The paradox of the quintuplets: Why the twin paradox of special relativity cannot be resolved

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1 Abstract

By introducing three reference frames in addition to the the two reference frames typically used to discuss Einstein's special relativity (i.e., a "laboratory" frame and a boosted frame), we can show the utter futility of trying to explain (or to resolve) the absurdities of the twin paradox.

2 Introduction

There are at least two versions of special relativity – one due to Einstein, and one being the work primarily of Lorentz. The version of special relativity due to Einstein has become universally accepted. But it is the contention of this paper that the version of special relativity due to Lorentz – often referred to as "Lorentz aether (ether) theory (LET/ LAT) – is the correct version. We will use the acronym ESR to denote Einstein's special relativity.

The reason for this conclusion (i.e., that the Lorentz version of special relativity is to be preferred over the Einsteinian version) is that Lorentz specifies a preferred reference frame (i.e., the reference frame in which the ether – and presumably the earth – is at rest) while Einstein allows a multiplicity of equally valid reference frames and explicitly rejects the idea of a preferred reference frame. (By rejecting the necessity of the preferred aether- based frame to support the propagation of electromagnetic radiation, Einstein derived the Lorentz transformation in an especially easy manner[1].)

[1]The rejection of the aether resulted in the paradox in Einstein's version of relativity that the velocity of light in all but the laboratory – or "stationary" frame is not isotropic. This can be seen by considering a ray of light propagating along the direction of the y axis (in the stationary or Lab frame) when the Lorentz transformation is derived based on a ray of light traveling along the x axis in frames that are boosted along the x axis. In all but the laboratory frame, i.e., in all of the boosted frames, the ray of light will travel with speed c along the y' axis but will have an additional component of velocity along the x' axis. Thus the vector sum of these components will give a velocity greater than c for all directions other than along the x' axis. (The observation of the anisotropy of light speed in different directions is related to the historical discussion of whether a spherical light pulse remains spherical – or assumes an elliptical shape – in boosted frames; it is also related to the Ehrenfest paradox.)

By retaining an ether with different compositions in the x and y axes, this anisotropy can be avoided in Lorentz ether theory.

Einstein's version of special relativity produces a plethora of paradoxes and problems. The "twin paradox" has plagued Einstein's theory of special relativity almost since the publication of Einstien's first paper on the subject in 1905. The Ehrenfest paradox is another problem without apparent resolution. Another paradox was pointed out in [1] .

As pointed out in reference [4], there have been countless attempts to resolve the twin paradox. As with almost all of the great philosophical questions (e.g., the proposition that man has free will), there are two groups – those who agree with the proposition and those who disagree. In the case of the twin paradox, there are those who feel that the paradox can be – and has been – resolved, and those who feel that the paradox is incapable of resolution. It is the purpose of this paper to show that the twin paradox is incapable of being resolved or explained.

Before proceeding, it should be mentioned that there are several versions of the twin paradox, just as there are several versions of Mach's principle and several versions of Einstein's equivalence principle. One version of the paradox involves only the behavior of clocks in laboratory and boosted frames as they travel away from each other in uniform translational motion (obviously after one frame has been accelerated in the process of being Lorentz boosted). The second version of the paradox involves a comparison of the age of the twins when they are reunited after one twin has left earth and returned home. In the first version, the paradox involves the symmetry of the velocity of the two frames. From the viewpoint of the twin who departs, the lab frame is traveling away from the twin and its clocks should be running slower, not vice versa. It is an obvious impossibility for the clocks in the boosted frame to be running slower than the clocks in the lab frame and vice versa as was pointed out in [2].

The presence of acceleration of the boosted frame is often invoked to prove why the twin in the boosted frame is younger than the twin in the lab frame upon returning. Yet, the twin in the lab frame is accelerated with respect to the twin in the boosted frame. To say that the acceleration of the boosted frame is the only acceleration that matters because it is the only acceleration that is felt by an observer is to invoke the aboslute nature of space with respect to acceleration[2].

This analysis will attempt to show that the first form of the paradox is incapable of resoution. The first form of the paradox will be paraphrased in the following way. Consider twin A in a "stationary" or lab frame and twin B in a boosted frame. From the viewpoint of twin A, the clocks in the frame of twin B are running slowly. But from the viewpoint of twin B, twin B is the stationary twin (after the cessation of the necessary acceleration), the frame of twin A is the boosted frame, and the clocks in the frame of twin A are running slowly. This is a logical impossibility. It is absurd to think that each of two clocks can be running slower than the other clock.

