THE REINTERPRETATION OF THE EINSTEIN DE HAAS EFFECT

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ABSTRACT

This publication contains a mathematical approach for a reinterpretation of the calculation of the magnetic moment for the Einstein de Haas experiment under the assumption of a magnetic field density from the elaboration "The reinterpretation of the 'Maxwell equations'[1]". The basis for this is Faraday's unipolar induction, which has proven itself in practice in combination with the calculation rules of vector analysis and differential calculus. The newly calculated "Maxwell equations" offer a generally valid calculation approach for the Einstein de Haas experiment and its problem that the difference between measurement and calculation is a factor of 2. This connection is established mathematically in this work.

It is shown that the magnetic moment can be derived mathematically by using one of the newly calculated basic equations of electrodynamics from the elaboration "The reinterpretation of the 'Maxwell equations'[1]". The gradient of the magnetic flux density $\nabla B$ and its mathematical consequences regarding the divergence of the magnetic flux density $\nabla \cdot B$ will play an important role here in this essay. By formulating that the trace of the gradient of the magnetic flux density $(Sp)\nabla B$ corresponds to the divergence of the magnetic flux density $\nabla \cdot B$ a direct connection of the magnetic flux density field itself with the field density of the magnetic flux density is revealed. It also explains and corrects the difference between measurement and calculation in the Einstein de Haas experiment. This is successful because: In this experiment, alternating current and alternating voltage were used to carry out the experiment [2]. Due to this fact, the "Maxwell equations" can be used for calculation and therefore also their new formulation from the article "The reinterpretation of the 'Maxwell equations'[1]".
1. INTRODUCTION

The Einstein de Haas experiment was carried out by Albert Einstein (March 14, 1879 - April 18, 1955) and Wander Johannes de Haas (March 2, 1878 - April 26, 1960), in 1915. The experiment showed how a magnetic moment is generated in a body. This effect is now better known as the "Einstein de Haas effect". The interpretation of this effect was that the elementary particles in the body generate a magnetic moment through rotation. The experiment was later repeated several times by different scientists. It turned out that the measurement result of the experiment is generally a factor of 2 larger than the corresponding calculation.

A solution to this problem is offered in the paper "The reinterpretation of the 'Maxwell equations'[1]". Therefore, the elaboration "The reinterpretation of the Maxwell equation'[1]" serves as the basis for this work. In particular, the newly formulated approach to induction and the associated magnetic field density are the core of the following chapters. Only the solution to the problem of factor 2, between measurement and calculation for the Einstein de Haas experiment, is focused on.

2. IDEAS AND METHODS

2.1 IDEA FOR REINTERPRETING THE "EINSTEIN DE HAAS EFFECT"

First of all, it must be clarified that "The reinterpretation of the Einstein de Haas effect" is not a reinterpretation but rather a reformulation of the calculation on the topic, since the effect itself does not need to be reinterpreted. The basic idea for the development: "The reinterpretation of the Einstein de Haas effect" is based on carrying out of the following experiments:


Based on the elaboration of "The reinterpretation of the 'Maxwell equations'[1]" and the associated mathematical requirement of a magnetic field density, the magnetic moment can now be reformulated.

All physical and mathematical descriptions used in this work are listed below.

\[
\begin{align*}
\vec{E} & = \text{electric field strength} \\
\vec{v} & = \text{velocity} \\
\vec{B} & = \text{magnetic flux density} \\
\times & = \text{cross product} \\
\vec{s} & = \text{distance} \\
t & = \text{time} \\
\delta & = \text{delta} \\
\text{rot} & = \text{rotation} \\
\text{div} & = \text{divergence} \\
\text{grad} & = \text{gradient} \\
\vec{m} & = \text{magnetic moment} \\
\vec{m}(t) & = \text{time-dependent magnetic moment} \\
I & = \text{electrical current strength} \\
i(t) & = \text{electrical current strength (alternating current)} \\
U & = \text{electrical voltage} \\
u(t) & = \text{electric voltage (alternating voltage)} \\
R & = \text{electrical resistance} \\
\vec{A} & = \text{area} \\
Sp & = \text{trace/track} \\
\end{align*}
\]

Unipolar induction according to Farady:

\[
\vec{E} = \vec{v} \times \vec{B} \tag{2.1.1}
\]

Magnetic moment:

\[
\vec{m} = I \cdot \vec{A} \tag{2.1.2}
\]

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2.2 BASICS OF VECTOR CALCULATION

In order to be able to derive the equation for the induction from the newly formulated equation for the reformulation of the Einstein de Haas experiment, the basics of vector calculation used for this are described in this chapter.

