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Abstract
In this paper I give a major update of quantum gravity framework project. The heuristic conceptual framework proposed in previous versions is expanded to include structure formation and consciousness in the universe. A Path Integral version of decoherence in curved space-time is introduced as major update. Then we discuss the philosophical insights into structure formation in the universe and consciousness. We introduce various mathematical concepts to describe structure formation in the universe and consciousness.

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1 Review and Introduction

In this paper, I do next update of the proposal for conceptual framework of quantum general relativity [3]. The previous quantum gravity framework of the papers are canonical in nature. But universe as described by the established laws is overwhelmingly covariant in nature. So in this paper update the framework to make it covariant. Here we will be also generalizing the entire project further to include consciousness and structure formation in the universe.

In section 2.0, I introduce the Lagrangian formulation of Lindblad type evolution equation. This is the generalization of the non-covariant formalism as introduced in the previous version [4]. In this paper, in general, I don’t do detailed study of the idea, as the formalism is not yet ready for applications. But simply formally discuss how to apply to some mathematical simple situations. The relevance of self-time and rest-frame foliation is relevant if the world is observed by an observer who converts mixed states into pure states. The Lagrangian formulation also requires a preferred foliation to be observed by an observer that converts mixed states into pure state.

In quantum gravity framework 3.0, I discussed the rest frame evolution, in which gravitational fields and other fields are least changing. Rest frame evolution is the most natural foliation in which the observer observes the world, and the observer is the universe itself observing itself. In section 3.3, I also discuss the covariant generalization of the rest frame evolution to be combined with the Lagrangian formulation of quantum gravity framework 4.0.

In section 4 we discuss the application of the path integral form of decoherence to cosmology. I introduce a decoherence function. I briefly discuss the application to various simple cases, and resolution of singularities.

In section 5, I start extending quantum gravity framework project to include consciousness and structure formation in the universe. I build on philosophical insights from my book [31]. In section 5.1, I discuss how global quantum reduction and rest-frame foliation is related to consciousness.

In section 5.2, I discuss the theoretical aspects for understanding relationship structures and consciousness. In section 5.3 I discuss how structures are related to conscious experience based on insights from neuroscience.

When the universe rises during big-bang expansion, initially it was mostly structureless. But eventually structures rises out of the universe at various scales: galaxies, stars, planets, matter, etc. Now matter which initially was inorganic, evolves into organic and eventually into living biological entities that consciously observes the Universe. To develop proper unification of concepts in fundamental physics we need sufficient conceptual foundation that describes the universe fully at conceptual level. This will also address structure formation in the universe and consciousness.

1 For the latest updates proper discussions, comments and issues, please visit www.qstaf.com. Much of the discussions, updates and supplementary downloadable materials regarding this project will be mostly available on www.qstaf.com, and other websites referred to such as the researchgate: the link for quantum gravity framework project is https://www.researchgate.net/project/Quantum-Gravity-Framework. Update information will be provided on social media also (www.qstaf.com/links).
Until we have discussing only the universe without relevance to conscious observer. But including con-
scious observer is important for many different reasons:

1. Quantum mechanics needs the observer in the quantum measurement process.

2. Many things about the universe such as the value of physical constants can be explained easily by
   presence of observers such as in anthropomorphic principle.

3. Matter naturally evolves into living thing and conscious arises as inherent property of matter in the
   universe.

So discussing the universe without discussing life and consciousness is incomplete. In this section we
will explore rise of structures in the universe, and consciousness. To understand the rise of structures and
consciousness, I built on insights from my book [31]. To understand grand unification of the human knowledge
based in this paper I refer to [30].

There are some important changes in this update, I don’t assume space or time is discretized. If I use
discrete model in this paper, it is only for explanatory purpose. Further research needs to be done regarding
this. The framework presented is not-yet ready for application, because it is not yet complete. The ideas
are brief and quite heuristic in this paper. The purpose of this paper is to establish an initial conceptual
framework for quantum gravity. Further research needs to be done to complete the framework. This research
will be further updated. Please follow the updates online in the sources mentioned above.

I apologize for typos and grammatical mistakes in this paper, and the previous papers related to this
paper. This paper is only a rough draft of work in progress.

We follow the following conventions in this article:

**Convention 1:** In any integral, the variables over which the integration is done is same those used in the
measure placed in the right most end of the integral, unless explicitly indicated otherwise.

**Convention 2:** Summation is assumed for all repeated Greek indices in the explicit elementary products
of the basic variables of the theories discussed.

**Convention 3:** In the differential measures of the integrals, the multiplication over all the suffixes and the
prefixes is assumed, for example $dx^\alpha dy_\gamma$, mean $\prod_{\beta, \gamma} dx^\alpha dy_\gamma$.

**Convention 4:** For functions with arguments that have suffixes, prefixes, and parameters: The function
depends on all the collection of the arguments for all different values of the suffixes, the prefixes and the
parameters. Example: $f(x^\alpha_\gamma(t), y_\beta) = f(X)$, where $X = \{x^\alpha_\gamma(t), y_\beta, \forall \alpha, \beta, \gamma, t\}$.

**Convention 5:** No other summation or multiplication of repeated indices is assumed other than those defined
in conventions 2 and 3. Examples: 1) there no summation in $f_\alpha(x^\alpha, y_\alpha)$, the three $\alpha$’s are independent, 2)
$(p^\beta_\gamma x_\alpha + f_\beta(x^\gamma, y_\alpha))dx^\alpha dy_\gamma = (\sum_\alpha p^\beta_\gamma x_\alpha + f_\beta(x^\gamma, y_\alpha))\prod_{\eta, \xi} dy_\eta dx^\xi$.

**Convention 6:** It is assumed that $\hbar = c = G = 1$, unless specified.

## 2 Path Integral Form of Decoherence

All the notations used in this section were defined in the in section 2.2 of previous update [3].
2.1 The theory for simple systems

The version of equations involving decoherence described in quantum gravity framework 3.0 [4] is not in path integral form unlike the other three proposals of the framework. For this purpose, in this section I will propose a path integral formulation of decoherence as an alternative. In this section, I will have work out different formalism of understanding density matrix and from there proceed to a path integral formulation of decoherence.

In this section let me work in flat space-time. Let me consider the density matrix as an element of the space of outerproducts of the Hilbert Space $\mathcal{H}$ and its Hermitian Conjugate space $\mathcal{H}^\dagger$ of a system. In the following let $\tilde{y}$ represent operators that only act on the variables that belong to the Hermitian conjugate space. Let me explain this in a simple example in one dimension. If $x$ and $\tilde{x}$ are element of $\mathcal{H}$ and $\mathcal{H}^\dagger$. The we have $\rho(x, \tilde{x})$ as a quantum state in $\mathcal{H} \otimes \mathcal{H}^\dagger$. Then the Hamiltonian evolution of the evolution is

$$\frac{d\rho(x, \tilde{x})}{dt} = iH\rho(x, \tilde{x}) - i\rho(x, \tilde{x})\tilde{H}$$

Above $\tilde{H}$ only acts on $\tilde{x}$. Because of the commutator in the left, the trace $\int \rho(x, x)dx$ is preserved in this evolution. If we choose $\rho(x, \tilde{x}) = \rho(\tilde{x}, x)$ and set the trace to be one, we then have that $\rho(x, \tilde{x})$ is a density matrix, as the above evolution preserves these conditions.

The path integral form of this is formally,

$$\langle \rho(x_1, \tilde{x}_1, t_1) | \rho(x_2, \tilde{x}_2, t_2) \rangle = \int_{\gamma, \tilde{\gamma}} \exp(iL(\gamma) - i\tilde{L}(\tilde{\gamma}))D\gamma D\tilde{\gamma}$$

where $\gamma$ and $\tilde{\gamma}$ are paths from $x_1$ to $x_2$ and $\tilde{x}_1$ to $\tilde{x}_2$ respectively, $D\gamma$ and $D\tilde{\gamma}$ are path integral measure corresponding to paths in $\mathcal{H}$ and $\mathcal{H}^\dagger$ space, and, 1 and 2 represents initial final states. If the $\rho(x, \tilde{x})$ is such that $\rho(x, \tilde{x}, t) = \psi(x, t)\tilde{\psi}(\tilde{x}, t)$, that is pure, then both the Hamiltonian and Path integral forms splits into two separate pieces.

$$\frac{d\psi(x)}{dt} = iH\psi(x, t)$$

$$\frac{d\tilde{\psi}(\tilde{x})}{dt} = -i\tilde{\psi}(\tilde{x}, t)\tilde{H}$$

$$< \psi(x_1, t_1) | \psi(x_2, t_2) >= \int_{\gamma} \exp(iL(\gamma))D\gamma$$

$$< \tilde{\psi}(\tilde{x}_1, t_1) | \tilde{\psi}(\tilde{x}_2, t_2) >= \int_{\tilde{\gamma}} \exp(-i\tilde{L}(\tilde{\gamma}))D\tilde{\gamma}$$

Basically we have doubled the Hilbert Space. The two evolutions are independent. This description is redundant if the states or pure or if there is no decoherence. Now we will introduce decoherence in the path integral formulation to make it covariant in quantum field theory sense.

