

A Theory of the Quantum Vacuum Structure

Rami Rom^(a)

Abstract: According to the standard model (SM) the quantum vacuum is not empty. However, both general relativity (GR) and SM do not describe the quantum vacuum content and structure. We propose a theory of the quantum vacuum structure based on pion tetrahedron tetraquarks that fill space with varying density determined by the local matter and electric charge densities. The quantum vacuum theory assumes that the valence quarks and antiquarks, u, d, \bar{u}, \bar{d} that form the pion tetrahedron tetraquarks are the building blocks of the universe where all other stable and unstable particles are comprised from these building blocks. The pion tetrahedron density drops exponentially moving away from massive and electrically charged objects. Motion of massive particles on the vacuum pion tetrahedron lattice is performed by quark exchanges via quantum tunneling through a double well potential barrier and the motion of massless particles are performed by excitations of internal degrees of freedom of the pion tetrahedron lattice. The Zitterbewegung force free trembling motion, the Zero-Point-Energy (ZPE) electromagnetic field fluctuations, the β decay and high-precision measurements of the electron mass, may be low energy QCD tracks that may prove the theory of the quantum vacuum structure.

Keywords: Pion tetrahedrons, Quantum Chromodynamics (QCD), Quantum Electrodynamics (QED), General Relativity (GR), quantum vacuum, Zitterbewegung, Zero-Point-Energy (ZPE).

(a) Email: romrami@gmail.com

1. The Vacuum Pion Tetrahedron Lattice Condensate

According to the standard model (SM), the mass of the elementary particles is due to the Higgs mechanism where a non-zero vacuum expectation value (VEV) spontaneously breaks the chiral symmetry of the otherwise massless particles solution of the Dirac Lagrangian. The SM assumes that the quantum vacuum is not empty but does not tell what is the content and structure of the vacuum that creates the non-zero VEV and the Mexican hat potential¹⁻⁵. The internal structure and spin of the elementary particles will be studied by the EIC⁶. Paraoanu described the quantum vacuum as an entity endowed with structure, which lies beneath the existential level of “real” matter⁷ and cites Einstein’s words - *“There is no such thing as an empty space, i.e. a space without field. Space-time does not claim existence on its own, but only as a structural quality of the field”*. In previous papers we described the electron and pion tetrahedron models and various quark exchange reactions of the pion tetrahedrons including β decay and quark confinement⁸⁻¹¹.

In this paper we focus on a theory for the quantum vacuum structure filled with pion tetrahedron tetraquark lattice. We note that the vacuum pion tetrahedrons are not “real” matter since they are composed of 50% antimatter that “annihilates” its other 50% matter component, but they have properties like mass, rotational and vibrational energy and electric dipole. Each lattice site contains a single tetraquark, $u\tilde{d}d\tilde{u}$, composed of two valence quarks and their antiquark pairs and having a tetrahedron structure. There are two pion tetrahedron chiral enantiomers obtained by exchanging the positions of two quarks at the tetrahedron vertices as shown below in line with Dirac/Weyl massless chiral spinors and further used by the effective field theories¹⁻⁵.

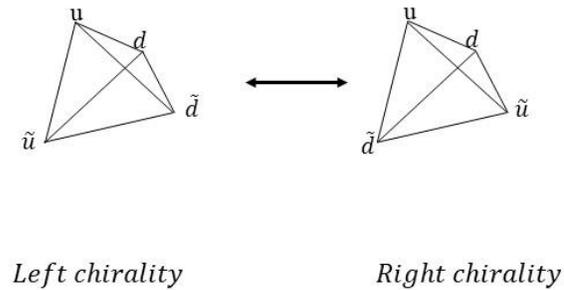


Figure 1 illustrates the two pion tetrahedron enantiomers where the \tilde{u} and \tilde{d} antiquarks exchange positions.

We assume that the pion tetrahedron lattice in empty space may be a simple cubic lattice. However, in the vicinity of a massive star, the pion tetrahedron lattice has a spherical symmetry with a cell dimension that changes according to the distance from the star. Each lattice site contains a single pion tetrahedron tetraquark, $u\tilde{d}d\tilde{u}$, composed of the two valence quarks and their antiquark pairs. The size of the tetrahedron edge may be a fraction of a femtometer while the pion tetrahedron lattice length in free space may be much larger, for example about $5.0 * 10^{-8}$ meter. In extreme gravitational fields, in the vicinity of a black hole for example, the pion tetrahedron lattice cell size may become extremely short, similar to the pion tetrahedron edge size of about $0.1 * 10^{-15}$ meter or less. Far away from any galaxy in cosmic voids, the pion tetrahedron lattice may be extremely diluted and the lattice cell size may be much larger, $\sim 1.0 * 10^{-3}$, meter for

example, and in these cosmic voids gravity¹²⁻¹⁴ the MOND acceleration limit $F = m \frac{a^2}{a_0}$, where $a \ll a_0$, and $a \sim \frac{GM}{r}$ instead of Newton's gravitational acceleration of $a = \frac{GM}{r^2}$ ⁹.