The ideas of reference [1] will be used to show the futility of explaining the first form of the twin paradox[3]. That reference demonstrated that the Lorentz transformation as used by Einstein is meaningless and useless because the introduction of many different frames (rather than consideration of the usual two reference frames, i.e., the lab and boosted frames) shows that many different rates

[2]Einstein's first paper on special relativity completely ignored the potential confusing effects of acceleration in the whole issue of relativity. This topic is one of several subtle issues that are completely ignored in Einsteins' first paper on relativity. An attempt to catalog several of these ignored subtle issues will be published in a forthcoming paper.

[3]And if the first form of the paradox can be shown to be incapable of resolution, the second form of the paradox is similarly hopelessly beyond resolution.

can be associated with (or predicted for) any certain clock in a given reference frame.

3 The basic argument

To show the fuitlity of trying to resolve the (first form of) the twin paradox, we imagine the existence of five inertial reference frames[4]. One frame is labeled with the letter L and serves as the "laboratory" (or "stationary") frame. Two frames are labeled A_R and A_L and the other two frames are labeled B_R and B_L. We will designate both A_R and A_L the "A frames"; similarly frames B_R and B_L will be denoted the "B frames". All five frames share a common x axis. Next we consider identical quadruplets. The quadruplets will remain unnamed and will be identified (when necessary) only by associating them twith their respective reference frame and will only be discussed if the need to consider aging during a round trip is needed.

We ensure that the origins of all five frames are conincident initially, at which time we ensure that all clocks and measuring rods are identical and have been synchronized in an unambigous manner (if possible). We do not consider the complications of synchronization conventions and assume that an unambiguous method for synchronization of clocks and measuring rods is possible and has been used in this case. We also avoid the consideration of whether the effects of acceleration can be ″nullified″ by synchronizing the clocks of each frame *after* the acceleration has ceased and uniform translational motion is the norm. If such synchronization is impossible (as it naievely seems to be), then because acceleration may introduce a retardation factor into the time shown by each of the clocks of a given frame, it is pointless to try to compare times as shown on two clocks of differing frames at a certain event in spacetime. Instead, only theoretical rates of the clocks can be compared. That is, if acceleration can potentially alter the careful synchronization of clocks (and measuring rods) achieved while the frames are coincident and at rest, we cannot compare the aboslute values of the time shown on any two clocks from differing frames at a certain spacetime event. (We could only compare clocks from differing frames after a round trip; this is similar to the paradox that special relativity is based on the invariance of the two- way speed of light rather than invariance of the one way speed of light.)

In what follows, we will scale all velocities v with lightspeed c and use the symbol $\beta \equiv v/c$ to represent the velocities.

The reference frames will be accelerated to final velocities $\beta = 0.75$ with respect to the frames adjacent to them. The frames with subscript L will be accelerated in the negative x direction and the frames with subscript R will be accelerated in the positive x direction.

Following [5], we will use the symbol $\boxplus$ to represent the relativistic addition of velocities v_{AB} and

[4] An inertial reference frame is any frame in which an object unaffected by any force will maintain uniform translational motion or rest. We define such a frame in this manner rather than saying that an inertial frame is one in which Newton's three laws of motion are true since Newton's three laws involve circular reasoning.

We will suppose that it is possible to define a prototype (i.e., a "boilerplate" or "primordial") inertial reference frame from which all others can be obtained via Galilean boosts. Whether this can be achieved is open to debate.

v_{BD} which obeys the formula

$$v_{AD} \quad = \quad \frac{V_{AB} + V_{BD}}{1 + \frac{V_{AB} \cdot V_{BD}}{c^2}}$$

$$\beta_{AD} \quad = \quad \frac{\beta_{AB} \ + \ \beta_{BD}}{1 \ + \ \beta_{AB} \cdot \beta_{BD}} \tag{1}$$

We will also use the fact that rapidity ϕ defined by

$$\tanh\,(\phi) \quad = \quad v/c \tag{2}$$

is additive. That is, for a series of collinear boosts,

$$\phi_{1,N} \quad = \quad \phi_{1,2} + \phi_{2,3} + \ldots + \phi_{N-1,N} \tag{3a}$$

$$\beta_{1,N} \quad = \quad \tanh \phi_{1,N} \tag{3b}$$

All boosts between adjacent frames will have the value $\beta = 0.75$. To obtain the value of the boosts between frames which are not adjacent we will use the values obtained via (3); these values are displayed in table 1.