**Proposition 1** The covariant evolution of state $\rho(t)$ of the system is formally given by

$$\langle \rho(t_1) | \rho(t_2) \rangle = \int_{\gamma, \tilde{\gamma}} \exp \left[ iL(\gamma) - i\tilde{L}(\tilde{\gamma}) - \beta d(\gamma, \tilde{\gamma})^2 \right] D\gamma D\tilde{\gamma}$$

where $d(\gamma_1, \gamma_2)$ is a measure of distance between field configuration paths $\gamma_1$ and $\gamma_2$ in the topological space of field configurations, such that $d(\gamma, \gamma) = 0$. The Lagrangian $L$ and distance $d$ are to be covariant under Lorentz transformations in special relativity. The $\beta$ is constant representing the strength of decoherence, which is small enough that the Hamiltonian evolution is not severely affected, $E_p$ is Planck mass.

The $d(\gamma, \tilde{\gamma})^2$ term helps to implement decoherence between the two paths. Let me call $d(\gamma, \tilde{\gamma})$ as the decoherence function. The differential form of this equation is heuristically,

$$dL = \beta \int d^4x d^4\tilde{x} d(\gamma, \tilde{\gamma})^2$$
\[ d\rho(x, \tilde{x}) = iH \rho(x, \tilde{x})dt - i\rho(x, \tilde{x})\dot{H}dt - \beta d(x, \tilde{x})\rho(x, \tilde{x}) \]

The presence of \( d(x, \tilde{x}) \) removes the cross terms over time and only preserves the diagonal terms.

Let me try to decide what must be \( d(\gamma, \tilde{\gamma}) \). Simplest choice is something like this in one dimensional case:

\[ d(\gamma, \tilde{\gamma}) = \int (x(t) - \tilde{x}(t))^2 dt \]

In full space-time situation, we want this to be coordinate independent in the general relativistic sense. We will give a choice in the next section.

3 Quantum gravity framework 4.0

3.1 Simple Quantum System

Consider a simple quantum system which is described by a Hamiltonian constraint only. Let the internal configuration space of the quantum system is of dimension \( d \), and is made of canonical variables \( p_\alpha \) and \( q^\alpha \). Let \( q^\alpha \) takes values in configuration space \( R^n \). Let \( m_{\alpha \beta} \), a function of \( q^\alpha \), is the metric in the internal configuration space. Hereafter I will use \( m_{\alpha \beta} \) and its inverse \( m^{\alpha \beta} \) (assuming it exists), to raise and lower indices. Usually \( m_{\alpha \beta} \) is simply a delta matrix \( \delta_{\alpha \beta} \) multiplied by mass \( m \).

Let me define a scalar product using the metric:

\[ \langle a, b \rangle = \frac{1}{2} a_\alpha b_\beta m^{\alpha \beta}. \]

I will assume \( m^{\alpha \beta} \) is positive definite for now.

We can make the following standard definitions:

\[ \text{Norm } |p| = +\sqrt{m^{\alpha \beta} p_\alpha p_\beta} \]

Unit Vector \( \hat{p}^\alpha = \frac{p^\alpha}{|p|} \)

3.1.1 Relative-Time Evolution

Given any smooth classical path \( \eta \) defined by \( q^\alpha(\tau) \) in the configuration space \( R^n \) We also assume the function \( q^\alpha(\tau) \) has smooth first and second order derivatives. We can always define the quantum evolution for a given Lagrangian as a function of \( q^\alpha \) and \( \dot{q}^\alpha \). Let \( L \) be the Lagrangian of the system which depends on \( q^\alpha \) and \( \dot{q}^\alpha \).

1. Define \( v^\alpha(t) = \dot{q}^\alpha(\tau) \) and \( p_\alpha = v^\beta m_{\beta \gamma}(q^\gamma(\tau)) \), where I have assumed \( m_{\alpha \beta} \) is a function of \( q^\gamma \).

2. Define a one parameter family of hyperplanes \( S(\tau) \) isomorphic to \( R^{n-1} \) orthogonal to \( p_\alpha(\tau) \) going through \( q^\alpha(\tau) \). If \( x^\alpha \) is the points on this plane, then it satisfies \( m_{\alpha \beta}(q^\gamma(\tau))(x^\alpha - q^\alpha)p^\beta = 0 \). We can denote the hyperplanes by \( S(\tau) = S(p_\alpha(\tau), q^\alpha(\tau)) \) as it depends on \( q^\alpha(\tau) \) and \( p_\alpha(\tau) \). \( S(\tau) \) describes a foliation of the configuration space if the surfaces don’t cross each other.

3. Define quantum states \( \rho(q^\alpha_\perp, \dot{q}^\alpha_\perp, \tau) \) on \( S(\tau) \). Here \( q^\alpha_\perp \) and \( \dot{q}^\alpha_\perp \) takes values in \( R^n \) but is restricted to \( S(\tau) \).
4. Define a single step path integral from $S(\tau)$ to $S(\tau + d\tau)$ for $\rho(q^a_{11}, \tilde{q}^a_{11}, \tau)$

$$G_{s+}(q^a_{11}, q^a_{12}, \tilde{q}^a_{11}, \tilde{q}^a_{12}; \eta, \tau, \Delta \tau) = \frac{1}{(2\pi)^{d-1}} \int_{q^a_2 = q^a_{12}, \tilde{q}^a_2 = \tilde{q}^a_{12}} \exp(i(L(\gamma) - \tilde{L}(\tilde{\gamma})) \Delta \tau) d\gamma d\tilde{\gamma}. \quad (3)$$

Here $\gamma$ stands for $q^a_3$. The path integral is evaluated between $S(\tau)$ and $S(\tau + d\tau)$ with boundary conditions as described above. We can use the relative path integral to define the quantum evolution of states on $S(\tau)$ of the configuration space:

$$\rho(q^a_{12}, \tilde{q}^a_{12}, \tau) = \int G_{s+}(q^a_{11}, q^a_{12}, \tilde{q}^a_{11}, \tilde{q}^a_{12}; \eta, \tau, \Delta \tau) \rho(q^a_{11}, \tilde{q}^a_{11}, \tau) dq^a_{11} d\tilde{q}^a_{11}$$

For this path integral formulation to genuinely describe the evolution of wavefunction we need to have $\eta$ smooth enough such that $S(\tau)$ don’t intersect other, at least in the region where the wavefunctions are finite.

#### 3.1.2 Relative-Time Decoherence

In previous versions [3] we discussed inclusion of this using diffusion equation method [11]. We will generalize the formalism to include relative quantum evolution. I define relative quantum decoherence evolution equation as follows.

$$G_{s+}(q^a_{11}, q^a_{12}, \tilde{q}^a_{11}, \tilde{q}^a_{12}; \eta, \tau, \Delta \tau) = \frac{1}{(2\pi)^{d-1}} \int_{q^a_2 = q^a_{12}, \tilde{q}^a_2 = \tilde{q}^a_{12}} \exp(i(L(\gamma) - \tilde{L}(\tilde{\gamma}) - \beta d(\gamma, \tilde{\gamma})^2) \Delta \tau) d\gamma d\tilde{\gamma}. \quad (4)$$

where $d(\gamma, \tilde{\gamma})$ is the decoherence function. Here $\gamma$ the paths and it stands for $q^a_3$. Here the evolution of $\rho(q^a_{11}, \tilde{q}^a_{11}, \tau)$ depends on $\eta$. That is why we refer to the function this Relative-Time decoherence, $\rho(q^a_{11}, \tilde{q}^a_{11}, \tau)$ defines probability density states at each value $\tau$ during relative-time decoherent evolution with respect to $\eta$.