We assume that the pion tetrahedrons in each site of the lattice are massive and respond to gravitational field like ideal gas in gravitational field with exponential density drop⁸

$$\rho(r) = \rho(r_0) e^{-\frac{GM_s m_\pi (r - r_0)}{r_0^2 k_b T}} \quad (1)$$

The pre-exponent density $\rho(r_0) = \frac{N}{V}$ may be estimated according to the mass of the black hole divided by the mass of a proton $N = \frac{M_{BH}}{M_p}$ divided further by the volume of the black hole Schwarzschild radius r_s cube or the star radius cube. We assume that the maximal density of the pion tetrahedron lattice is obtained when the number of pion tetrahedrons are equal to the number of protons in the star or BH.

$$\rho(r_0) = \frac{3M_s}{M_p^4 \pi r_s^3} \frac{\# \text{ pions}}{m^3} \quad (2)$$

For the Sagittarius A black hole, the pion tetrahedrons density on its Schwarzschild radius is $6.55 * 10^{32} \frac{\# \text{ pions}}{m^3}$ and for the sun on its surface it is $8.431 * 10^{29} \frac{\# \text{ pions}}{m^3}$. For a single proton it is $7.799 * 10^{17} \frac{\# \text{ pions}}{m^3}$, which is a huge number of pion tetrahedrons but 6 orders of magnitude smaller than Avogadro's number $6.023 * 10^{23}$.

2. Electrons and the Vacuum Pion Tetrahedron Lattice

Electrons in the proposed quantum vacuum theory are non-elementary composed particles made of tetraquarks having two configurations we refer to herein as a right chiral $\tilde{u} du\tilde{u}$ and a left chiral $\tilde{u} dd\tilde{d}$ which are dynamical conserved quantities. Adding an electron to the pion tetrahedron lattice is done by replacing one pion tetrahedron with one of the two chiral electron tetrahedron and then using a double well potential Hamiltonian¹⁵ that represents an exchange of

quarks between the electron and the pion tetrahedron on adjacent sites that transform the chiral electron tetrahedron to a pion tetrahedron and vice versa as shown in equation 1 below for a left chiral electron where the u and d quarks are exchanged as shown in figure 1 and equations 1 and 2 further below.

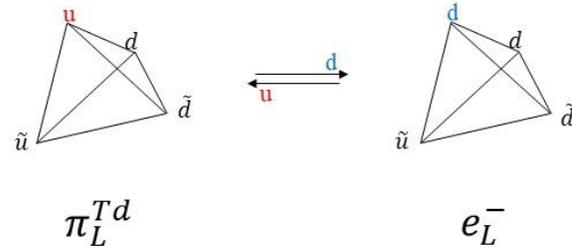


Figure 2 illustrates an electron and a pion tetrahedron where a single quark flavor is exchanged (u and d).

$$\tilde{u}d\tilde{d}u (\pi^{Td})_i + \tilde{u}d\tilde{d}d (e^L)_j \rightarrow \tilde{u}d\tilde{d}d (e^L)_i + \tilde{u}d\tilde{d}u (\pi^{Td})_j \quad (1)$$

In the case of the right chiral electron the \tilde{u} and \tilde{d} antiquarks are exchanged, and the equation is

$$\tilde{u}d\tilde{d}u (\pi^{Td})_i + \tilde{u}d\tilde{u}u (e^R)_j \rightarrow \tilde{u}d\tilde{u}u (e^R)_i + \tilde{u}d\tilde{d}u (\pi^{Td})_j \quad (2)$$

Since the quark exchange reactions are symmetric, the reactants on the left hand side and the products on the right hand side are identical, the usage of the double well potential Hamiltonian is justified¹⁵.

$$\hat{H} = \frac{\hat{p}^2}{2m} + m \lambda (x^2 - a^2)^2 \quad (3)$$

The mass m is the electron rest mass, a is the pion tetrahedron lattice cell size and the double well potential parameter λ determines the barrier height $V_0 = m \lambda a^4$. Based on Dirac equation zitterbewegung force free trembling motion, we assume that in free space the potential barrier height $V_0 = \hbar\omega = 2m_e c^2$, and hence the frequency $\omega = \frac{2m_e c^2}{\hbar}$ is equal to Dirac's equation zitterbewegung¹⁶. The approximate ground state energy inside the well $E_0 = \frac{1}{2}\hbar\omega = m_e c^2 = 5.11 * 10^6 eV$ is equal to the electron rest mass energy.

Figure 3 below illustrates the double well model for the electron and pion tetrahedron quark flavor exchange reaction in lattice sites i and j in the ground state. The quark exchange reaction and the double well potential is replicated to all lattice sites and the electron motion in the ground state is hence by quantum tunneling through the potential barrier V_0 , which is twice the electron rest mass energy and represents the threshold for electron-positron pair production. Note that the electron tetraquark on both sides of the double well is identical and hence the electron configuration ($\tilde{u} dd\tilde{d}$ or $\tilde{u} du\tilde{u}$) is dynamically conserved while the exchanges occur with the zitterbewegung frequency on the pion tetrahedron lattice.