Scaled velocities to be added	Resulting β	Corresponding γ	Corresponding clock rate factor
$0.75 \boxplus 0$	0.7500	1.512	0.66
$0.75 \boxplus 0.75$	0.9600	3.571	0.28
$0.75 \boxplus 0.75 \boxplus 0.75$	0.9942	9.298	0.11
$0.75 \boxplus 0.75 \boxplus 0.75 \boxplus 0.75$	0.9992	25.005	0.04

Table 1: Boost factors with corresponding gamma and clock rate factors

The boost velocities are achieved by accelerating each frame (except the lab frame L) with the same acceleration (but for different time periods). We will assume that the final velocities of the various frames are achieved within a certain time period given by T_{boost}. After time T_{boost} as shown by the lab frame's clock, all of the frames will be in uniform translational velocity with respect to each other. As mentioned above, the possibility that acceleration might affect the syncronization of the clocks introduces a subtle complication into the analysis. If this possibility is accepted, the clock rates can no longer be determined by a one - time observation at a certain event in space time. The rates can only be compared after returning to the same pont in space time after a round trip; repeated observations of the clocks can then be made to ascertain elapsed time. The effect of acceleration can then be "diluted" if the trips are long enough. However, we will only be interested in comparing the clock *rates* (and not the absolute times shown on the various clocks) as derived theoretically by the Einstine time dilation equation.

Also, another subtle "paradox within a paradox" is introduced: While the A and B frames are accelerating away from the lab frame L, the L frame is also accelerating away from the other frames. Its clock will be running differently as seen by the other frames; however, the observer in frame L senses no acceleration. Can the observer really be said to be accelerating if the observer is unaware of the acceleration, i.e., if the observer does not "feel" the acceleration ? We also have the possibility that the observers in the A frames can be accelerating away from the corresponding B frames (i.e., A_L is accelerating away from B_L and A_R is accelerating away from B_R) while an observer in the frames would be feeling an acceleration toward the corresponding frames (i.e., the twin in A_L feels an acceleration toward frame B_L).

We will ignore these complications and just consider the clock rates (as opposed to the actual times shown on the clocks) from the standpoint of a "super- observer" not subject to the constraints of the speed of light and time. That is, we just consider the clock rates, not the aboslute value of the time shown on each. We wil assume that we compare the clock rates at a time (as shown on the lab frame L clock) which is far later than the time T_{boost} necessary to achieve the final boost velocities[5].

According to Einstein's special relativity (ESR), the only factor that determines the *relative* rate of a clock in a boosted frame as compared to the clock rate in a "stationary" or laboratory frame is the Lorentz boost factor β as it appears in the *gamma* factor defined by[6]

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{4}$$

To validate this statement, note that the only factor involved in determining the ratio of Δt_L and Δt_B in the famous equation for "time dilation" is the gamma factor:

$$\Delta t_B = \gamma \cdot \Delta t_L \tag{5}$$

In (5), the subscripts L and B refer to the lab frame and the boosted frame, respectively and t refers to time and Δ t refers to a time interval.

We can easily transform (5) into the following two equations:

$$\Delta T_B = \gamma \cdot \Delta T_L \tag{6a}$$

$$f_B \equiv T_B^{-1} = \gamma^{-1} f_L \tag{6b}$$

In (6), T represents a period of oscillation and f represents the corresponding frequency of oscillation. Since $\gamma \geq 1$ and $\gamma^{-1} \leq 1$, the frequency (and thus the "rate") of a Lorentz- boosted clock will always be less than that of the corresponding clock in the related lab ("stationary") frame.

As seen from reference frame L (the lab frame), the clocks in the A frames (i.e., frames A_R and A_L), will have the same rates but will have a slower rate than the clock in frame L. As seen from reference frame L (the lab frame), the clocks in the B frames (i.e., frames B_R and B_L), will have identical rates but will have a slower rate than the clock in frame L and also slower rates than the clocks in the A frames. This situation is shown in figure 1.

[5] Another method to remove the possible effect of acceleration on the absolute times shown by the clocks is to find some way to calibrating the clocks at the origin of the frames after the accelerations are over and the steady state boost velocities have been achieved. This would involve sending a common signal to each frame as the origins were simultaneously superimposed at a single common point along the x axis.

[6] As pointed out in [1], the explanation of the *absolute* rate of the passage of time in a given frame is a mystery. The factors which determine the "absolute" value of the length of measuring rods in a given frame is also a mystery.

As seen from reference frame B_L, none of the five clocks will have a rate matching the rate of one of the other clocks. All of the other clocks will have unequal rates. In fact, the order of the magnitude of the rates will be given by $\text{Rate}_{B_L} > \text{Rate}_{A_L} > \text{Rate}_L > \text{Rate}_{A_R} > \text{Rate}_{B_R}$. This is shown in figure 2.

Therefore, we see that there is a great conflict between the rates of time as interpreted by the observers in frames L and B_L. This conflict is an unsolvable problem for those commited to Einstein's use of the Lorentz transformation. This is why the first form of the twin paradox is incapable of being resolved. If the first form of the twin paradox is absolutely incapable of being resolved, the second form is equally incapable of being resolved.