#### 3.2 General Curved Space Time

Let’s now apply the formalism that we discussed in the previous subsections to field theory on general curved space time. We will see in this update of quantum gravity framework, all constraints including Hamiltonian constraint need not be explicitly needed for formulating dynamics.

Assume we have an initial hypersurface. To each point on $x$ we can apply the theory for single point systems. There will be internal fields at each point $q^a_x$. Let $L$ be the Lagrangian which depends on $q^a_x$ and $\tilde{q}^a_x$. We are using simplified version of the fields to make discussion easy. There will be one classical curve $\eta^a_x(\tau_x)$ for each point, smooth up to second derivative, one parameter family of hyperplanes $S^\tau(\eta^a_x(\tau_x), q^a_x(\tau_x))$ in the configuration space of fields at each point, and one free (dummy) parameter $\tau_x$ for each point. For each point the physics is identical to the single point system discussed in the previous section. Only major difference is that the Lagrangian contains interaction terms as functions of the $q^a_x$ of adjacent points.

Let me assume that space is discretized for simplicity and is made of countable number pieces of volume elements such as in cubic lattice. I am assuming this discretization only for simplicity and explanatory purpose.

Let $B$ be the number of lattice points, and for simplicity let us assume $B$ is finite. Let $\Delta V$ be the coordinate volume associated to the coordinate volume element associated to each lattice element of the 3D manifold.

#### 3.2.1 Relative-Time evolution

Now consider the path integral defined in the previous section in equation (3). For each system at $x$, we have one curve $\eta_x$ assigned. Then we have the combined one step relative path integral is
Proposition 2 The relative time evolution instantaneous path integral defined as function of \( \eta_x, \tau_x \) is as follows:

\[
G_{s+}(q^\alpha_{s+1}, 3, q^\alpha_{s+2}, 3, \overline{q}^\alpha_{s+1}, 2; \eta_x, \tau_x, \Delta \tau_x) = \frac{1}{(2\pi)^{3BD}} \int_{q^\alpha_{s+1}=q^\alpha_{s+2}, 3, \overline{q}^\alpha_{s+1}=\overline{q}^\alpha_{s+2}} \prod_x \{\exp(iL(\gamma) - iL(\tilde{\gamma}))d\gamma d\tilde{\gamma}\},
\]

where \( \gamma \) are the paths in configuration space and it stands for \( q^\alpha_{s+1} \). This formulation does not contain constraints at all. In this form it is not necessary to have constraints. We can ask why do you need a relative time evolution since one don’t have Hamiltonian constraint. Because in the next section we will see that decoherence defined depends on \( \eta_x \).

We can use the relative path integral to define the quantum evolution of states on \( S(\tau) \) of the configuration space:

\[
\rho(q^\alpha_{s+1}, 2, \overline{q}^\alpha_{s+2}, 2, \tau + d\tau) = \int G_{s+}(q^\alpha_{s+1}, 1, q^\alpha_{s+2}, 2, \overline{q}^\alpha_{s+1}, 2; \eta_x, \tau_x, \Delta \tau_x) \rho(q^\alpha_{s+1}, 1, \overline{q}^\alpha_{s+1}, 1, \tau)dq^\alpha_{s+1}d\overline{q}^\alpha_{s+1}
\]

3.2.2 Relative-Time Decoherence

We can generalize the single system form of Lagrangian form of decoherence to \((3+1)D\) case.

Now we can define relative decoherence as the following:

Proposition 3 The relative decoherent evolution of the \( \rho \) is given by the following path integral as functional of \( \eta_x, \tau_x \), where \( d(\gamma, \tilde{\gamma}) \) is the decoherence functional and \( \beta \) is the decoherence constant.

\[
G_{s+}(q^\alpha_{s+1}, 1, q^\alpha_{s+2}, 2, \overline{q}^\alpha_{s+1}, 1, \overline{q}^\alpha_{s+2}, 2; \eta_x, \tau_x, \Delta \tau_x) = \frac{1}{(2\pi)^{3BD}} \int_{q^\alpha_{s+1}=q^\alpha_{s+2}, 2, \overline{q}^\alpha_{s+1}=\overline{q}^\alpha_{s+2}} \exp \left[ iL(\gamma) - iL(\tilde{\gamma}) - \beta d(\gamma, \tilde{\gamma})^2 \right] D\gamma D\tilde{\gamma}
\]

This evolution depends on \( \eta_x \) for each point. So, this is relative-time decoherence. The choice of \( \eta_x \) needs to be discovered by further research. One of the best choice of \( \eta_x \) is the self-time evolution in which \( \eta_x \) is the classical expectation value of \( q^\alpha_{s+1} \). This is what I referred to as the rest-frame evolution in the configuration space of fields. This will be later discussed in this section.

In quantum gravity framework 2.0 and 3.0, we had that the decoherence part was in Hamiltonian evolution form, while the other three components of the framework were in path integral approach. Now the form of relative decoherence is path integral like the other three components of the framework.

Since the path integral directly deals with evolution of the density matrix, there is need to take the square of the wavefunction. Summing of the product of density matrix with other operators will give the expectation values. For example, if \( A \) is an operator, a function of the \( \overline{q}^\alpha_{s+1}, 1 \), and their conjugate momenta’s, then, the expectation value is,

\[
<A> = \frac{\text{tr}(\rho A)}{\text{tr}(\rho)}
\]

The quantum states can be derived from \( \rho \) by diagonalizing it:

\[
\rho = \sum \lambda_i |\lambda_i\rangle <\lambda_i|
\]

where \( |\lambda_i\rangle \) are the probable states with the probability of \( p_i \), associated to a hypersurface. And it is dependent on \( \eta_x, \tau_x \). So, evolution of \( \rho \) describes a relative probable evolution of states. We discuss the next
3.2.3 Global quantum reduction

Let me define \( d\tau_x = n_x(\tau) \, d\tau \), where the \( n_x(\tau) \) are continuous functions of \( \tau \), one of them for each lattice point \( x \). The repeated application of the one-step path integral for infinitesimal \( d\tau \) evolves the quantum state along the spatial hypersurfaces. The \( n_x(\tau) \) functions defines the various ways to foliate the discretized geometry, whose topology is \( B \) point \( \otimes \) \( 1D \). Here \( n_x(\tau) \) is essentially is the lapse. Now depending on the choice of \( n_x(\tau) \) we will have different foliations of the classical space-time geometry relating to the quantum geometry.

The evolution defined by relative time decoherence evolution generates a time dependent quantum state \( \rho(q_{\tau,\perp}^a, \dot{q}_{\tau,\perp}^a, \tau) \) which evolves from the initial quantum state \( \rho(q_{\tau,\perp}^a, \dot{q}_{\tau,\perp}^a, 0) \). If we express each step in Hamiltonian form, we can include relative decoherence discussed also in this evolution. This evolution evolves the initial state \( \psi_x \) continuously to generate an entire quantum space time. But this evolution depends on \( \eta_x \) and \( n_x(\tau) \).

The relative decoherence formulation helps calculate the density matrix, it simply converts any pure state into mixed state. Continuous reduction due to observation requires continuous probabilistic reduction of mixed states into pure states. This once again depends on foliation. The sequence of continuously reduced pure states in one foliation is not equivalent to a sequence of pure state and they depend on \( \eta_x \) and \( n_x(\tau) \).

Now there are two ways to understand the relative decoherence formulation used evolve \( \rho \) over a region of space time.

**Proposition 4 Proposal 3.1: Observer less Interpretation-** We can also use \( \hat{\rho} \) to calculate averages and other statistical values.

