation the cosmological constant and the curvature are made equal to zero. Thus, according to equation (1) this critical density is given by:

\[
H^2 = \frac{8\pi G \rho_c}{3}
\]

\[
\rho_c = \frac{3H^2}{8\pi G} = 0.92 \times 10^{-26} \text{ Kg/m}^3
\]

We transform equation (1) in the following way,

\[
H^2 = \left( \frac{a'}{a} \right)^2 = \frac{8\pi G (\rho_m + \rho_\Lambda + \rho_k)}{3}
\]
where $\rho_m$ is the energy density related to mass, $\rho_\Lambda$ is the energy density of vacuum space, related to the cosmological constant by the equation $\rho_\Lambda = \Lambda c^2/8\pi G$ and $\rho_k$ is the energy density related to the curvature of space-time $\rho_k = Kc^2/8\pi G$.

We propose to calculate $\rho_m$ from recent experimental data:

To determine $\rho_k$ we use, among others, data from the Planck mission [3], there the parameter $\Omega_k = Kc^2/H^2$ was measured as curvature.

Substituting this expression in equation (1) and dividing both sides of the equation by $H^2$, results in the equation that we are going to use to calculate the energy density $\rho_m$:

$$1 = \frac{8\pi G}{3H^2}(\rho_m + \rho_\Lambda) + \Omega_k$$  \hspace{1cm} (2)

2.- Experimental data to use

The energy density of the vacuum space $\rho_\Lambda$ is referred to [2]:

$$\rho_\Lambda = (0.603 \pm 0.013) \times 10^{-26} \text{ kg/m}^3$$

The curvature of space-time is referred to [3]:

$$\Omega_k = 0.001 \pm 0.002$$

$$-0.001 \leq \Omega_k \leq 0.003$$

The Hubble parameter, is referred to [4]:

$$H_0 = 73.52 \pm 1.62 \text{ km/s por mega parsec, igual a } 2.35 \times 10^{-18} \text{ m/s por metro}$$

3.- Calculation of the parameter $\rho_m$

According to equation (2)

$$1 = \frac{8\pi G}{3H^2}(\rho_m + \rho_\Lambda) + \Omega_k$$

doing:

$$G = 6.67 \times 10^{-11} \text{ N.m}^2/\text{Kg}^2$$

result:

$$\rho_m = 0.3848 \times 10^{-26} \text{ Kg/m}^3$$

4.- Calculation of the associated uncertainty

We calculate the uncertainty associated with the energy density parameter $\rho_m$ in accordance with the regulatory recommendations contained in [5]:
### Experimental data

<table>
<thead>
<tr>
<th>$X = X \pm K\sigma$</th>
<th></th>
<th>Associated uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K=2$</td>
<td>$\rho_\Lambda = (0.603 \pm 0.013) \times 10^{-26} \text{ kg/m}^3$</td>
<td>$\sigma_\Lambda = 1.1%$</td>
</tr>
<tr>
<td>$H_0 = 73.52\pm1.62 \text{ sec}^{-1}$</td>
<td>$\sigma_{H} = 1.1%$</td>
<td></td>
</tr>
<tr>
<td>$K=2$</td>
<td>$\Omega_k = 0,001\pm0,002$; $\Omega_\Lambda = -0,001; \rho = 0,3867$</td>
<td>$\sigma_{\Omega_k} = 0.5%$</td>
</tr>
<tr>
<td></td>
<td>$\Omega_k = -0.003; \rho = 0.3828$</td>
<td>$\sigma_\rho = 1.6%$</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 0.0019$</td>
<td>$\sigma_\rho = 0.0019$</td>
</tr>
</tbody>
</table>

### Result:

$$\rho = \rho \pm 2\sigma$$

$$\rho = \rho_m = (0.385 \pm 0.012) \times 10^{-26} \text{ Kg/m}^3$$

### 5. Conclusions

In the $\Lambda$CDM model, this parameter $\rho_m$ represents the value of the energy density in space due to the existence of gravitational mass. It is therefore associated with the mass of the cosmos (baryonic mass and dark matter). The first thing that surprises us about our result is its lower value than the experimental value of the vacuum energy density of $0.603 \times 10^{-26} \text{ Kg/m}^3$. Our calculation of the parameter $\rho_m$, with a value of $0.385 \times 10^{-26} \text{ Kg/m}^3$ seems to indicate that in addition to the energy due to mass, there is another type of energy in the cosmos that could well be dark energy and that it does contribute to the density of real energy measured experimentally in the value of $\rho_\Lambda$. According to our results, this dark energy density would be at least of the order of the difference between $\rho_\Lambda$ and $\rho_m$, that is, $0.218 \times 10^{-26} \text{ Kg/m}^3$.

Thus, the result of our calculation leads us to the need for the existence of a necessary energy beyond that corresponding to that created by the gravitational masses, the existence of dark energy responds to this need.

### Referencias


[3] Planck Collaboration, Aghanim, N., Akrami, Y. et al. 2020, September,