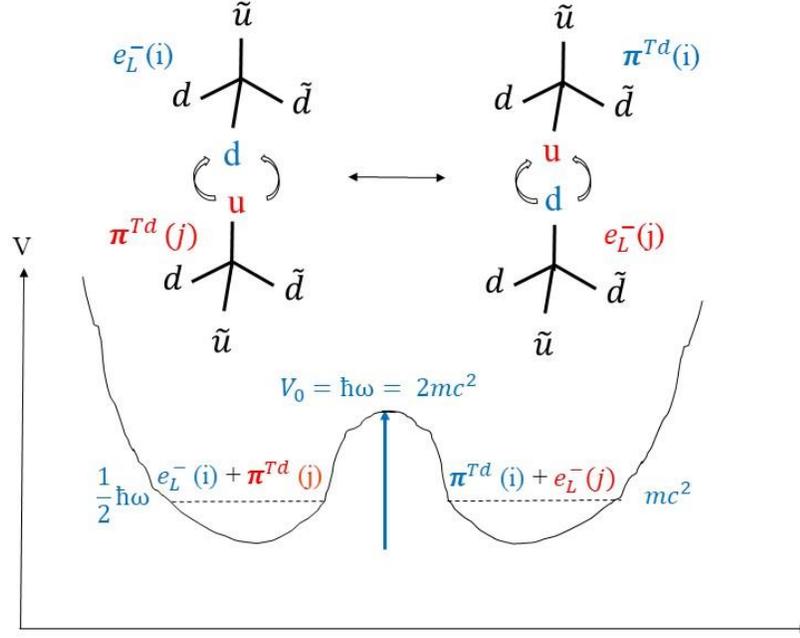


Figure 3 illustrates the double well potential model for the electron and pion tetrahedron quark flavor exchange reaction in lattice sites i and j in the ground state. Note that the potential barrier V_0 is twice the electron rest mass energy and represents the threshold for electron-positron pair production.

The tunneling probability, T , from the first to the second potential well in the ground state through the potential barrier is¹⁵

$$T = e^{-\frac{8ma^3\sqrt{2\lambda}}{3\hbar}} = e^{-\frac{32V_0}{3\hbar\omega}} \quad (4)$$

ω is the ground state frequency in each well separately given by

$$\omega = \frac{2\pi}{\tau} = \sqrt{8\lambda a^2} \quad (5)$$

We may assume that the barrier height potential V_0 may vary in space according to the gravitational field for example and that $2m_e c^2$ is its absolute minimal value and accordingly we

may determine the electron velocity in space on the pion tetrahedron lattice according to the equation below. The velocity of the electron tetrahedron from site i to j due to the flavor exchange wave is calculated by the distance between the sites, a , divided by the time period, τ , and multiplied by the tunneling probability in the ground state T .

$$v_e = \frac{a}{\tau} T = \frac{a\omega}{2\pi} e^{-\frac{32V_0}{3\hbar\omega}} \quad (6)$$

Since the electron velocity is limited by the speed of light, we get the following expression for the hoping velocity from site to site

$$\frac{a\omega}{2\pi} = c e^{\frac{32}{3}} \quad (7)$$

And hence the electron velocity is

$$\frac{v_e}{c} = e^{-\frac{32}{3}(\frac{V_0}{\hbar\omega} - 1)} \quad (8)$$

In the case that $V_0 = \hbar\omega$ the electron velocity is maximal and equals to the speed of light, this means that in reality V_0 must be bigger than $\hbar\omega$.

The electron does not follow a classical trajectory, quarks are exchanged between the pion tetrahedron sites by tunneling through a potential barrier and effectively create the motion of the electron as a delocalized cloud. We assume that in some small Compton length range, $V_0 = \hbar\omega$, and the electron speed is equal to the speed of light as in Dirac's equation zitterbewegung and semi-classical electron models¹⁶⁻¹⁷. Out of this small region that may be in a shape of a ring or a torus, $V_0 > \hbar\omega$, the electron speed is smaller than the speed of light. The double well potential barrier height for the quark flavor exchange in lattice sites i and j may be a function of the quark states $V_0(d_i, u_j)$. The pion tetrahedron quarks may rotate or vibrate with long range correlation creating local electric and magnetic fields since the quarks are charged.

The double well potential with $\frac{V_0}{\hbar\omega} = 1.0$ and with $\frac{V_0}{\hbar\omega} = 2.0$ are shown below. With larger potential barrier V_0 value the wells are steeper, the tunneling probability through the barriers is smaller, the quark flavor exchange wave propagation is slower and the electron ground state wavefunction is more localized inside the well.