For example, we can calculate the following: classical metric \( g_{\alpha\beta} \) of the corresponding classical geometry using

\[
g_{\alpha\beta} = \frac{\text{tr}(\hat{\rho} g_{\alpha\beta})}{\text{tr}(\hat{\rho})}.
\]

In the trace we sum over all \( \gamma = \tilde{\gamma} \). This removes the dependence on \( \eta_x \) and \( n_x(\tau) \). Now we can use the \( G \) for continuous evolution over a finite space-time region to calculate to correlations between values of \( q_x^a \) in different space-time points. This is similar to calculation of propagation amplitudes using Feynman diagrams. Only difference is we don’t need to do squaring to calculate the probability amplitudes, as \( G \) deals with evolution of density matrix. We sum over all free value sum over \( \gamma = \tilde{\gamma} \) and remove the dependence on \( \eta_x \) and \( n_x(\tau) \). Here is an example:

\[
< q_{x,\perp}^a q_{x,\perp}^a >_{\Delta \tau_x} = \int q_{x,\perp}^a q_{x,\perp}^a 2^G_{x,\perp}(q_{x,\perp}^a, q_{x,\perp}^a, q_{x,\perp}^a; \eta_x, \tau_x) dq_{x,\perp}^a dq_{x,\perp}^a\
\]

where \( < q_{x,\perp}^a q_{x,\perp}^a >_{\Delta \tau_x} \) is the correlation between \( q_{x,\perp}^a \) and \( q_{x,\perp}^a \) separated by time parameter \( \Delta \tau_x \).

**Proposition 5 Proposal 3.2: Global Quantum Reduction** - The quantum evolution and reduction process occurs along a spatial foliation such that the \( C^1 \) smooth functions \( n_x(\tau) \) and \( \eta_x \) take smooth values, such that relative probability weight is given by \( \exp(-c_r \tilde{\Upsilon} - c_r \tilde{\Upsilon}) \), where \( c_r \) is a fundamental constant, where \( \Upsilon \) is \( \Upsilon(q_x^a, n_x(\tau), \eta_x) \) is measure discussed above, and \( \tilde{\Upsilon} \) is \( \tilde{\Upsilon}(q_x^a, n_x(\tau), \eta_x) \) corresponds to \( q_x^a, c_r \) and \( n_x(\tau) \) are to be discovered and verified experimentally.

Now the Lagrangian density is now of the form:

\[
L_4 = \mathcal{L}(\gamma) - i \mathcal{L}(\dot{\gamma}) + i \beta d(\gamma, \dot{\gamma})^2 + i c_r \tilde{\Upsilon}(\gamma, n_x(\tau), \eta_x) + ic_r \tilde{\Upsilon}(\tilde{\gamma}, n_x(\tau), \eta_x)
\]

\( L_4 \) is total Lagrangian described including the decoherence and global reduction fields. The constant \( c_r \) need to be small enough that the imaginary terms don’t disturb the usual Lagrangian quantum evolution. I have assumed \( \tilde{\Upsilon} \) as function of \( (g_{\mu\nu}, n_x(\tau), \eta_x) \) only. But in reality, could be function other variables depending on the fields we are dealing with. The value of \( \exp(-c_r \tilde{\Upsilon} - c_r \tilde{\Upsilon}) \) for different \( (g_{\mu\nu}, n_x(\tau), \eta_x) \) gives...
relative probability weight for each of these values. In addition to the probabilistic nature of the theory due to \( \rho \), we also have additional statistical nature due to the probability weights \( \exp(-c_r \bar{Y} - c_s \bar{T}) \). The physical interpretation of these probability weights depends on \( \bar{Y} \). In quantum gravity framework 3.0 and 4.0, we discussed various possible choices for \( \bar{Y} \) depending on various physical motivations. One of important case is the rest frame foliation introduced in [4]. Later we will discuss the covariant generalization of this field.

3.2.4 Determinism, Continuum Limit and Scale invariance

**Proposition 6** The fourth postulate is unchanged is same as the previous versions [3] and [4]. Only generalization is we need two \( \sigma_x \) instead of one in the Lagrangian.

\[
\mathcal{L} \rightarrow \mathcal{L} + i \sum_{x,s} \frac{1}{2} \sigma_x(q^0_{y,s}) + i \sum_{x,s} \frac{1}{2} \sigma_x(q^0_{y,s}) \tag{7}
\]

such that \( \sigma_x \) are

1) smooth real functions of the variables \( \tilde{q}^0_{x,s} \) with a lower bound,
2) functions of quantum variables at \( x \) and adjacent (or nearby) quantum systems to point \( x \), and
3) are increasing functions as \( |q^0_{x,s} - q^0_{y,s}| - > \infty \).

In [3] we discussed scale invariance. Full understanding of determinism, continuum Limit and scale invariance requires extensive study of the other two principles. This is one possible future course of research.

3.3 Covariant Rest Frame foliation

Let’s do the generalization of rest frame foliation to the covariant formulation. The propagator was defined formally defined in the section on global quantum reduction. We need to discuss the dependence of the \( \bar{Y} \) on \( \eta_x \) and \( n_x(t) \). First let us look at \( \eta_x \). Let me assume that dependence on these two is additive.

\[
Y(\gamma, n_x(\tau), \eta_x) = Y(\gamma, 0, \eta_x) + Y(\gamma, n_x(\tau), 0)
\]

Let me define

\[
\begin{align*}
Y_1(\gamma, \eta_x) &= Y(\gamma, 0, \eta_x) \\
Y_2(\gamma, n_x(\tau)) &= Y(\gamma, n_x(\tau), 0)
\end{align*}
\]

The \( \eta_x \) determines the time flow in the configuration space of fields at each point. The most natural general proposal for dependence of \( \eta_x \) is as follows:

\[
Y_1(\gamma, \eta_x) = |\eta_x^0 - q_x^0|^2
\]

This basically restricts the possible values of \( \eta_x \) to be close to expectation values of \( q_x^0 \). The norm squared is calculated in the internal metric of the fields.

Now let us focus on \( Y(\gamma, n_x(\tau), 0) \). The self-time constrained evolution and rest frame foliation discussed in ([4]) are in which global reduction could occur naturally. It was defined by

\[
Y_3 = \int \left( \frac{<\pi^{ab}_{x} \pi_{f x}^{ab} >}{c_g} + \sum_f \frac{1}{2} E_f^2 \right) \frac{1}{\sqrt{h}} d\mathcal{X}
\]

where the suffix 3 indicates it quantum framework version 3.0. The above proposal is not covariant. So we need to generalize this fully into an appropriate form.

To generalize this idea, we need a time-like killing field. In Schwarzschild metric we have the time-like killing field, which defines good time parameter in weak gravitational fields. This would be good around planets. But if we go to initial state of universe the universe was expanding and so there is no time like
killing fields. The appropriate form is a conformal killing field, which was discussed as one of the options in the previous version ([3]). It can define the natural time parameter in both in the universe’s expansion phase or around spherically symmetric matter field. This is the most covariant form.

Let $T^\gamma$ be a time vector field that generates a one parameter family of space-time diffeomorphism, such that a given initial surface $S_1$ is mapped to a different surface $S_2$ of the foliation. So, specifying $T^\gamma$, assuming it is integrable, is another way to define the foliation. Now instead of $n_x$ we are going to use $T^\gamma$. The relation between $n_x$ and $T^\gamma$ is not so obvious. We want $T^\gamma$ such that it can detect movement of the metric up to a scaling factor, and also give foliation locally, even though it may not be integrated globally. If we have specific choice of $T^\gamma$ in a region then normal surfaces to $T^\gamma$ gives that foliation. For example, $t =$ constant surfaces in Schwarzschild or inflationary universe. We need to replace $n(x)$ by $T^\gamma$ in our theory to describe evolution in all the four parts of quantum gravity framework 4.0 defined in this section.

Now let try to find a possible choice of function for $\Upsilon_2(\gamma, T^\gamma)$. Let me define tensor $C_{\alpha \beta}$ defined as a function of space-time metric $g_{\mu \nu}$ by

$$C_{\alpha \beta}(g_{\mu \nu}, T^\gamma) = \mathcal{L}_T(g_{\alpha \beta}) - \frac{1}{4} (g^{\gamma \delta} \mathcal{L}_T(g_{\gamma \delta})) g_{\alpha \beta},$$

where $\mathcal{L}_T$ is the lie derivative along $T^\alpha$. For a vector $T^\alpha$ to be conformal killing, $C_{\alpha \beta}$ is to be zero.

