QCD’s Low-Energy Footprint

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Abstract: QCD has a low-energy footprint that affects electrons, atoms, and gravity through the vacuum quark and gluon pion condensate dynamics. We propose that electron motion occurs via quark and antiquark flavor waves of electrons and pions condensate gas. We propose that a polarized pion tetrahedrons sphere of about 1.1% of the Bohr radius may cause the hydrogen atom ground state hyperfine energy split. The magnetoresistance and spin torque of ferromagnetic layers may be explained by the proposed electrons and pion tetrahedron gas model. We expand the electrons and pion tetrahedrons gas model to a virtual cosmological scale box and propose that the pion tetrahedrons condensate density should be added to Einstein’s equation energy-momentum tensor.

Keywords: Quantum Chromodynamics (QCD), Electron-Ion Collider (EIC), Pion tetrahedrons, QCD vacuum, Spin Waves, Magnetic Materials, Spin Orbit Torque (SOT), Spin Transfer Torque (STT), magnetic tunnel junction (MTJ), General Relativity (GR), Energy-momentum tensor, Friedmann equation.
1. The Electron and Pion Tetrahedron Clouds

The Electron-Ion Collider (EIC) will explore the most fundamental building blocks of the visible matter and reveal the properties of the strong force that binds the protons and neutrons together to form the nuclei\(^1\). The EIC will explore the three-dimensional distributions in coordinate space and in momentum space of the quarks and gluons and will allow further exploration of the way in which the nucleons and nuclei mass, spin, and mechanical properties emerge from the fundamental interactions of the partons, and how these properties are distributed in their confined spaces. The EIC program will include exotic meson spectroscopy aimed to observe the exotic meson states through photoproduction.

In this paper we argue that the strong force (QCD) has a low-energy footprint that affects electrons, atoms, and gravity through the quark and gluon pion condensate dynamics. In previous papers\(^2,3,4\), the \(u\bar{d}d\bar{u}\) pion tetrahedron and the electron tetrahedron models were presented. We assumed that the \(u\bar{d}d\bar{u}\) pion tetrahedrons fill space and form a non-uniform condensate with an atmospheric like density drop. The \(u\bar{d}d\bar{u}\) pion tetrahedron mass may be calculated by measuring the \(\beta\) decay rate variability\(^3\). We assumed that the attraction between quarks and antiquarks may be the source of gravity\(^2\). The pion tetrahedrons density in space vary according to the gravitational field and the gravitational force is transferred by interaction of quarks with the non-uniform pion tetrahedron condensate.

We further assumed that the electron may be a tetraquark tetrahedron, two quarks determine its electric charge, and two quarks determine the electron spin. High frequency quark exchange reactions may transform the electron tetrahedrons to pion tetrahedrons and vice versa and form together electron clouds with fixed spin state\(^2\). The \(u\bar{u}\) and \(d\bar{d}\) quark pairs assumed to be part of the electron tetrahedrons, allow forming chemical bonds between electron pairs and proton-neutron pairs in the nuclei forming the \(u\bar{d}d\bar{u}\) pion tetrahedron that act as a QCD glue. We assumed
that the valence quarks and antiquarks ($u, d, \bar{d}$ and $\bar{u}$) are conserved particles and may only be exchanged between reactants forming products.

In this paper we propose that electron tetrahedrons and pion tetrahedrons form together electron clouds in thermal equilibrium due to high frequency quark exchange reactions that occur between the electrons and the pion tetrahedrons forming a quark and antiquark flavor waves. We further assume that two electron tetrahedrons scattering having opposite spins create a pion tetrahedron transition state complex, the QCD glue, that dissipate heat to the pion tetrahedron condensate generating an electric resistance.