First of all, three meta-vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are introduced at this point. The three meta-vectors will be used in the following basic mathematical description. In Equation 2.2.1, these three meta-vectors are used to represent the cross product.

\[ \vec{c} = \vec{a} \times \vec{b} \]  

(2.2.1)

In equation 2.2.1, the rotation operator ($\text{rot}$) is now applied to both sides of the equation. This creates equation 2.2.2.

\[ \text{rot} \vec{c} = \text{rot} (\vec{a} \times \vec{b}) \]  

(2.2.2)

Now the right-hand side of equation 2.2.2 is rewritten according to the calculation rules of vector calculation. This results in equation 2.2.3.

\[ \text{rot} \vec{c} = \text{rot} (\vec{a} \times \vec{b}) = (\text{grad} \vec{a}) \vec{b} - (\text{grad} \vec{b}) \vec{a} + \vec{a} \text{ div} \vec{b} - \vec{b} \text{ div} \vec{a} \]  

(2.2.3)

On the right side of equation 2.2.3 two vector gradients arise, to be exact $\text{grad} \vec{a}$ and $\text{grad} \vec{b}$ . In addition, two vector divergences arise, to be exact $\text{div} \vec{a}$ and $\text{div} \vec{b}$ . From equation 2.2.3, for equation 2.1.1 follows, by applying the rotation operator ($\text{rot}$), the equation 2.2.4.

\[ \vec{E} = \vec{v} \times \vec{B} \]  

(2.1.1)

\[ \text{rot} \vec{E} = (\text{grad} \vec{v}) \vec{B} - (\text{grad} \vec{B}) \vec{v} + \vec{v} \text{ div} \vec{B} - \vec{B} \text{ div} \vec{v} \]  

(2.2.4)

The relationship between the expressions $\text{grad} \vec{a}$ and $\text{div} \vec{a}$ is described by equation 2.2.5.

\[ (Sp)(\text{grad} \vec{a}) = \text{div} \vec{a} \]  

(2.2.5)
The connection of equation 2.2.5 also applies to the connections of equations 2.2.6, 2.2.7 and 2.2.8. Equations 2.2.7 and 2.2.8 refer to equation 2.2.4.

\[(Sp)(\nabla \vec{b}) = \text{div} \vec{b} \quad (2.2.6)\]

\[(Sp)(\nabla \vec{B}) = \text{div} \vec{B} \quad (2.2.7)\]

\[(Sp)(\nabla \vec{v}) = \text{div} \vec{v} \quad (2.2.8)\]

Equation 2.2.7 will still play an important role in the reformulation of the magnetic pole moment \(\vec{m}\). First, however, the magnetic pole moment \(\vec{m}\) is explained in Chapter 2.3.

### 2.3 THE MAGNETIC POLE MOMENT

Since there are a number of formal descriptions of the magnetic pole moment \(\vec{m}\), of which only the one used by Einstein and de Haas is needed to meet the goal of this work, only this will be discussed [2]. Equation 2.1.2 describes this magnetic pole moment \(\vec{m}\). In equation 2.1.2, \(I\) stands for the electric current and \(\vec{A}\) stands for the area that is penetrated by the magnetic field in the direction of the magnetic pole moment \(\vec{m}\).

\[\vec{m} = I \cdot \vec{A} \quad (2.1.2)\]

The formulation described in Equation 2.1.2 states that the magnetic pole moment \(\vec{m}\) is calculated by multiplying the area \(\vec{A}\) that is penetrated by the magnetic field with the electric current \(I\) that encloses this area.

However, in the Einstein de Haas experiment an alternating current \(i(t)\) was used, which means that equation 2.1.2 must be reformulated into equation 2.3.1.

\[\vec{m}(t) = i(t) \cdot \vec{A} \quad (2.3.1)\]

Starting from equation 2.3.1, it will now be shown why only half of the measured value for the magnetic pole moment \(\vec{m}\) can be calculated by using the "Maxwell equations". For this purpose, the newly formulated "Maxwell equations" from the elaboration "The reinterpre-
tation of the 'Maxwell equations'[1]" will be used, which results in a calculated value for the
time-dependent magnetic pole moment $\vec{m}_t$, that also corresponds to the actual measured
value for the time-dependent magnetic pole moment $\vec{m}_t$.