For measuring the smallness of $C_{\alpha \beta}$ consider the most obvious norm:

$$\int C_{\alpha \beta} C^{\gamma \delta} \sqrt{g} d^4 x = \int g^{\alpha \gamma} g^{\beta \delta} C_{\alpha \beta}(g_{\mu \nu}, T^\gamma) C_{\gamma \delta}(g_{\mu \nu}, T^\gamma) \sqrt{g} d^4 x$$

The second line makes the depends on $g^{\alpha \gamma}$ and $T^\gamma$ to be explicit. Since the metric is Lorentzian, the measure is not positive definite. So the smallness of $\int C_{\alpha \beta} C^{\gamma \delta} \sqrt{g} d^4 x$ does not imply the smallness of components of $C_{\alpha \beta}$. To surmount this, metric can be Euclideanized so that the norm is positive definite.

$$\Upsilon_2(\gamma, T^\gamma) = \int g^{\alpha \gamma} g^{\beta \delta} C_{\alpha \beta}(g_{\mu \nu}, T^\gamma) C_{\gamma \delta}(g_{\mu \nu}, T^\gamma) \sqrt{g} d^4 x$$

where $g_{\mu \nu}^E$ is the Euclidean version of the Lorentzian metric $g_{\mu \nu}$, and $g_{\mu \nu}^E$ is the inverse of $g_{\mu \nu}^E$. This was discussed in the previous version. But this approach seems to unnatural and not simple. The most natural form is

$$\Upsilon_2(\gamma, T^\gamma) = \int |g^{\alpha \gamma} g^{\beta \delta} C_{\alpha \beta}(g_{\mu \nu}, T^\gamma) C_{\gamma \delta}(g_{\mu \nu}, T^\gamma)| \sqrt{|g|} d^4 x$$

In case of the covariant decoherence we also have the $\tilde{g}_{\mu \nu}$ field, for which can define another norm,

$$\Upsilon_2(\gamma, T^\gamma) = \int |\tilde{g}^{\alpha \gamma} \tilde{g}^{\beta \delta} C_{\alpha \beta}(\tilde{g}_{\mu \nu}, T^\gamma) C_{\gamma \delta}(\tilde{g}_{\mu \nu}, T^\gamma)| \sqrt{|\tilde{g}|} d^4 x$$

The transition probability is peaked when $T^\gamma$ is close to the conformal killing vector of the metric fields $g_{\mu \nu}$ and $\tilde{g}_{\mu \nu}$.

Also we can include the electric fields in $\Upsilon$ if use Kaluza-Klein unification of gauge fields with gravity. These suggest that $\Upsilon$ and $\Upsilon_3$ are closely related. It will be quite interesting to study how in the linear limit $\Upsilon(\tilde{g}_{\mu \nu}, T^\gamma, n)$ defined above leads to the rest frame foliation.

Now the total Lagrangian density is as follows:

$$\mathcal{L}_4 = \mathcal{L}(\gamma) - i \mathcal{L}(\tilde{\gamma}) + i \beta d(\gamma, \tilde{\gamma})^2 + ic_\tau \Upsilon_1(\gamma, \eta_x) + ic_\tau \Upsilon_2(\gamma, T^\gamma) + ic_\tau \Upsilon_2(\tilde{\gamma}, T^\gamma) + \frac{1}{2} \sigma_x(\gamma) + i \frac{1}{2} \sigma_\tau(\tilde{\gamma})$$
where I have included the terms for smoothness defined in equation (7). In theory with this Lagrangian density we need to use $T^\gamma$ to describe foliation of space-time locally. We do this analysis in detail in a future version.

4 Application to Quantum Gravity

Let us discuss a specific application to quantum gravity. A simple possibility for the decoherence term is the following:

**Proposition 7**: The covariant distance operator is:

$$d = c_h \int (g_{ab} - \tilde{g}_{ab})(g^{ab} - \tilde{g}^{ab})(g\tilde{g})^{\frac{1}{4}} dt d^3 x$$

This is a simple proposal with measures the distance between two metrics which is diffeomorphism invariant on simultaneous diffeomorphism of both tilde and non-tilde space-times. The $g_{ab}$ and $\tilde{g}_{ab}$ are the metrics on the tilde and non-tilde space-times. $g$ and $\tilde{g}$ are the determinants. $(g\tilde{g})^{\frac{1}{4}}$ is for maintaining coordinate independence of the integral giving equal importance to non-dual and dual space. This is a very formal definition.

In density matrix formulation the evolution heuristically is as follows:

$$\dot{\rho} = \int \left\{ i[H, \rho] - c_h g^{\frac{1}{4}} \left( 8\rho - g^{ab}\rho g_{ab} - g_{ab}\rho g^{ab} \right) g^{\frac{1}{4}} \right\} d^3 x$$

(8)

where $\rho = \rho\{\{q^2_x, \tilde{q}^2_x; S_x, \forall x, \}, t\}$, and $H$ is the effective Hamiltonian density. Above equation is an operator equation. To understand the derivation of the decoherence terms from path integral, please note that from the path integral:

$$(g_{ab} - \tilde{g}_{ab})(g^{ab} - \tilde{g}^{ab}) = 8 - g_{ab}\tilde{g}^{ab} - g_{ab}\tilde{g}^{ab}$$

The $g^{\frac{1}{4}}$ factors form the density for the volume measure.

4.1 Examples

4.1.1 Cosmology

For isotropic and homogenous cosmology the density evolution equation reduces to the following, with scale factor $a$ as the time , :

$$g_{ab} = diag(-1, a^2, a^2, a^2)$$

$$\dot{\rho} = i[H, \rho] + c_h a^{3/2} \left( 6\rho - 3a^{-2}\rho a^2 - 3a^2\rho a^{-2} \right) a^{3/2}$$

(9)

where $a$ is the scale factor acting as the only configuration variable. Explicitly

$$\dot{\rho}(a, a') = i \langle a | [H, \rho] | a' \rangle + c_h a^{\frac{3}{2}} a'^{\frac{3}{2}} \rho(a, a')(a - a')(a^{-1} - a'^{-1})$$

The $H$ need to be derived using symmetry conditions. Below are two cases, with the first for expanding universe with only a cosmological constant, and second the universe with scalar field $\phi$ included. The $\pi$ below are conjugate momentas.
\[
L = \pi_a \dot{a} - N dt c_g \left\{ \left( -\frac{\pi_a^2}{a} - ak \right) + \Lambda \alpha^3 \right\}
\]

\[
H = \left( -\frac{\pi_a^2}{a} - ak \right) + \Lambda \alpha^3
\]

\[
L = \pi_\phi \dot{\phi} - N dt \left\{ c_g \left( -\frac{\pi_a^2}{a} - ak \right) + \frac{\pi_\phi^2}{2a^3} + \frac{a^3 \dot{\phi}^2}{2} + \Lambda \alpha^3 \right\}
\]

\[
H = c_g \left( -\frac{\pi_a^2}{a} - ak \right) + \frac{\pi_\phi^2}{2a^3} + \frac{a^3 \dot{\phi}^2}{2} + \Lambda \alpha^3
\]

The notations have standard interpretation and were introduced in the previous version [3].

### 4.1.2 Example: Spherically symmetric space.

Consider the spherically symmetric case of spherically symmetric macroscopic matter of radius R with centre mass at \(x\):

\[
g_{ab} = \text{diag}(-1 + \phi, 1 + \phi, 1 + \phi, 1 + \phi)
\]

where \(\phi\) is gravitational potential. Here I don’t assume the spherically symmetric matter is a point particle, but certain mass of order of Planck mass or more than that. We also assume the matter distribution is uniform, so that there is no spike in the gravitational field. A detailed calculation yields the following:

\[
\int (g_{ab} - \tilde{g}_{ab})(g^{ab} - \tilde{g}^{ab})(\tilde{g}^{ab}) \frac{1}{2} d^3 x \approx 4 \int |\phi(y - x) - \phi(y - x')|^2 d^3 y
\]

Then the quantum mechanics of the macroscopic spherical matter at center of mass is described by a simple model as follows:

\[
\dot{\rho}(x, x') = i \langle x | [H, \rho] | x' \rangle - 4c_{h\rho} \int |\phi(y - x) - \phi(y - x')|^2 d^3 y
\]

\(H\) can be derived from the Hamiltonian analysis of the standard Hamiltonian. Based on private calculations, I believe, the decoherence integrals both in cosmology and spherically symmetric case seems to be convergent and seems promising. Further analysis will be discussed in the future.