Figure 1 illustrates an electron tetrahedron and pion tetrahedron scattering exchanging d and u quarks, transforming the electron tetrahedron to a pion tetrahedron and vice versa. High frequency repetition of the quark exchange reactions form an electron cloud that conserve charge and spin and may be seen as a quark flavor wave.
We assume that when an electron is accelerated, for example by an electric field, a plurality of pion tetrahedrons that surround it are accelerated with the electron and have similar velocity in thermal equilibrium due to the rapid quark exchange reactions. The electron cannot be separated from its surrounding pion tetrahedron cloud, and the electron may be seen as a many-body cloud composed of a plurality of pion tetrahedrons that perform rapid d and u quark exchange reactions with the electron tetrahedron. The thermal equilibrium with the pion tetrahedron condensate assumption applies not only to electrons. Protons and neutrons participate also in the rapid quark exchange reactions with the pion tetrahedrons. We assume that protons and neutrons of atom nuclei in a solid state are surrounded by the vacuum pion tetrahedrons since the pion tetrahedrons antiquarks are attracted to the quarks of the nuclei. Since the density of matter inside the solid is much higher than in a gas, the non-uniform pion tetrahedron condensate density inside a solid may be higher and may follow the lattice symmetry. In the next sections we describe the electrons and pion tetrahedrons polarization and motion in a hydrogen atom and an electron and pion tetrahedron gas model that may be used to explain the magnetoresistance and spin torques in magnetic materials.

2. Electron Tetrahedrons and the Hydrogen Atom
Werner Heisenberg introduced in 1925 matrix mechanics trying to explain the electron trajectories \((x(t), p(t))\) for the hydrogen atom in an electromagnetic field arguing that the electrons do not move in classical orbits as Niels Bohr suggested\(^5\). Quantum mechanics explained electron dynamics in terms of wave functions and discrete energy levels according to equations that were derived a year later by Erwin Schrodinger and in 1928 by Paul Dirac. The wave mechanics gave the correct spectra and the probability to find the electron in the atom but it did not provide a detailed description of the electron motion, its spin and the spin-spin interaction with the proton.
The hydrogen atom hyperfine structure Hamiltonian may be written in terms of Dirac’s exchange operator, $\hat{P}^{Ex}$, the exchange energy parameter $A$ and the Pauli spin matrices for the electron $\sigma^e$ and the proton $\sigma^p$: 

$$\hat{H}_{\text{hyperfine}} = A \sigma^e \sigma^p = -A \sum (2 \hat{P}^{Ex} - 1)$$

(1)

Dirac’s exchange operators, $\hat{P}^{Ex}$, exchange two particle spins. If their spins are in the same state, the overall spin state remains unchanged but if their spin states are anti-parallel the two spins are switched.

$$\hat{P}^{Ex} |P^\uparrow, e^\uparrow\rangle = |P^\uparrow, e^\uparrow\rangle$$

(2a)

$$\hat{P}^{Ex} |P^\downarrow, e^\uparrow\rangle = |P^\uparrow, e^\downarrow\rangle$$

(2b)

$$\hat{P}^{Ex} |P^\uparrow, e^\downarrow\rangle = |P^\downarrow, e^\uparrow\rangle$$

(2c)

$$\hat{P}^{Ex} |P^\downarrow, e^\downarrow\rangle = |P^\downarrow, e^\downarrow\rangle$$

(2d)

The hyperfine Hamiltonian matrix in the basis set of the four possible spin states, $|P^\uparrow, e^\uparrow\rangle$, $|P^\uparrow, e^\downarrow\rangle$, $|P^\downarrow, e^\uparrow\rangle$, $|P^\downarrow, e^\downarrow\rangle$ is given by the following 4*4 matrix:

$$\hat{H}_{i,j}^{\text{hyperfine}} = \begin{bmatrix}
A & 0 & 0 & 0 \\
0 & -A & 2A & 0 \\
0 & 2A & -A & 0 \\
0 & 0 & 0 & A
\end{bmatrix}$$

(3)

Solving the Schrodinger equation for the hyperfine Hamiltonian gives the ground state solution as an antisymmetric combination of the two anti-parallel spin states, $1/\sqrt{2} (|P^\uparrow, e^\downarrow\rangle - |P^\downarrow, e^\uparrow\rangle)$, with the ground state energy $-3A$ and the three other states have the same positive energy $A$. The total energy split is $\Delta E_{\text{hyperfine}} = 4A$. The hyperfine energy split is the source for the 21 cm line emission by hydrogen atom clouds of $5.87 \times 10^{-6}$ eV. It is about six order of magnitude smaller than the difference between the hydrogen electronic ground state $E_{n=0}^{\text{hydrogen}}$ and its first excited states $E_{n=1}^{\text{hydrogen}}$ which is 10.2 eV.
Dirac’s spin exchange operator $\hat{P}^\text{Ex}$ are used also in spin lattice models to exchange spins of two adjacent lattice sites generating spin waves with energies $^6\hbar\omega_k = 2A(1 - \cos(kb))$, where for small $k$ values, $kb << 1$, equals to $\hbar\omega_k = Ak^2b^2$ and an effective mass may be calculated.