2.4 DERIVATION OF THE FORMULA FOR THE MAGNETIC MOMENT

In the following chapters, the time-dependent magnetic moment $\vec{m}_t$ is connected to Hea-
viside's "Maxwell equations", specifically to the law of induction. This is done in order to cre-
ate the conditions for subsequently connecting the time-dependent magnetic moment $\vec{m}_t$
with the newly formulated "Maxwell equations" from the elaboration: "The reinterpretation of
the 'Maxwell equations'[1]". These new "Maxwell equations" can be used to explain why the
measurement result from the experiments on the time-dependent magnetic moment $\vec{m}_t$
assumes twice the value from the associated calculation.
The derivation adequately explains this discrepancy by introducing a magnetic field density
$(\text{div } \vec{B})$.

2.4.1 THE MAGNETIC MOMENT AND "THE MAXWELL EQUATIONS"

In order to explain the time-dependent magnetic moment $\vec{m}_t$, a simple technical setup is
first used here theoretically, in which the electric current and the area play a role. Considering
a simple loop of wire through which an electric current flows, this current creates a magnetic
field, twisted at a 90° angle, around and through the loop of the wire. The strength of this ma-
gnetic field depends on the strength of the electric current and the size of the area of the wire
loop. The area enclosed by the wire loop therefore contains a part of the magnetic field gene-
rated by the electric current, to be exact the part that is relevant for calculating the magnetic
moment. The magnetic moment is now a vector that is perpendicular, at a 90° angle, to the
surface enclosed by the conductor loop. If the conductor loop is now subjected to an alterna-
ting current, both the magnetic field and the magnetic moment change direction depending on
time, with the frequency of the alternating current by 180°. In order to derive the time-depen-
dent magnetic moment $\vec{m}_t$, a comparison is made at this point. The starting point for the
derivation of the magnetic moment will be equation 2.3.1 in combination with the "Maxwell
equations", first according to the well-known simplified formulation by Oliver Heaviside and
then according to the formulation from the elaboration "The reinterpretation of the 'Maxwell
equations'[1]". The differences between the two formulations are highlighted. In a first step,
however, a formulation must be found that connects the „Maxwell equations“ with the time-dependent magnetic moment $\vec{m}_{(t)}$. To do this, the basic formula for the magnetic moment from equation 2.3.1 is used as an introduction.

\[ \vec{m}_{(t)} = i_{(t)} \cdot \vec{A} \]  \hspace{1cm} (2.3.1)

If Ohm's law applied to the time-dependent electric current $i_{(t)}$, the expression from equation 2.4.1 is created.

\[ i_{(t)} = \frac{u_{(t)}}{R} \]  \hspace{1cm} (2.4.1)

The time-dependent electrical voltage $u_{(t)}$ can now be reformulated as $-\frac{\delta \vec{B}}{\delta t} \cdot \vec{A}$. It is assumed here that the area $\vec{A}$ enclosed by the conductor is constant and points vectorially in the same direction as the resulting time-dependent magnetic flux density $\frac{\delta \vec{B}}{\delta t}$. If this expression for the time-dependent voltage $U_{(t)}$ is inserted into equation 2.4.1, equation 2.4.2 results.

\[ i_{(t)} = \left( -\frac{\delta \vec{B}}{\delta t} \cdot \vec{A} \right) \]  \hspace{1cm} (2.4.2)

Now the formulation for the time-dependent electric current $i_{(t)}$ from equation 2.4.2 can be inserted back into equation 2.3.1 for the time-dependent magnetic moment $\vec{m}_{(t)}$, resulting in equation 2.4.3.

\[ \vec{m}_{(t)} = \left( -\frac{\delta \vec{B}}{\delta t} \cdot \vec{A} \right) \cdot \vec{A} \]  \hspace{1cm} (2.4.3)
In this way, the formulation for the time-dependent magnetic moment $m_{(t)}$ was connected to the "Maxwell equations". Here this happens specifically using the expression $-\frac{\delta \vec{B}}{\delta t}$. This expression represents part of the law of induction.

At this point the time-dependent magnetic moment $m_{(t)}$ would be sufficiently described, taking into account the "Maxwell equations" according to Heaviside. In the next chapter, equation 2.4.3 is used and the calculation for the time-dependent magnetic moment $\vec{m}_{(t)}$ is carried out, taking into account the newly formulated "Maxwell equations" from the elaboration "The reinterpretation of the 'Maxwell equations'[1]" improved.