### 4.1.3 Resolution of singularities

When we go towards the singularities the metric weight \(h\) goes towards zero. The Hamiltonian has inverse \(h\) factors. So, the Hamiltonian formalism is not much use near the singularities. In such case Lagrangian formulation is useful to study physics near singularities.

### 5 Universe, Consciousness and structure formation

In this section we now discuss ideas regarding consciousness and structure formation. We will discuss how consciousness involves quantum gravity through rest frame foliation. We introduce concepts to understand structure formation and how consciousness is linked to it.
5.1 Consciousness and Framework 3.0

5.1.1 Consciousness and Global Quantum Reduction

Now we will try to understand the physical relevance of the global quantum reduction proposal. When we are involving only Schrödinger evolution, it really doesn’t matter what foliation you are using in space-time or what foliation we are using in the configuration space of fields at each point to evolve the wavefunction. But if we include decoherence as discussed in framework 2.0, then for that foliation matters. Continuous reduction of a quantum system is given by the Bloch equations in the Lindblad form [26] governing evolution of density matrix (reviewed in [27]):

\[
\dot{\rho}_\tau = i[\hat{\rho}_\tau, \hat{H}] + \sum_{m,x} (2\hat{L}_{m,x}\hat{\rho}_\tau \hat{L}_{m,x}^* \hat{L}_{m,x} - \hat{L}_{m,x}^* \hat{L}_{m,x} \hat{\rho}_\tau - \hat{\rho}_\tau \hat{L}_{m,x}^* \hat{L}_{m,x}),
\]

(11)

with

\[
\hat{\rho}_\tau = \frac{M(|\psi_\tau \rangle \langle \psi_\tau|)}{\langle \psi_\tau | \psi_\tau \rangle},
\]

Even if we use the path integral form of decoherence described in this paper, foliation still matters. We expand \(\hat{\rho}_\tau\) into a sum of pure states. Essentially \(|\psi_\tau \rangle \langle \psi_\tau|\) is a sequence pure states got by probabilistically reducing \(\hat{\rho}_\tau\) at each instant into pure state. This sequence of states \(|\psi_\tau \rangle\) obtained this way depends on the foliation. That is, this conversation of mixed to pure state is the measurement process which depends on the foliation. For this we need to find the most natural foliation that nature uses to it, if that is the way this happens.

We have discussed possible foliations for global reductions in the previous paper [3]. Now there are three questions to be addressed:1) whether the reduction process occurs along a preferred foliation, 2) what is the choice of the foliation along which the reduction occurs, and 3) whether this can be addressed as experimental questions.

The answer to the first question is 'NO', if we take into account the spirit of general relativity: Basic laws of physics are supposed to be independent of foliation in which we analyze the process of evolution. But the answer is yes if we take into account the presence of conscious observers as human beings. We observe the world through our brain. The process of observation receives information from the environment to be entered into quantum state of matter in relevant regions. This is entangled with quantum state of matter around the observer. **During observation, observer converts mixed to pure states, and it is registered in his consciousness.** This is lesson from quantum mechanics, both in theory and experiment.

Now to answer the third question, yes we can come certain answers for the first two questions. Consider the first two facts:

1) When an observer observes a quantum superposed state it is probabilistically projected into a single state. This process is decoherence described by the second proposal: conversion of mixed states into pure states.

2) Second we know that observer observes in his rest frame.

3) Choice of foliations described for global quantum reduction are those in fields and matter are relatively at rest. We will discuss this in detail after the observation.

Combining these three ideas we can come to the following proposal:

**Proposition 8** Global quantum reduction is process of continuous observation of conscious observers.

Now we would like to go into more details regarding this. We observe the world through our brain. The information we perceive is distributed throughout the brain. The synchronized pattern of firing of neurons in our brain is perceived by conscious information. But somehow they combine together give us a 4d picture of the world, which is seen from a particular reference frame. The reference frame is usually the co-moving reference frame of the observers. But we see the observer as collection of neurons in human brain, then we
ask the question what precisely is the hypersurface foliation used by neurons to synchronize themselves? In other words what is the foliation along which we observe the world?

Each neuron is sitting still in the human brain. If we keep our head still in an inertial reference frame, then all our neurons have the same four velocity, then we the orthonormal hypersurfaces to this four velocity is clearly is the foliation along which the neurons synchronize. Then the information in neurons at each of this hypersurface gets binded together somehow and presented as instantaneous sequence of perceptions to the conscious observer.

But usually observer keeps shaking his head. Then in this case each neuron no longer have the same four velocity, so the reference frames and the orthonormal hypersurfaces of each of each neuron are slightly different. Even though differences are very small in relativistic context, but yet we have a conceptual problem. How do we specify this velocity in field theory concept? In case of a moving head the hypersurfaces are slightly curved, because each neuron is at a different velocity. So in general the hypersurface along which the brain organizes information are curved.

Why should the conscious observer observe the universe from the hypersurfaces (reference frame) decided by the four velocity of the neurons of the head. Many other reference frames are equally possible. The question is how does the brain (neurons) choose this reference frame? My question is more technical, how the brain sense its reference frame? or the sequence of spatial hypersurfaces associated to the reference frame? This is the fundamental question that we need to answer. This is the spatial hypersurface that is relevant to humans. Brain is matter, and this matter seems to sense the hypersurface along which it moves to organize the information it receives. Most neuroscientists consider consciousness as fundamental capability of many of the lower mammals. Many of important scientists such as Wigner, consider that even an electron may have an elementary conscious capability. If we want to link quantum measurement to consciousness, understanding the link between the hypersurfaces and dynamics of brain matter becomes more relevant and this requires further research. In this section we will discuss some proposals.

But how do we specify these hypersurfaces using quantum field theory. The four velocity of the brain is not a fundamental concept in field theory. We need to specify the hypersurfaces using the fundamental quantum field theory, so that it can be specified at each point of the universe in an objective manner. For this, consider the set up used for studying continuum canonical general relativity. Consider space time with metric $g_{\alpha\beta}$ and one parameter spacial foliation $S_t$, where $S_t$ is the spacial hypersurface for a given $t$. This foliation can be specified by function $t(x)$, $x$ is a point on space time, with $t(x) = \text{constant}$ describes the surface $S_t$. We can choose $t$ to be the time coordinate. Consider the vector field, $T^\gamma = (\frac{\partial}{\partial t})^\gamma$. $T^\gamma$ generates a one parameter family of space-time diffeomorphism, such that a given initial surface $S_t$ is mapped to a different surface $S_{t+}$ of the foliation. So specifying $T^\gamma$, assuming it is integrable, is another way to define the foliation. Let $n^\gamma$ is unit normal to the hypersurfaces.

There are many fundamental quantities that can be used to calculate $T^\gamma$: Energy momentum tensor $T^{\alpha\beta}$, extrinsic curvature $K_{\alpha\beta} = L_n(h_{\alpha\beta})$, or the lie derivative of the metric $L_{\alpha\beta} = L_T(g_{\alpha\beta})$. All these three has special relationship to the hypersurfaces along which a conscious observer observes the universe. $T^{\alpha\beta}$ for moving matter is parallel to the direction in which it is moving. $K^{\alpha\beta}$ is zero along moving observer, if the gravitational field is of its own. Usually, Lie derivative of the metric is zero along a moving matter, if the gravitational field is of its own.

Let first consider the energy momentum tensor $T^{\alpha\beta}$, which is a field theory concept. Consider the classical expectation value. $T^{0\beta}$ gives the direction along which matter energy travels. But human brain is a noise environment. $T^{0\beta}$ differs from point to point in a random manner. Also $T^{\alpha\beta}$ is zero if there is no matter. So it is difficult to link $T^{\alpha\beta}$ to the hypersurfaces in a one to one manner.