The two aspects above related to Dirac’s exchange operator, i.e. the hyperfine energy split and spin waves are adapted here for the description of the hydrogen atom’s proton and electron surrounded by the pion tetrahedron condensate forming a quark and antiquark flavor waves. We propose that electron dynamics may be explained by rapid quark exchange reactions that occur between the electrons and the vacuum pion tetrahedrons we describe as quark and antiquark flavor waves in analogy to spin waves.

We assumed in a previous paper that the electron tetrahedrons have two quark compositions that determine their spin state$^2$. One electron tetrahedron has the composition $d\bar{u}\bar{d}d$ (spin up configuration) and the second electron tetrahedron composition is $d\bar{u}\bar{u}u$ (spin down configuration). The two electron tetrahedrons are surrounded by the vacuum pion tetrahedrons $\bar{u}d\bar{d}u$ forming together electrons clouds in thermal equilibrium. The exchanged quarks for the spin up electrons are the $d$ and $u$ quarks, while the exchanged quarks for the spin down electrons are the $\bar{u}$ and $\bar{d}$ quarks.

Since the proposed quark flavor exchanges between the electron tetrahedron and pion tetrahedron may be seen as switching between two quantum states, we adapt the spin lattice model for the QCD vacuum and assume that each lattice site may be in one of the two states, the pion tetrahedron state or the electron tetrahedron state. The transition between the two states occurs by exchanging a $d$ quark with a $u$ quark or a $\bar{d}$ and a $\bar{u}$ quarks as shown in figure 2 below.
Figure 2 illustrates quark exchange reactions for the spin up and the spin down electrons with the vacuum pion tetrahedrons. The exchanged quarks for the spin up electrons are the d and u quarks, while the exchanged quarks for the spin down electrons are the $\bar{u}$ and the $\bar{d}$ quarks.

We propose a hydrogen atom model with two co-centric spheres filled with pion tetrahedrons that are polarized according to the spins of the proton and the electron in each sphere. There are 4 possible spin polarized states as shown below in figure 3. We assume that the vacuum is filled with a virtual lattice that contains in each site a pion tetrahedron or an electron tetrahedron. Near the proton in the inner sphere with a radius, $a_s$, the pion tetrahedron condensate is polarized according to the proton spin, and in the outer sphere, the pion tetrahedrons are polarized according to the electron spin.
Figure 3 illustrates the hydrogen atom model with two co-centric pion tetrahedron polarized spheres, one in the proton vicinity and the other further away from it.

We assume that similar to Dirac’s exchange operators, quark flavor exchange operators exchange the two particles in each neighboring lattice sights i and j. The exchange is performed by quark exchange reactions where $d$ and $u$ quarks are exchanged for the electron spin up tetraquark, $|e^+_j\rangle = |d\bar{u}\bar{d}d\rangle$, and the $\bar{d}$ and $\bar{u}$ anti quark for the electron spin down tetraquark $|e^+_j\rangle = |d\bar{u}\bar{u}u\rangle$. The quark flavor exchange operators for the electron spin up and spin down are:

\begin{align*}
\hat{P}_{ij}^{Flavor\ Exchan ge} & \left| \pi_i^{Td}, e^+_j \right\rangle = \left| e^+_i, \pi_j^{Td} \right\rangle \quad (4) \\
\hat{P}_{ij}^{Flavor\ Exchan ge} & \left| e^+_i, \pi_j^{Td} \right\rangle = \left| \pi_i^{Td}, e^+_j \right\rangle \quad (5) \\
\hat{P}_{ij}^{Flavor\ Exchan ge} & \left| \pi_i^{Td}, e^+_j \right\rangle = \left| e^+_i, \pi_j^{Td} \right\rangle \quad (6) \\
\hat{P}_{ij}^{Flavor\ Exchan ge} & \left| e^+_i, \pi_j^{Td} \right\rangle = \left| \pi_i^{Td}, e^+_j \right\rangle \quad (7)
\end{align*}
If the two particles in sites \( i \) and \( j \) are pion tetrahedrons, the exchanged state is the same. We do not consider the exchange of two electrons \( |e^\dagger_i, e^\dagger_i\rangle \) since hydrogen atom has only one electron.