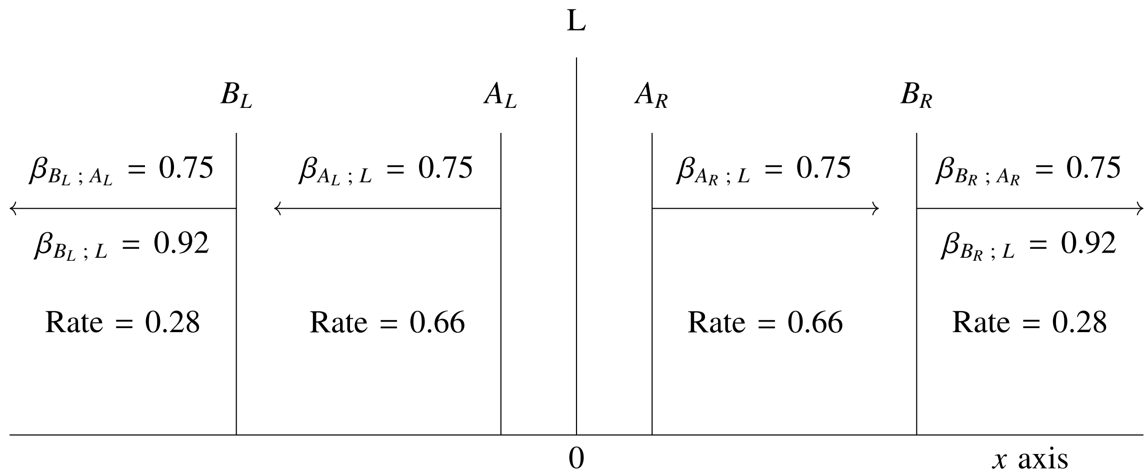

Figure 1: The scenario as seen from reference frame L, the lab ("stationary") frame. The rates shown are fractional clock rates as compared to the rates of clocks in lab frame L.

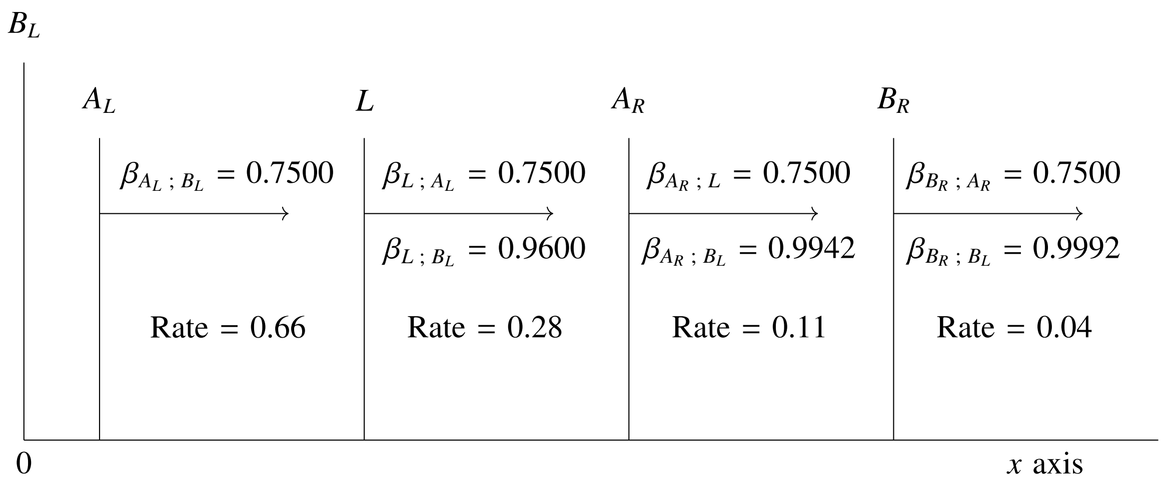

Figure 2: The scenario as seen from reference frame B_L. The rates shown are fractional clock rates as compared to the rates of clocks in lab frame B_L

4 Discussion

We have shown, using a simple application of the ideas of [1] that the twin paradox in both of its classic forms is an unsolvable problem. In other words, there is no possible solution to the twin paradox in either of its forms. By slightly complicating the considerations of the twin paradox by introducing quintuplets, i.e., by actually resorting to five identical reference frames subject to different accelerations (or no acceleration at all in the case of frame L), we have shown that the clock in each frame must be displaying time at different mutually exclusive rates. This is clearly impossible unless one invokes the use of an absurd world that is not based on logic or universal truth. This is the paradox of the quintuplets: That clocks can (and in ESR, do) have various rates, depending on which reference frame is used to derive their time dilation factor.

5 Acknowledgements

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