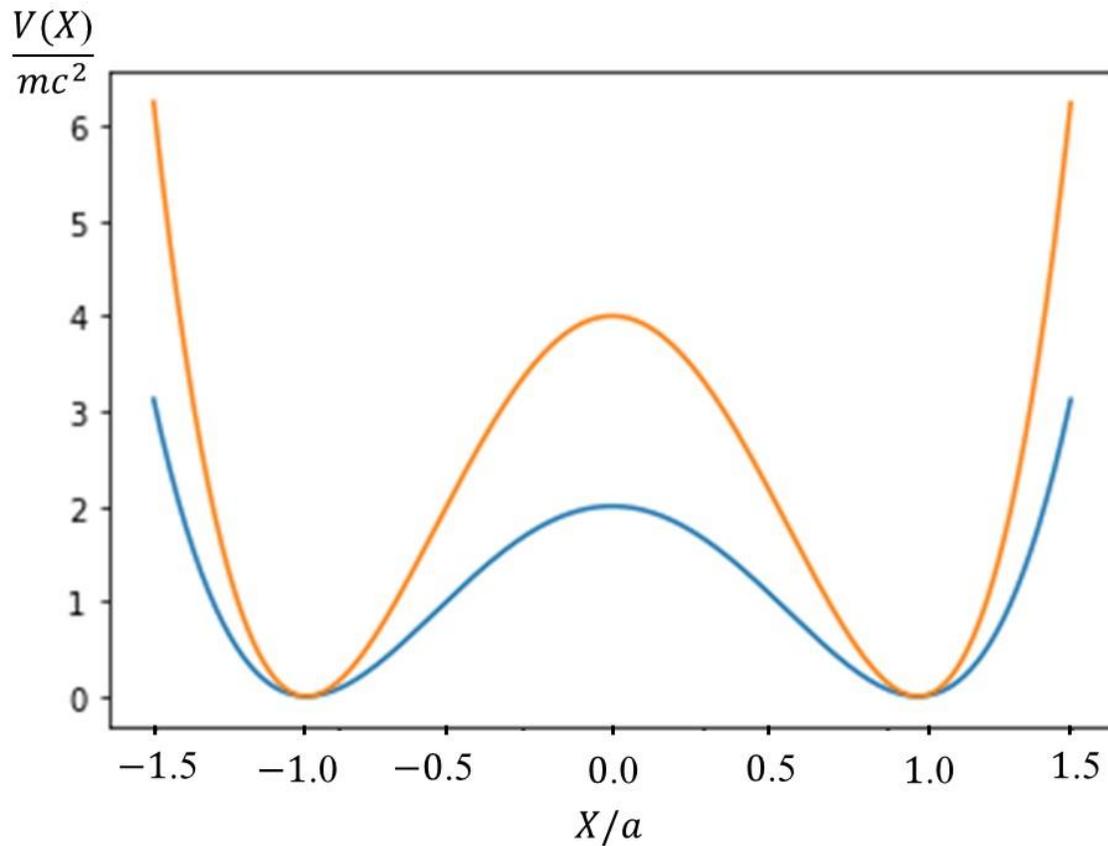


Figure 4 illustrates the double well potential with two values of the barrier height $V_0 = 2m_e c^2$ and $V_0 = 4m_e c^2$ with $a = 5.2045 * 10^{-8} m$.

The frequency ω times the pion tetrahedron lattice cell length a is a constant (equation 7 above)

$$\omega a = 2\pi c e^{\left(\frac{32}{3}\right)} = 8.0811 * 10^{13} \quad (9)$$

Using the zitterbewegung frequency, in the Compton region where the potential barrier height is minimal $V_0 = \hbar\omega$, we can calculate the pion tetrahedron lattice cell length in free space

$$a = \frac{8.0811 * 10^{13}}{\omega} = \frac{\hbar 8.0811 * 10^{13}}{2m_e c^2} \text{ meter} \quad (10)$$

$$a = \frac{8.0811 * 10^{13}}{1.5527 * 10^{21}} = 5.2045 * 10^{-8} \text{ meter} \quad (11)$$

Note that with $V_0 = \hbar\omega$, and the zitterbewegung frequency $\omega = \frac{2m_e c^2}{\hbar}$, the barrier height is $V_0 = 2m_e c^2$, which is the threshold for production of an electron-positron pair. The potential parameter λ is given by $\lambda = \frac{V_0}{m_e a^4} = \frac{2c^2}{a^4} = 2.4498 * 10^{46} \frac{1}{m^2 sec^2}$

The first order correction to the ground state energy of $E_0^{(0)} = \frac{1}{2}\hbar\omega$ in the double well is¹⁵

$$E_0^{(1)} = \frac{3\hbar^2}{32m_e a^2} = 2.637 * 10^{-6} eV \quad (12)$$

The electron mass may be measured with high precision¹⁸ where small deviations due to the time periodic variable interaction with the vacuum pion tetrahedron density may be measurable¹¹.

Assuming that the mass of the pion tetrahedron is about 6 orders of magnitude smaller than the electron, the pion tetrahedron density in free space may be estimated roughly.

$$\rho_{pion \ tetrahedron} = \frac{10^{-6} m_e}{(5.204 * 10^{-8})^3} = 6.46 * 10^{-15} \frac{k_g}{m^3} \quad (13)$$

The estimated density of the universe is $9.9 * 10^{-27} \frac{k_g}{m^3}$, which is equivalent to 5.9 protons in meter cube. Only 4.7% of the total density is due to visible matter which is about $4.33 * 10^{51} k_g$.

The estimated volume of the visible universe is $9.322 * 10^{78} m^3$. If we assume that the pion tetrahedron density in the universe is uniform its mass will be about $6.023 * 10^{64} k_g$, 13 orders of

magnitude larger than the visible mass. However, we assume that the pion tetrahedron density is dense close to matter particles and is extremely diluted far from matter for example in the cosmic web voids. The pion tetrahedron mass in the universe is probably much smaller than $6.023 * 10^{64} k_g$, however, due to the huge volume of the universe, the pion tetrahedron mass should not be neglected.