We assume that if the proton and electron spins are parallel, the two polarization spheres are separated by a polarization wall like in a magnetic domain, while if the proton and electron spins are anti-parallel, the electron moves freely in the two co-centric spheres. Hence, the hyperfine energy split is explained by the larger volume allowed for the quark and antiquark flavor waves to move in case the spins are anti-parallel. In the other case when the two spins are parallel, a polarization domain wall exist that limits the space for the electron motion and increase the energy.

We can calculate the radius of the internal sphere, \( a_s \), using the hydrogen atom hyperfine energy split of \( 5.87 \times 10^{-6} \) eV and the spheres’ volumes -

\[
\Delta E^{\text{hyperfine}} = \frac{4}{3} \pi a_0^3 N f \varepsilon - \left( \frac{4}{3} \pi a_n^3 - \frac{4}{3} \pi a_s^3 \right) N f \varepsilon = \frac{4}{3} \pi a_s^3 N f \varepsilon
\]  

(8)

\( N \) is the density of cells in the condensate lattice, \( f \) is the exchange frequency and \( A \) is the exchange energy. We do not know the values of the three parameters, \( N, f \) and \( \varepsilon \), however, we can calculate the ratio of the hyperfine energy and the difference between the ground state and the first excited states of the hydrogen atom that remove these parameters.

\[
E_1^{\text{hydrogen}} - E_0^{\text{hydrogen}} = \frac{4}{3} \pi a_{n=1}^3 N f \varepsilon - \frac{4}{3} \pi a_{n=0}^3 N f \varepsilon
\]  

(9)

\[
\frac{\Delta E^{\text{hyperfine}}}{E_1^{\text{hydrogen}} - E_0^{\text{hydrogen}}} = \frac{a_s^3}{a_{n=1}^3 - a_{n=0}^3} = \frac{5.87 \times 10^{-6}}{10.2}, a_{n=1} = 4 a_{n=0}, a_{n=0} = 0.529^{-10}
\]  

(10)

Accordingly, the radius of the small polarization sphere, \( a_s \), is \( 6.346^{-13} m \) and the ratio of the two spheres’ radii is \( 1.199\% \) -

\[
\frac{a_s}{a_0} = \frac{6.346^{-13}}{0.529^{-10}} = 0.01199
\]  

(11)
The hydrogen atom hyperfine energy split is extremely small, however, in ferromagnets due to the long-range magnetic interaction, the spin-spin interaction energy becomes bigger and more significant. In the next section the magnetoresistance of the electrons and pion tetrahedrons gas model is described.

3. The Electron and Pion Tetrahedron Gas Resistance
In contrast to most electronic devices and circuits where the electric charge is the main player, magnetic materials use the electron spin to store data and the spin-orbit coupling to switch between spin states. For example, hard drive discs use magnetic domains formed by typically 10 nanometer size ferromagnetic Co-Pt-Cr grains to maintain stable spin states. Magnetic read heads include typically two ferromagnetic layers, one with a fixed magnetization and a second with a varying magnetization sensitive to the magnetic domains maintained by hard drive discs. The current that flows through the magnetic read head depends on the spin alignment of two ferromagnetic layers that produce the giant magnetic response (GMR) discovered by Fert and Grünberg.

Sinova et al described Mott’s double scattering spin polarization proposed in 1929 inspired by Dirac’s electron theory and magnetic tunnel junctions (MTJ). First scattering of an electron beam on heavy atom target creates a spin polarized beam that is scattered again by a second heavy atom target creating two currents that can be measured with different chirality.

We propose the following electron and pion tetrahedron gas resistance model that may be used to explains MTJs and Mott’s double scattering. We assume that scattering of two electron tetrahedrons with the same spin that share pion tetrahedron clouds are elastic collisions with no heat dissipation –