Lets consider the other two cases $K_{\alpha\beta} = L_n(h_{\alpha\beta})$ and $L_{\alpha\beta} = L_T(g_{\alpha\beta})$. Consider an static observer moving in free space in an inertial reference frame, with his neurons moving in the inertial reference frame. The observer generates a static field around him. By symmetry, the two Lie derivatives are zero if $T$ is given by the time like killing field $(\frac{\partial}{\partial t})^\gamma$ along which the field moves. Then we can determine the hypersurfaces along which $\int_V K_{\alpha\beta}K^{\alpha\beta}$ or $\int_V L_{\alpha\beta}L^{\alpha\beta}$ is zero or the smallest, where integration is done over region of the observer and along a hypersurface of the foliation. But $L_{\alpha\beta}L^{\alpha\beta}$ is not positive definite because the metric has $(-+++)$ signature. Then $K_{\alpha\beta}K^{\alpha\beta}$ which is positive definite seems to be the best possible choice. Smaller the $K_{\alpha\beta}K^{\alpha\beta}$, small the variation in $h_{ab}$ the spatial metric along the direction movement of the observer. This seems to the most appropriate surface along which the observer moves. The quantity that is directly related to $K_{\alpha\beta}$ in canonical formalism is the canonical momentum $\pi_{\alpha\beta}$. 


\[ \pi_{\alpha \beta} = \sqrt{h} (K_{\alpha \beta} - Kh_{\alpha \beta}) \]

In case of observer in gravitational field of a celestial object, \( K_{\alpha \beta} K^{\alpha \beta} \) is minimal along the static foliations of the body. So the foliations are determined by the gravitational field in the region in which the observer lives. But there is the electric field of the conscious matter that we also one need to take into account. This was discussed in [4]. Before in this paper we discussed the possible covariant generalization of the rest frame foliation using conformal killing vector.

5.1.2 Conscious and Rest frame foliation

As we discussed before usually observer keeps moving his head. Then in this case each neuron is no longer have the same four velocity, so the reference frames and the orthonormal hypersurfaces of each of each neuron are slightly different. In case of a moving head the hypersurfaces in which the observes the universe is slightly curved, because each neuron is at a different velocity. So, in general the hypersurface along which the brain organizes information are curved. The most natural form is the rest frame foliation, in which moving matter is most at rest.

So, we have the following proposal:

**Proposition 9** The rest frame foliation is the foliation in mixed quantum state of universe is converted into pure quantum state continuously and in which a conscious observer observes the universe

In our theory both conversion of mixed state to pure state happens all over the universe. This doesn’t mean that universe is consciously observing itself. In conscious matter such as in people or living organisms, the conscious observation is used for conscious observation and behavioral choice.

The rest frame foliations are quite natural for an observer, or any object such as camera, or a microorganism, to observe the world as a sequence of events. How to calculate this foliation? and make useful for study of movement needs to be studied further. Further relevance of this foliation will be discussed further in the full version of this paper to published later. In the previous paper [3], I have discussed some experimental test to understand the effect of foliation dependence of decoherence. It needs to be further studied.

5.2 Structures and Conscious observers

5.2.1 Relational Harmonic Structures

In the book [?] I have argued that purpose of the universe is creation of relational harmonic structures. The competition between gravity and entropy creates these structures. In our theory the gravity provides the necessary organizing force, while stochastic component (decoherence mechanisms) creates the entropy. In between these two we have other forces such as electromagnetic, weak and strong nuclear forces. These create the relationship structures out of particles to create composite particles such as atoms and molecules. These relationship structures further create complex structures such as bulk matter. Here we describe a formula to measure relational complexity which has also been described in the book [?]. Let \( \{x_i\} \) be the free variables relating to a quantum system, say N atoms. Then the joint probability distribution is

\[ P_f = P_f(\{x_i\}) \]

From this we can derive the probability distribution for each variable.

\[ P_i(x_i) = \sum_{\{x_j, \forall j \neq i\}} P_f(\{x_j\}) \]

The Shannon information associated to the entire system is
The Shannon information associated to each variable is

$$I_i = I(P_i) = - \sum_{x_i} P_i(x_i) \log_2 P_i(x_i)$$

Then the sum of all this information is

$$I_{sT} = \sum_i I_i$$

The mutual information that is associated due to connectivity between the variables is as follows.

$$I_m = I_{sT} - I_f$$

The average information per variable is

$$I_s = \frac{I_{sT}}{N}$$

where $N$ is the number of variables.

The average independent information per variable that is associated to the variables subtracting the mutual information is as follows:

$$I_i = I_s - I_m$$

Then the strength of the relationships network in the system is the geometric mean of the last two.

$$I_r = \sqrt{I_m \cdot I_i}$$

We normalize this information as follows:

$$I_r = \frac{1}{I_s} \sqrt{I_m \cdot I_i}$$

$I_r$ measures the strength of relationship structure. If $I_r$ is maximum it means, there is the maximum relational complexity. That is they not only have as much self-independence as possible, but also as much mutual connection in the system. Smaller the $I_r$, either mutual connection reduces or self independence reduces.

The most typical application of $I_r$ is the measure of relational complexity of living things. Assume $x_i$ is the locations of each of the individuals of a colony of species. If $I_r$ is maximum for a living colony it means they have both balance of both independence at the sametime they are mutually connected.

For matter in free space, $I_i$ is the independent information associated to each degree of freedom of particles, measures the effect of self-entropy. $I_m$ measures the connectivity between free particles, in planetary scale is a measure of gravitational clumping. $I_r$ is the geometric mean of these two is the measure of relationship structure as a balance between defensive and connective factors as discussed in the book [?].

We can also measure mental complexity of the neural network of a living brain. Here we can measure $x_i$ to be the voltage inside the axon of the neuron. $I_r$ is low mean either the neurons are two independent
or they are too connected. More the information a brain stores more will be the complexity of information with both interconnectivity and relational independence.

If there are entities with their state described by variables $x_1$ and $x_2$ then we can calculate the connectivity as follows: It can be calculated as follows:

$$C_{12} = \frac{I_{m,12}}{I_{s,12}}$$

Consider we have a system with many entities describe by variables $x_i$. We divide the system into two regions by a cross-sectional area $\Sigma$. Then let $k$ and $k'$ refer to pair of connected entities across the surface $\Sigma$. Then the total connectivity between the two parts on the two sides of $\Sigma$ is given by

$$C_{\Sigma} = \sum_k C_{kk'}$$

The fractional connectivity is given by

$$c_{\Sigma} = \frac{C_{\Sigma}}{N}$$

where $N$ is the number of links across $\Sigma$.

The concept of connectivity can be used to understand how integrated a system is. If it can be divided into two parts by a surface $\Sigma$ with $C_{\Sigma} = 0$, then it is not connected. Smaller the $c_{\Sigma}$ is, less connected the parts are.

5.2.2 Perception and Consciousness

Human brain is a conscious structure. It perceives the world as a sequence of perceptions. The perceptions are nothing but a combination of sensations, which I would like to refer to as a sensation complex. This has been described in chapter 3 of [1]. We need to understand the link of sensational complex to physics. Let me frame the following axioms.

- Each quantum of reality, like a particle, is capable of elementary sensation and the sensation happens when it split into or merges with a different quantum. The extent of sensation depends on the energy involved in split or merger.

- When one or more quantum particles bond together they become a bigger observer and their elementary sensation merge to form a bigger sensational complex. The elementary sensational complex is bound together by quantum entanglement and electromagnetic forces to become the bigger sensational complex.

- Qualia are sensational complex mapped to objects by a living entity through it memory systems and reproduced on demand by it. The objects could be external object such as tree or stone. The objects could be another sensation complex such as a word or image or sound.

- The time of flow of sensation is given by the rest frame foliation. The sensational complex information on each of the hypersurface of the rest frame foliation, is an instant of consciousness.

How quantum systems put together technically becomes the qualia observed by living brains needs to be researched. This may involve understanding the role of cortical columns in the cerebral cortex of living brain. This cortical columns may be considered as pixels of the sensational complex observed by the living brains.

We can use above list of ideas to give some mathematical description. But before going into that please note that in this paper consciousness is described as sequence of sensational complex that is all. There is nothing deeper than that. For example, when a human being becomes self-aware, he doesn’t not feel
everything about himself. He just feels the sensational complex that is provided in the brain based on the
information stored in the brain, to give a sense of self-awareness in combination with other sensations such as
visual, tactile, sound sensations at that time. We look at something and feel that we know it, it just
a sensation generated by the brain as feedback based on what it remembers. When we pay attention to
something basically our brain processes that information isolating it from the rest of the environment. Once
the processing is done, it provides the necessary sensational complex to identify it, along with a sense of
knowing it. Our brain over a course of time has evolved to generate its own sensational complex to give
a sense of self, feedback about the immediate environment, also feedback about its internal mental states
which in themselves is a sensational complex, and mechanical biological mechanisms to use the information
it learned to promote its own survival.