3. The β Decay Quark Equation and the Electron Tetrahedron Model

We assumed that electrons are composed of tetraquarks, and we did not attempt to provide a proof for the proposed electron model⁸⁻¹¹. Here we want to show that β rays, which were discovered in 1899 by Ernest Rutherford gives a significant support for the proposed quark content of the electron tetrahedron model. In 1900, Becquerel measured the mass-to-charge ratio (m/e) for radioactive beta particle emission and found that m/e ratio for beta particles is the same as Thomson's cathode ray electrons and suggested that β rays are electrons¹⁹⁻²⁰. At that time, protons and neutrons and their internal structure were not known and the question how an electron is emitted from a nucleus made of protons and neutrons that contains quarks and gluons only⁶ was not asked.

We propose to write down a quark based β decay equation and get an expression for the electron and the neutrino that propagate on the pion tetrahedron lattice (assuming that the valence quarks and antiquarks are conserved).

$$udd (n) + u\tilde{d}d\tilde{u} (\pi^{Td}) \rightarrow udu (p^+) + \tilde{u} d \tilde{d} d (e_L^-) + \tilde{\nu}_\sigma \quad (14)$$

Note that the β decay according to equation 14 describes a second order kinetic reaction where a collision between a neutron and a pion tetrahedron triggers the reaction. The second order kinetics classification is significant since if the density of the pion tetrahedrons is reduced, for example in the cosmic voids, the rate of the β decay will be reduced too. Hence the β decay reaction

rate is not a first order kinetics constant and it should depend on the local gravitational field since the pion tetrahedrons are massive and their density drops like the atmospheric density drops away from earth surface⁸.

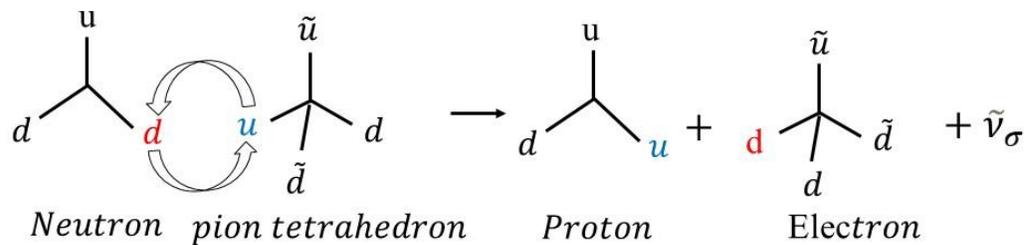


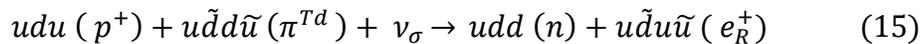
Figure 5 illustrates the β decay as a quark exchange reaction between a neutron and a pion tetrahedron as a second order kinetics reaction.

The electron tetrahedron $\tilde{u} d \tilde{d} d$ (e_L^-) and the anti-neutrino $\tilde{\nu}_\sigma$ are energetic spin half fermions that propagate on the pion tetrahedron lattice. The electron tetrahedron propagates via quark exchange reactions that occur by tunneling through the double well potential barrier while the anti-neutrino propagation may be an internal motion of the pion tetrahedrons degrees of freedom in each lattice site, a vibration or rotation waves that propagate in the speed of light on the pion tetrahedron lattice.

Note that the second configuration right chiral electron, $d\tilde{u}u\tilde{u}$, described above in equation 2, is not obtained by the β decay according to equation 14, which means that the β ray electrons are created with a preferred chirality. The weak nuclear force interaction observed in β decay breaks the parity symmetry as observed experimentally by C. S. Wu in 1957²⁰. The proposed electron tetrahedron model made of tetraquarks and the β decay quark equation 14 with the assumed quark conservation rule provides a reaction mechanism for the β decay process that may explain the chirality of the emitted electrons and also predicts a second order kinetics and a dependence on the gravitational field of the decay rate that may be observed⁸.

The reason for the β decay electron chirality polarization may be due to the quark equation 14 that generates polarized electrons with a specific chirality¹⁹⁻²⁰.

The β^+ decay transform a proton to a neutron and emits a positron with the opposite chirality comparing to the electron emitted in the β^- decay. The β^+ decay is similarly a second order kinetic reaction triggered by a pion tetrahedron and conserve the valence quark numbers that are only rearranged differently in both reactions.



4. Positrons and the Vacuum Pion Tetrahedron Lattice

According to the theory of the quantum vacuum structure, positrons are tetraquark tetrahedrons like the electron tetrahedrons but having a $u\tilde{d}$ quark pair with a plus charge instead of the $\tilde{u}d$ quark pair with a negative charge of the electron. Two positron configurations can be described with right and left chirality similar to the electrons and have additional $d\tilde{d}$ or $\tilde{u}u$ quark pairs as shown below in figure 6. Quark exchange reactions of the positron tetrahedrons with pion

tetrahedrons analog to the electron tetrahedrons and pion tetrahedrons are symmetric reactions where the products are identical to the reactants.