\[
\begin{align*}
\bar{u}d\bar{d} + \bar{u}d\bar{d} & \rightarrow \bar{u}d\bar{d} + \bar{u}d\bar{d} \\
\bar{u}\bar{d}\bar{u} + \bar{u}\bar{d}\bar{u} & \rightarrow \bar{u}\bar{d}\bar{u} + \bar{u}\bar{d}\bar{u}
\end{align*}
\]
However, the scattering of two opposite spin electron tetrahedrons occurs via a pion tetrahedron transition state complex \((\bar{u}d\bar{d}d\bar{u}u\bar{u}d^\dagger, \text{equation } 13a)\) and is a non-elastic reaction between the two electron tetrahedrons where heat is dissipated to the pion tetrahedron condensate causing a friction for the electric current flow and switching the electrons’ spin as shown in equation 13b (the \(d\bar{d}\) and \(\bar{u}u\) are exchanged) -

\[
\bar{u}d\bar{d} + \bar{u}d\bar{u} \rightarrow \bar{u}d\bar{d}\bar{u}u\bar{u}d^\dagger \tag{13a}
\]

\[
\bar{u}d\bar{d}\bar{u}u\bar{u}d^\dagger \rightarrow \bar{u}d\bar{u}u + \bar{u}d\bar{d} \tag{13b}
\]

Note that both \(\bar{d}\) and \(d\) quarks are exchanged with \(\bar{u}\) and \(u\) quarks of the second electron. The quark exchange reaction via the pion tetrahedron transition state complex cause heat dissipation and electrical resistance that is explained typically by scattering of electrons on the solid impurities. The electron and pion tetrahedron gas model resistance may be an additional spin dependent scattering.

The MTJ read process may be explained in terms of the electron and pion tetrahedron gas resistance model as follows: the initial non-polarized electron current has equal number of spin up and spin down electron tetrahedrons, \(\bar{u}d\bar{d}\bar{d}\bar{d}\) and \(\bar{u}d\bar{u}u\). The non-polarized current flows through a first ferromagnetic layer we assume is polarized and occupied mostly by \(\bar{u}d\bar{d}\bar{d}\) electron tetrahedrons. The scattering of two electron tetrahedrons of the same type is elastic while the scattering of opposite spins creates a transition state complex of the two electrons with a pion tetrahedron “QCD glue” as illustrated in figure 4. The collision process dissipates heat and cause a selective delay for one electron tetrahedron configuration. The outcome is that the electron current has now higher percentage of \(\bar{u}d\bar{d}\bar{d}\) electron tetrahedrons that were elastically scatters with no delay, e.g. the electric beam is polarized.
The polarized current enters the second ferromagnetic layer and now the outcome will depend on the second ferromagnetic layer spin state. If it is dominated by $\bar{u}d\bar{d}d$ electron tetrahedrons, a favorable parallel ferromagnetic layer configuration (P), the quark exchange reactions will be elastic and a high current will be seen. However, if the second layer magnetization is in anti-parallel configuration (AP), more electron tetrahedron collisions with spin flips will occur, heat will be dissipated, and the current will be delayed. The magnetoresistance is higher and the electric current is lower.

$$d\bar{u}\ d\bar{d} + d\bar{u}\ u\bar{u} \quad \longleftrightarrow \quad d\bar{u}\ u\bar{d}\bar{d}\ d\bar{u}$$

Figure 4 illustrates the pion tetrahedron transition state complex “QCD glue” formed in a collision between two electrons with opposite spins.

The electron and pion tetrahedron gas resistance model provides a mechanism that may explain why the magnetoresistance of two parallel ferromagnetic layers (P) is lower than the magnetoresistance of the antiparallel (AP) configuration. The pion tetrahedron condensate in the
ferromagnetic layer is polarized favorably and allows coherent elastic scattering of electron tetrahedrons with the same spin with no heat dissipation.

Magnetic Random Access Memory (MRAM) is an emerging non-volatile semiconductor memory technology\textsuperscript{12} expected to replace traditional computer memory based on complementary metal-oxide semiconductors\textsuperscript{13}. MRAM surpasses other types of memory devices in terms of nonvolatility, low energy dissipation and fast switching speed. Future developments in MRAM are based on spin-transfer torque. Spin transfer torque corresponds to the interaction of a spin polarized electronic current with the local magnetization. Magnetic moment is transferred from the conduction electrons to the magnetization resulting in a change of the magnetization orientation\textsuperscript{14,15}.

MRAM cells include a magnetic tunnel junction (MTJ) that provides the write, read and bit storing functionality, essentially using two magnetic layers, reference layer (RL) and the free layer (FL), separated by a magnesium oxide (MgO) tunnel barrier. The two-bit storage states are the parallel (P) and antiparallel (AP) magnetization orientations\textsuperscript{16,17}. Topological insulators provide spin polarized current surface states due to the topology of the bulk band structure\textsuperscript{18}. The interplay of electron transport and spin dynamics provide a method to electrical control the magnetization state in Weyl semimetals\textsuperscript{19}.

