One Theorem complementary to the Fundamental Theorem of Arithmetic.

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0- Abstract:
In this paper I want to show a complementary theorem of the Fundamental Theorem of Arithmetic. Using the delta notation (Δ) I was able to deduce a generic formula involving prime numbers and natural numbers.

1- Introduction:
Gauss Theorem of Arithmetic can be expressed in a simple way as follows: Every natural number can be expressed as product of \( p_i \) primes with powers in exactly one way:

\[
(1) \quad n = \prod_{i=1}^{k} (p_i)^{\alpha_i}
\]

For \( \alpha_i \) belong to Naturals.
We will start here our work.

2- Delta notation tool:
As I shown previously in some papers [1]:

\[
(2) \quad \Delta_k a_i = a_k \div a_{(k-1)} \div ... \div a_2 \div a_1
\]

This is the divisory notation or just Delta notation for operators.

3- Deduction of the formula:
First we start in the FTArith formula:

\[
(3) \quad n = \prod_{i=1}^{k} (p_i)^{\alpha_i}
\]

Now we use the delta notation in both sides:

\[
(4) \quad \Delta_k n_i = \Delta_k \prod_{i=1}^{k} (p_i)^{\alpha_i}
\]
We simplify:

\[ \prod_{i=1}^{k \Delta} n_i = (p_i)^{\alpha_i} \]  

(5)

Because being opposite serial operators we can cancel one with the other, we now correct powers:

\[ \left( \prod_{i=1}^{k \Delta} n_i \right)^{\frac{1}{\alpha_i}} = \left( (p_i)^{\alpha_i} \right)^{\frac{1}{\alpha_i}} \]

(6)

Simplify again:

\[ p_i = \sqrt[\Delta]{\prod_{i=1}^{k \Delta} n_i} \]

(7)

And now we can state the Theorem:

**Theorem:** Every prime number can be expressed as \( \alpha_i \) root of i-th natural numbers in serial division by other powers of prime numbers inside the root, in infinite different ways. For \( \alpha_i \) and \( i \) belong to Naturals.

**4- Some easy examples:**

Being \( p, q, r \in \text{Primes} \):

\[ p = \sqrt[\alpha]{p^a} \]

(8)

For \( a \geq 2 \), \( p^a \in \mathbb{N} \) but \( p^a \notin \text{Primes} \).

\[ p = \sqrt[\alpha]{p^a \cdot q^b \div q^b} \]

(9)

For \( a \geq 2 \), \( b \geq 2 \), \( p^a, q^b \in \mathbb{N} \) but \( p^a, q^b \notin \text{Primes} \).

\[ p = \sqrt[\alpha]{p^a \cdot q^b \cdot r^c \div q^b \div r^c} \]

(10)

For \( a \geq 2 \), \( b \geq 2 \), \( p^a, q^b, r^c \in \mathbb{N} \) but \( p^a, q^b, r^c \notin \text{Primes} \).

**5- Bibliography:**