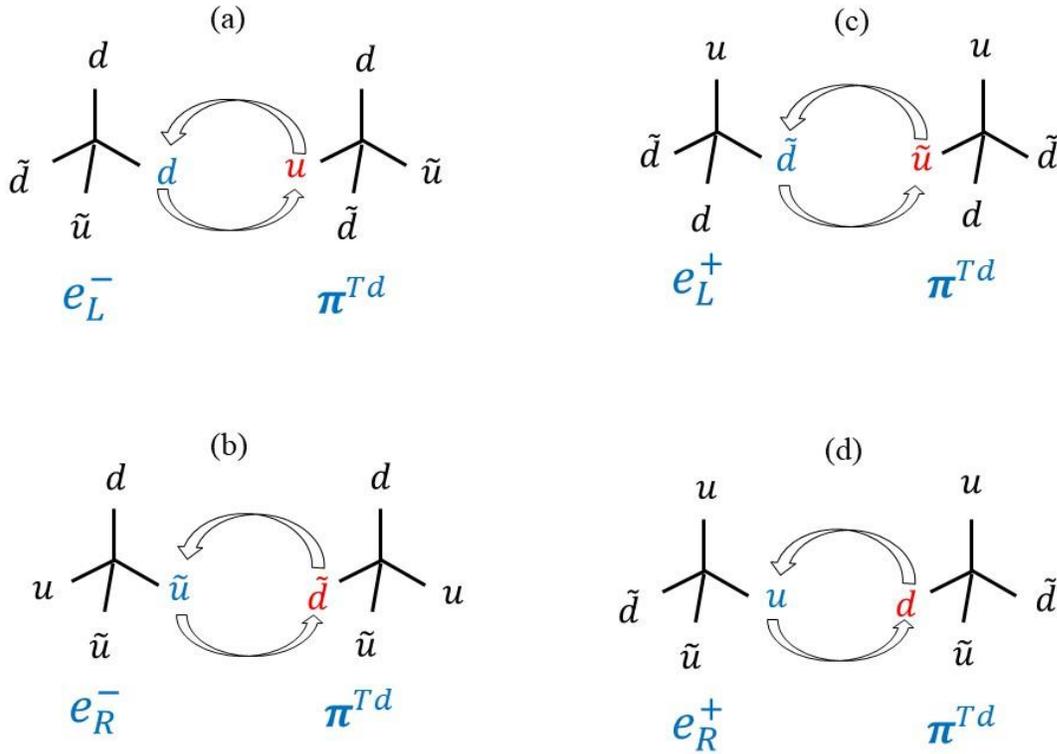


Figure 6 illustrates electron (a) and (b) and positron (c) and (d) enantiomers exchanging quarks with pion tetrahedrons with symmetric reactions such that the electron and positron enantiomers transform to pion tetrahedrons and vice versa in each exchange reaction.

5. Electron-Positron Annihilation and the Vacuum Pion Tetrahedron Lattice

Electron-positron annihilation according to the theory of the quantum vacuum structure is a condensation of an electron tetrahedron and a positron tetrahedron into two pion tetrahedrons as shown in equation 16 below.



Hence if an electron tetrahedron in site i on the lattice collide with a positron on site j on the lattice the outcome is that in both sites i and j after the collision there will be two pion new tetrahedrons and the electron and the positron are annihilated and their charge and rest mass is transferred to the pion tetrahedron lattice.

6. Electron Pairs, Pion Tetrahedron Glue and Pauli Exclusion Principle

An electron-electron pair may be created according to the theory of the quantum vacuum structure by forming a pion tetrahedron that acts a QCD glue as shown in equation 17 below. The spin of the pair is annihilated creating a boson pair like a Cooper pair in BCS superconductivity theory²¹.

$$\tilde{u}d\bar{d}d(e_L^-) + \tilde{u}d\tilde{u}u(e_R^-) \rightarrow \tilde{u}d\bar{d}d\tilde{u}u\tilde{u}d(e - e_{\downarrow\uparrow}^{pair}) \quad (17)$$

According to BCS theory the electron pairing interaction is mediated by phonons, the motion of the solid-state lattice ions, that creates the attraction between the electron pairs. Here we suggest that the pion tetrahedron acts like a QCD glue connecting the electron pairs in addition to the contribution of the observed lattice phonons, e.g. the isotopic effect.

We propose further that the electron-electron pair interaction described in equation 17 via the pion tetrahedron QCD glue may be the underlying electron pair attraction mechanism also in chemical bonds in atoms and molecules. When two electron tetrahedrons with opposite chirality collide in site i and j in the pion tetrahedron lattice, they are attracted to each other by the formation of the pion tetrahedron QCD glue and they will continue a correlated pair motion on the pion tetrahedron lattice. The attraction is a pair attraction, a third electron cannot correlate similarly with the electron pair and this mechanism may be the underlying mechanism for the Pauli exclusion principle²².