According to the electron and pion tetrahedron gas resistance model described here, we propose a mechanism for the spin torque, where the spin switching in the magnetic free layer (FL) occurs due to electron tetrahedron scattering quark exchanges via the pion tetrahedron transition state complex as shown in figure 4 above and figure 5 below.
Figure 5 illustrates spin transfer torque by electron-electron scattering, where the upper electron represents a beam of spin polarized electrons $e_{\sigma}^-(l)$ that flows through a free magnetic layer (FL) and are scattered by the FL electrons $e_{-\sigma}(FL)$ via a pion tetrahedron transition state complex that decays and flip the electrons’ spins.

The upper electron represents a beam of spin polarized electrons $e_{\sigma}^-(l)$ that flows through a free magnetic layer (FL) and are scattered by the FL electrons $e_{-\sigma}(FL)$ that we assume have an opposite magnetization state (AP). The two electron tetrahedrons exchange two quarks via the pion tetrahedron transition state complex and both flip their spin states. Assuming the initial spin state of the free layer was anti-parallel configuration (AP), $e_{-\sigma}(FL)$, after the scattering its spin is flipped up $e_{\sigma}^-(FL)$ while the spin of the incoming polarized electrons $e_{\sigma}^-(l)$ that created the spin transfer torque is switched to the down state $e_{-\sigma}^-(l)$. The FL magnetization state is switched
from $e^{-\sigma}(FL)$ to $e^{\sigma}(FL)$. Both $\bar{d}$ and $d$ quarks need to be exchanged with the $\bar{u}$ and $u$ quarks for the spin exchanges.

A competing resilience against switching the magnetization state of the whole magnetic domain will occur where adjacent ferromagnetic electrons with opposite spins via the surrounding pion condensate will try to flip back the torqued spin.

\[
\bar{u}d\bar{u}u(e^{-\sigma}) + \bar{u}d\bar{d}d(e^{\sigma}) \rightarrow \bar{u}d\bar{d}d\bar{u}\bar{u}\bar{d}\bar{d} \tag{14a}
\]

\[
\bar{u}d\bar{d}d\bar{u}\bar{u}\bar{d}\bar{d} \rightarrow \bar{u}d\bar{u}u(e^{\sigma}) + \bar{u}d\bar{d}d(e^{-\sigma}) \tag{14b}
\]

However, if the torque of the incoming electric current is strong enough, an inversion of the magnetic domain will occur and the opposite spin electrons will be pushed out from the inverted magnetic domain. Kateel et al.\textsuperscript{20} showed that shaping the SOT channel to create a bend below the MTJ device causes an efficient and deterministic inversion of the spin of the FL magnetic domain.

Note that the double quark flavor exchange reaction creates a pion tetrahedron transition state complex “QCD glue” that dissipate heat to the pion tetrahedron condensate and may also create small vibrations of the pion tetrahedron lattice sites that are probably extremely small and negligible in low energies. However, in an extremely high-energy events, like black hole or neutron star binary mergers, the vibrations of the QCD ground state pion tetrahedron condensate may be the carriers of gravitation waves. In the next section we use a virtual box with a cosmological scale and propose that the pion tetrahedron density should be taken into account in Einstein’s energy-momentum tensor $T^{\mu\nu}$.

4. The Pion Tetrahedrons and Friedmann Equation

Let’s imagine a virtual box in intergalactic space that has a very low density of free electron and pion tetrahedron gas and equal density of protons and neutrons gas for neutrality. Next, let’s
increase the box such that it includes for example the Milky way galaxy with all its visible mass \( M_{\text{Milkyway}} \) and in it its center the supermassive Sagittarius A black hole with its mass \( M_{\text{Sgr A}} \).

