$\underline{\textit{Parametric solutions to sum \& difference of two cubes}}$

Oliver Couto

University of Waterloo, Waterloo, Ontario, Canada

Email: matt345@celebrating-mathematics.com

ABSTRACT

$$Since\ Fermat's\ equation, [(a^3 + b^3) = (c)^3]$$

$$does\ not\ have\ a\ solution, we\ are\ considering\ the\ below\ two$$

$$Diophantine\ equations: :$$

$$(a^3 + b^3) = w(c)^3 \quad ------(1)$$

$$(a^3 - b^3) = w(c)^3 \quad ------(2)$$

Also, equation (2) above has been discussed in the book by Tito Piezas (Ref. # 3)

$$Parametric\ solutons\ to\ equations\ (1)\ \&\ (2)\ above:$$

$$(a^3 - b^3) = w(c)^3$$

$$a = m^3 - 3mn^2 - n^3$$

$$b = 3mn(m + n)$$

$$c = (m^2 + mn + n^2)$$

$$w = (m^3 - 6mn^2 - 3m^2n - n^3)$$

$$(m,n) = (3,2)$$

$$(a,b,c,w) = (-17,-90,19-107)$$

$$(a^3 + b^3) = w(c)^3$$

$$a = m^3 - 3mn^2 + n^3$$

$$b = 3mn(m-n)$$

$$c = (m^2 - mn + n^2)$$

$$w = (m^3 - 6mn^2 + 3m^2n + n^3)$$

$$(m,n) = (3,2)$$

$$(a,b,c,w) = (18,-1,7,17)$$

$$(a^3 - b^3) = w(c)^3 \;\; ------ \;\; (1)$$

From the internet we have the below solution:

From a known solution of (1) *namely* : $(x^3 + y^3) = w(z)^3$, *another solution is shown below:*

$$a = x(x^3 + 2y^3),$$

$$b = y(2x^3 + y^3)$$

$$c = z(x^3 - y^3)$$

for: $(x,y,z) = (2,1,1)$ & $w = 9$ *we get*:

$$20^3 - 17^3 = 9(7)^3$$

$$(a^3 + b^3) = w(c)^3 \;\; ------- \;\; (1)$$

$$a = p^3 - q^3$$

$$b = q^3$$

$$c = p^3$$

$$w = p^6 - 3p^3q^3 + 3q^6$$

$$For: (p, q) = (2,1) we\ get:$$

$$(a, b, c, w) = (26,1,3,651)$$

$$(a^3 + b^3) = w(c)^3 - - - - - - - - - - - - (1)$$

$$(a, b, c) = [(7 + t), (2 - t), 3], \qquad \&$$

$$w = (t^2 + 5t + 13)$$

$$For: t = 12\ we\ get\ (a, b, c, w) = (19, -10,3,217)$$

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