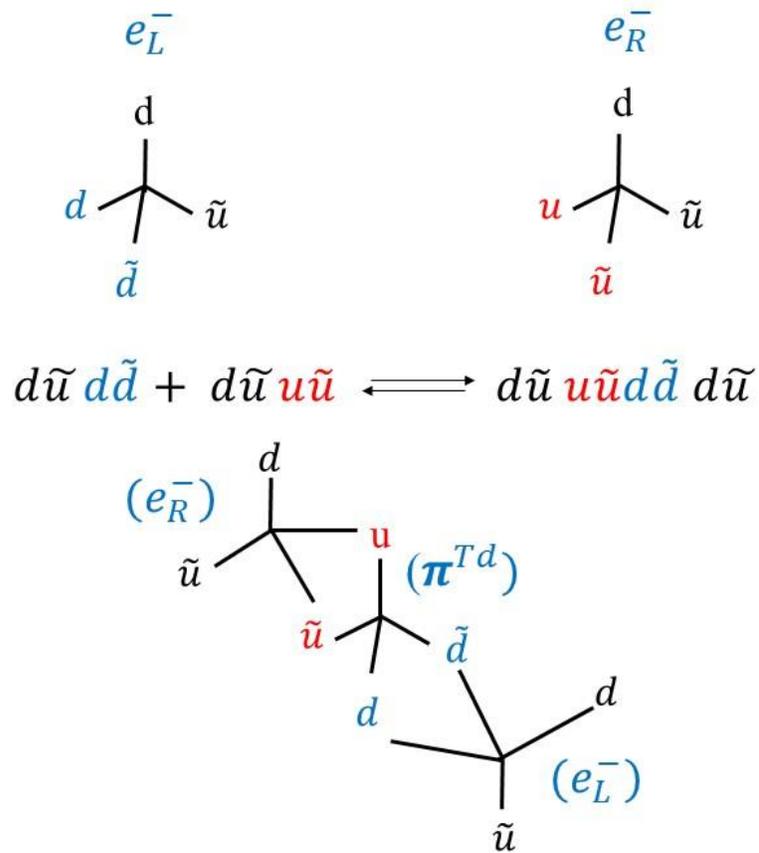


Figure 7 illustrates electron pairing mechanism forming a pion tetrahedron that acts like a QCD glue

7. The Pion Tetrahedron Glue in the Nucleus

According to the theory of the quantum vacuum structure, the pion tetrahedron may act as a QCD glue between a proton and neutron inside the nucleus as illustrated in figure 8 and 9 below. The pion tetrahedron allows a double quark exchange between that transform the proton to a neutron and vice versa.

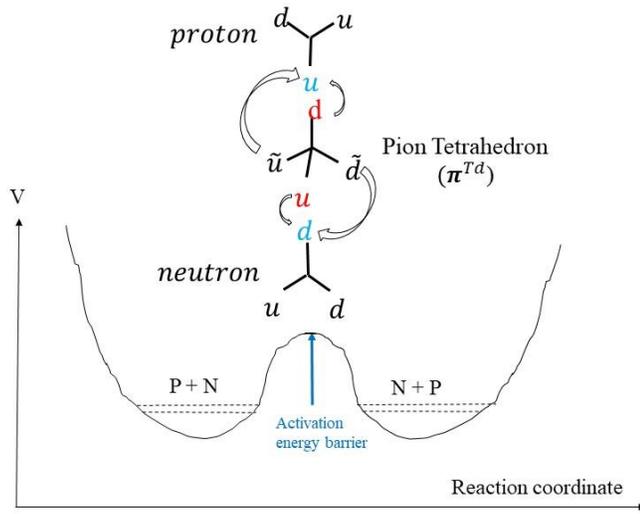


Figure 8 illustrates a proton and a neutron exchanging quarks with a pion tetrahedrons that acts like a QCD glue. Since the quark exchange reaction is symmetric transforming the proton to a neutron and vice versa, a double well potential is created.

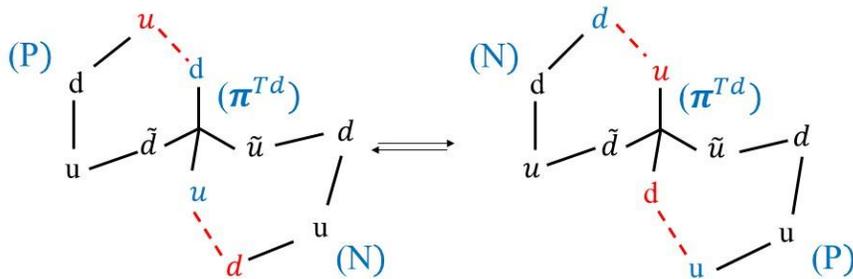


Figure 9 illustrates a proton and a neutron exchanging quarks with a pion tetrahedrons that acts like a QCD glue that provides the quarks for the double exchange reaction.