Let \( H_i(t) \) be the time dependent Hamiltonian associated with each quantum particle in a living brain.
Let energy \( e_i = \langle H \rangle \) is the expectation value. Then the sensational strength of the sensational complex is
given by

\[
s_i(t) = \frac{de_i(t)}{dt}.
\]

The qualia is essentially described by the interconnectivity between the various entities in the brain.
Consider the brain of a living thing. If \( \Psi(\{e_i(t)\}) \) is the wavefunction of the system, then the joint probability
density is

\[
P(\{e_i(t)\}) = |\Psi(\{e_i(t)\})|^2
\]

Using \( P(\{e_i(t)\}) \) we can calculate various information associated with respect to the system, and we
can define the connectivity and relational strength \( I_r \). The fractional connectivity on any division tells how
integrated the sensational complex is. If the fractional connectivity is zero it means that the system is
made of two more of conscious entities. Relational strength indicates how strong the mental development
of the system is. More the relational strength is, more complex is the information stored with strong
interconnectivity with them.

In case of neuron based brains, we don’t need to deal will each individual atom. But we can deal with
neuronal level information. Let \( s_i(t) \) represents average rate of change of voltage of the axon of human
brain. It tells the strength of sensational complex at the neuron. Our perceptual content is stored in our
neural network as bonds between atoms. This energy level fluctuates as electric field in the brain changes.
We perceive many frames of information in the brain, one frame per gamma cycle. As per our proposal in
last section, this gamma cycles are synchronous with the rest frame evolution of our brain. Let \( P_{js} \) be the
probability distribution values of \( s_i(t) \) during the \( \Delta T \) time interval of the Gamma Oscillation beginning at
\( T, T + \Delta T > t > T - \Delta T \). This probability can be calculated from the neural network itself.

\[
I_i(T) = \int_{s_i} P_i(s_i, T) \log_2 P_i(s_i, T) ds_i
\]

where integral is performed over the range of possible values of potential of each neuron.
\[ I_J(T) = - \sum_{s_j, \forall j} P_{J_0}(\{s_j\}) \log_2 P_{J_0}(\{s_j\}) \prod_{s_j, \forall j} ds_j \]

\[ I_s(T) = \sum_i I_i(T) \]

\[ I_m(T) = I_s(T) - I_J(T) \]

\( I_m(T) \) measures the sensational complex content of gamma oscillation during time interval starting at instant \( T \). Each gamma cycle defines a different instant of sensation.

**Proposition 10** \( I_m(T) \) is a measure of consciousness of a system.

This is our fifth postulate. It is one candidate of many other possible definitions available in the literature. For example, [zzz]

### 5.3 Universe and Relationship structures

#### 5.3.1 Theory

One of the important questions regarding consciousness is its relation to its free will. In the book [??] we proposed that macroscopic superposition is the best possible way to link physics to free will. If neural network of living brain becomes superposition of many possible feature courses \( M_i \), the quantum state of the entire neural network is

\[ |M > = \sum_i |M_i >, \]

\( M_i \) is the \( i \)th possible mental state. Eventually it decoheres, possibly based on the decoherence model that I have given in this paper to take one of the possible mental state \( |M_i >. \) But the important question if any new physics is involved in the decoherence to link consciousness to free will. That is whether the decoherence is not consistent with quantum mechanics, but slightly modified under the influence of free will in the brain. Further research needs to be done on this.

Now I would like to propose an interesting proposition based on the unification scheme developed in my work in [30] and also based on ideas in the previous section. Consider the measure of relational structure defined \( I_r \) before.

**Proposition 11** The transition probability of a synchronous system is an increasing function of the past state and decreasing function of the future state.

The last proposition is a mathematical statement of what has been discussed in chapter two of [31] and [30]. One possible realization is that in terms of relative time propagator

\[ G_{s+}(q_{s+,1}^0, q_{s+,1}^0, \tilde{q}_{s+,1}^0, \tilde{q}_{s+,1}^0; \eta_x, \tau_x, \Delta \tau_x) - g(q_{s+,1}^0, q_{s+,1}^0, \tilde{q}_{s+,1}^0, \tilde{q}_{s+,1}^0) \exp(-c_r \tilde{I}_{12} - c_r \tilde{I}_{12}) \exp(-d_r I_{r1} + d_r I_{r2}) \]

Where the arrow indicates that \( G_{s+} \) depends on various parts of its various similar to what has been written on the left hand side. \( \tilde{I}_{12} \) is restriction of \( I \) for global reduction to the slice containing the two hypersurfaces of space-time separated in time parameters \( \tau_x \) by \( \Delta \tau_x \). Most general possible way to understand the relationship structure dependence of a system transition amplitudes is to all a new term to the action of the system:

\[ S - \to S - i\xi(I_r) \]
where $\xi$ is an increasing function of $\xi(I_r)$. This will make transition amplitudes increase with increase in $I_r$.

Let's analyze the implications of the last proposal on the universe. In other words a synchronized system tries to become more relationally harmonic. Since synchronization is also as measure of relational harmonic nature in, synchronization also tend to increase. So, to put together, synchronization and relational harmonic structures tend to increase together. It is one possible implication of the above proposition. This can help understand the evolution of relationship structure of the universe: The formation of galaxies, stars, crystals, organic molecules, DNA and life so on. This has been discussed in [31] and [30], that universe tends to evolve with evolution of relationship structures.

5.3.2 Future of the universe

Now in the universe we can see three forces that shape its structure:

1. Entropy: This continuous to increase as universe evolves.
2. Blackholes: They tend to become stronger and stronger, influence the matter around.
3. Harmonic Relationship structures: Harmonic Relationship structures tends to increase as universe evolves.

So in the universe these three forces tend to shape it structures. Now if harmonic relationship structures are related to consciousness then it is the third force that shapes the future of the universe in addition to entropy and blackholes. Consciousness fights against the effects of gravity and entropy to keep the structures in the universe. The future of universe is determined by the battle between these three factors. The ultimate state of the universe is determined by who wins the fight.

5.3.3 Consciousness and Experience

Synchronization, that is consciousness, is also as a measure of relational harmonic nature. Now if you assume the extent of $I_r$ is a measure of consciousness, from the last proposal, a conscious system tries become more relationally harmonic. In other words conscious tends to increase itself.

In general in a relational harmic structure defined in space and time, the synchronization and the spatial relational structure tells about the extend of consciousness. Based on EEG studies of human brain, the extent of sychronization tells about the extend of conscious activation, while the spatial relational structure tells about information content.

If a conscious system is entangled with the environment, then the relational structure of the environment tends to be reflected in the mental state of the system. This means the conscious system tries to promote the increase in complexity of relational harmonic structure of the environment. So to understand the psychological experience I have the following proposal:

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**Proposition 12**  
1) Various different harmonic relationship states corresponds to different feelings, emotions or qualia of a conscious system.  
2) $I_r$ is a measure of the pleasure sensation by the synchronous system.  
3) Increase in $I_r$ is felt as happiness and decrease in $I_r$ is felt as sadness

This proposition has various parts that helps understand consciousness as discussed in chapter two of [31]. There are various important research needs to be done in relation to this proposal.

- Map the relation between various harmonic relationship structures in human brain and various sensory information. This is a pure experimental study which might probably lead to new theory

- Which of the brain structures and phenomena evolved due to pure wants. As I have discussed in book [30] artistic want are of pure wants. Its relates to the proposition that nature wants to increase harmonic relationship structures. Consciousness promotes it, and it is felt by it a positive feeling if $I_r$ increases and negative feeling if $I_r$ decreases.
• Which of the brain structures and phenomena evolved due to pure needs. Human brain evolved to accommodate pleasure and pain depending on where the needs are met or not. Usually neurotransmitters are involved in such phenomena. What is the mechanism, and chemistry and physics behind it?

• How pure needs and wants interact, to create life activities?

Answering these questions will lead to better understanding consciousness and how to develop a physics of it.

6 Conclusion

This paper describes only the heuristic setup. Application of this setup to study decoherence in simple context needs to be done. Even though, I believe the formalism is ready for application, but further research need to be done in future to develop the formalism before applying it. Further update to this paper will be done soon.

References


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