The Einstein-Hilbert action with a cosmological constant \( \Lambda \) is\(^{21,22,23} \)

\[
I = \int d^4x \sqrt{-g} \left( R - \frac{2}{16\pi G} \Lambda + L_M \right)
\] (15)

Where \( L_M \) is the matter Lagrangian. Einstein’s field equation is obtained by the requirement that the action will be an extremum with respect to variation of the metric tensor \( \delta g^{\mu\nu} \)-

\[
R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} + \Lambda g^{\mu\nu} = 8\pi G T^{\mu\nu}
\] (16)

The energy-momentum tensor \( T^{\mu\nu} \) is given by-

\[
T^{\mu\nu} = (\rho + p) u_\mu u_\nu + p g^{\mu\nu}
\] (17)

Where \( u_\mu \) is the tangent velocity 4-vector, \( \rho \) is the matter-energy density and \( p \) is the pressure.

\[
\rho = \frac{M_{\text{Sgr A}}}{V} + \frac{M_{\text{Milky} - \text{way}}}{V} + \frac{M_{\text{Intra-gas}}}{V} + \frac{E M}{V} + \frac{M_{\tau d}}{V}
\] (18)

The matter-energy density includes the matter and electromagnetic radiation densities of the Milky-way galaxy and its surrounding intergalactic space. The pion tetrahedrons are not regular matter particles since they are composed of equal parts of matter and antimatter quarks, however, since pion tetrahedrons exchange quarks with matter particles\(^2 \), interact with the electromagnetic field\(^3 \) and with other pion tetrahedrons, they contribute energy and momentum to Einstein’s equation energy-momentum tensor \( T^{\mu\nu} \). The pion tetrahedron density \( \frac{M_{\tau d}}{V} = < \)

\[
\Psi_u^\dagger \Psi_d^\dagger \Psi_\bar{u} \Psi_{\bar{d}} > \pi \]

represents the pion tetrahedron tetraquarks, \( \bar{u}d\bar{d}u \), condensate. \( \Psi_u^\dagger \) is an up quark creation operator, \( \Psi_d^\dagger \) is a down quark creation operator, \( \Psi_\bar{u} \) and \( \Psi_{\bar{d}} \) are the anti-up and anti-down quark creation operators.
Using the state equation, $\bar{p} = w\bar{\rho}$, with $w = -1$, for the expanding universe and Friedmann equation with $k=1^{22}$, the following expression for the matter-energy is obtained -

$$H(t)^2 = \frac{a^2}{a^2} = \frac{8\pi G}{3} \bar{\rho} + \frac{\Lambda}{3} + \frac{1}{a^2}$$ \hspace{1cm} (19)

The matter-energy density $\bar{\rho}$ is given in terms of the other variables as -

$$\bar{\rho} = \frac{3 \left( H(t)^2 - \frac{\Lambda}{3} + \frac{1}{a^2} \right)}{8\pi G}$$ \hspace{1cm} (20)

The pion tetrahedron condensate density $\frac{M_{\piTd}}{V}$ is given by equations 7 and 9 as -

$$\frac{M_{\piTd}}{V} = \frac{3 \left( H(t)^2 - \frac{\Lambda}{3} + \frac{1}{a^2} \right)}{8\pi G} - \left[ \frac{M_{Sgr A}}{V} + \frac{M_{Milky-way}}{V} + \frac{M_{Intra-gas}}{V} + \frac{EM}{V} \right]$$ \hspace{1cm} (21)

Equation 21 gives an estimate for the milky way galaxy pion tetrahedron condensate density $\frac{M_{\piTd}}{V}$. Since the mass of the pion tetrahedron is expected to be low, few order of magnitude smaller than the electron, the calculation of its density requires high accuracy estimations of all the variables on the right-hand-side of equation 21. A more direct method for calculating the pion tetrahedron mass is based on measurements of the time periodic variability of the $\beta$ decay half-life times of radioactive nuclei’s in the perihelion and aphelion points of earth’s trajectory around the sun$^3$.

Note that the pion tetrahedron density is expected to be non-uniform and have an atmospheric like drop far from galaxy centers$^2$ and that it may grow with time since the pion tetrahedrons are bosons that may be duplicated exponentially by Corley and Jacobson’s black hole laser effect$^{24}$. The pion tetrahedron density growth and the separation of matter and antimatter particles next to black holes event horizons may increase the energy density and pressure expanding the universe without adding dark matter and energy, where the additional antimatter may be trapped under the event horizon of the supermassive black holes$^{25}$. 
References


https://www.feynmanlectures.caltech.edu/III_15.html


https://www.spintec.fr/introduction-to-spin-transfer-torque/