8. Summary

According to the theory of the quantum vacuum structure, pion tetrahedron tetraquarks fill space with varying density that depends on the matter included in each space region. The pion tetrahedron lattice allows propagation of massive particles like electrons and positrons via rapid quark exchanges with the zitterbewegung frequency, and massless photons via vibrations and rotations of the pion tetrahedrons with no quark exchanges. The theory of the quantum vacuum structure assumes that the valence quarks and antiquarks, $u, d, \tilde{u}, \tilde{d}$ are the building blocks of the universe and that other stable and unstable particles are comprised from these building blocks. The Zitterbewegung force free trembling motion, the Zero-Point-Energy (ZPE) electromagnetic field fluctuations, the β decay and high-precision measurements of the electron mass, may be the low energy QCD tracks that may prove the proposed theory of the quantum vacuum structure.

References

- [1] Lee, T., (2012), “Vacuum quark condensate, chiral Lagrangian, and Bose-Einstein statistics”, <https://arxiv.org/abs/1206.1637>
- [2] Brodsky, S. B., Shrock, R., (2008), “On Condensates in Strongly Coupled Gauge Theories”, <https://arxiv.org/abs/0803.2541>
- [3] Brodsky, S. B., Roberts, C. D., Shrock, R., Tandy, P.C., (2010), “Essence of the vacuum quark condensate”, <https://arxiv.org/abs/1005.4610>
- [4] Buballa, M., Carignano, S., (2014), “Inhomogeneous chiral condensates”, <https://arxiv.org/abs/1406.1367>
- [5] Byers, N., (1998), “E. Noether’s Discovery of the Deep Connection Between Symmetries and Conservation Laws”, <https://arxiv.org/abs/physics/9807044>

- [6] Burkert, V.D. et al, (2022), “Precision Studies of QCD in the Low Energy Domain of the EIC”,
https://www.researchgate.net/publication/365850432_Precision_Studies_of_QCD_in_the_Low_Energy_Domain_of_the_EIC
- [7] Paroanu, G.S., (2014), ”The Quantum Vacuum”, <https://arxiv.org/abs/1402.1087>
- [8] Rom, R., (Apr 2023), “The Quantum Chromodynamics Gas Density Drop and the General Theory of Relativity Ether”, Journal of High Energy Physics, Gravitation and Cosmology, 9, No. 2. <https://www.scirp.org/journal/paperinformation.aspx?paperid=124153>
- [9] Rom, R., (Apr, 2024), “Non-Uniform Pion Tetrahedron Aether and Electron Tetrahedron Model”, Journal of High Energy Physics, Gravitation and Cosmology.
<https://www.scirp.org/journal/paperinformation?paperid=132602>
- [10] Rom, R., (Jan 2024), “The Pionic Deuterium and the Pion Tetrahedron Vacuum Polarization”, Journal of High Energy Physics, Gravitation and Cosmology, 10, No. 1.
<https://www.scirp.org/journal/paperinformation?paperid=130928>
- [11] Rom, R., (May 2024), “QCD’s Low Energy Footprint”, <https://vixra.org/abs/2403.0128>
- [12] Keenan, R.C., Barger, A.J, Cowie, L.L., (2013) “EVIDENCE FOR A ~ 300 MEGAPARSEC SCALE UNDER-DENSITY IN THE LOCAL GALAXY DISTRIBUTION”, The Astrophysical Journal, 775:62 (16pp), 2013, September 20.
- [13] Banik, I., (Nov 20, 2023), “Do we live in a giant void? It could solve the puzzle of the universe’s expansion”, <https://theconversation.com/do-we-live-in-a-giant-void-it-could-solve-the-puzzle-of-the-universes-expansion-216687#:~:text=When%20we%20measure%20the%20expansion,area%20with%20below%20average%20density>).
- [14] Mazurenko, S., Banik, I., Kroupa, P., Haslbauer, M., (Nov 20, 2023), “A simultaneous solution to the Hubble tension and observed bulk flow within $250 h^{-1}$ Mpc”,
<https://arxiv.org/abs/2311.17988>
- [15] Grabovsky, D, (2021), “The Double Well”,
<https://web.physics.ucsb.edu/~davidgrabovsky/files-teaching/Double%20Well%20Solutions.pdf>
- [16] Santos, I. U., (2023), “The zitterbewegung electron puzzle”,
https://www.researchgate.net/publication/374257062_The_zitterbewegung_electron_puzzle
- [17] Davis, B.S., (2020), “Zitterbewegung and the Charge of an Electron”,
<https://arxiv.org/abs/2006.16003>
- [18] Sturm, S., et al, (2014), “*High-precision measurement of the atomic mass of the electron*”,

<https://arxiv.org/abs/1406.5590>

[19] Frauenfelder, H., et al, (Apr, 1957), "Parity and the Polarization of Electrons from Co60", Phys. Rev. 106, 386.

[20] Lashomb, P.R, (Thesis, 2015), "MEASURING PARITY VIOLATION IN COBALT-60 DECAY", <https://dspace.houghton.edu/server/api/core/bitstreams/2c5d4610-06ad-43c1-b4fe-beb8e92c9e39/content>

[21] Bardeen, J., Cooper, L.N., Schrieffer, J.R., (1957), "Theory of Superconductivity", Phys. Rev. 108, 1175. <https://journals.aps.org/pr/abstract/10.1103/PhysRev.108.1175>

[22] Kaplan, I. G., (2019). "Pauli Exclusion Principle and its theoretical foundation", <https://arxiv.org/pdf/1902.00499>