### 2.4.2 THE MAGNETIC MOMENT AND "THE REINTERPRETATION OF THE 'MAXWELL-EQUATIONS'"

In the last chapter (Chapter 2.4.1) the magnetic moment was connected to the "Maxwell equations" according to Oliver Heaviside. Equation 2.4.3 shows this fact. In connection with the "Maxwell equations" according to Heaviside, the time-dependent magnetic moment $\vec{m}_{(t)}$ is adequately described by equation 2.4.3, but not according to the newly formulated "Maxwell equations" from the elaboration "The reinterpretation of the 'Maxwell Equations'[1]". Equation 2.4.3 therefore serves as the basis for this chapter. The term $-\frac{\delta \vec{B}}{\delta t}$ in particular will undergo a mathematical and physical reformulation.

$$m_{(t)} = \left( -\frac{\delta \vec{B}}{\delta t} \cdot \vec{A} \right) R \cdot \vec{A}$$  \hspace{1cm} (2.4.3)

First, the term $-\frac{\delta \vec{B}}{\delta t}$ is isolated from equation 2.4.3 and Heaviside's induction law is derived from it. This is shown by equation 2.4.4.

$$\text{rot} \vec{E} = -\frac{\delta \vec{B}}{\delta t}$$  \hspace{1cm} (2.4.4)

Since $-\frac{\delta \vec{B}}{\delta t}$ represents a vector in equation 2.4.4, it can also be represented in its component notation. This is represented by the formulation from Equation 2.4.5.
\[
\text{rot } \vec{E} = -\begin{vmatrix}
\frac{\delta B_x}{\delta t} \\
\frac{\delta B_y}{\delta t} \\
\frac{\delta B_z}{\delta t}
\end{vmatrix}
\] (2.4.5)

In the next step, the individual components \( \frac{\delta B_x}{\delta t} \), \( \frac{\delta B_y}{\delta t} \) and \( \frac{\delta B_z}{\delta t} \) from equation 2.4.5 are each added twice to the value 0. This is shown by equation 2.4.6.

\[
\text{rot } \vec{E} = -\begin{vmatrix}
\frac{\delta B_x}{\delta t} + 0 + 0 \\
0 + \frac{\delta B_y}{\delta t} + 0 \\
0 + 0 + \frac{\delta B_z}{\delta t}
\end{vmatrix}
\] (2.4.6)

If the individual terms from equation 2.4.6 are now multiplied by the value 1, equation 2.4.7 results. The value 1 is equated here with the expressions \( \frac{\delta x}{\delta x} \), \( \frac{\delta y}{\delta y} \) and \( \frac{\delta z}{\delta z} \).

\[
\text{rot } \vec{E} = -\begin{vmatrix}
\frac{\delta B_x}{\delta t} \cdot \frac{\delta x}{\delta x} + 0 \cdot \frac{\delta y}{\delta y} + 0 \cdot \frac{\delta z}{\delta z} \\
0 \cdot \frac{\delta x}{\delta x} + \frac{\delta B_y}{\delta t} \cdot \frac{\delta y}{\delta y} + 0 \cdot \frac{\delta z}{\delta z} \\
0 \cdot \frac{\delta x}{\delta x} + 0 \cdot \frac{\delta y}{\delta y} + \frac{\delta B_z}{\delta t} \cdot \frac{\delta z}{\delta z}
\end{vmatrix}
\] (2.4.7)

If the expression from equation 2.4.8 is now applied to equation 2.4.7, equation 2.4.9 is created.

\[
0 = \frac{\delta B_x}{\delta t} = \frac{\delta B_y}{\delta t} = \frac{\delta B_z}{\delta t}
\] (2.4.8)
\[
\text{rot } \vec{E} = - \left( \begin{array}{c} \frac{\delta B_x}{\delta t} \cdot \frac{\delta x}{\delta x} + \frac{\delta B_z}{\delta t} \cdot \frac{\delta y}{\delta y} + \frac{\delta B_x}{\delta t} \cdot \frac{\delta z}{\delta z} \\ \frac{\delta B_y}{\delta t} \cdot \frac{\delta x}{\delta x} + \frac{\delta B_z}{\delta t} \cdot \frac{\delta y}{\delta y} + \frac{\delta B_y}{\delta t} \cdot \frac{\delta z}{\delta z} \\ \frac{\delta B_z}{\delta t} \cdot \frac{\delta x}{\delta x} + \frac{\delta B_y}{\delta t} \cdot \frac{\delta y}{\delta y} + \frac{\delta B_z}{\delta t} \cdot \frac{\delta z}{\delta z} \end{array} \right) \quad (2.4.9)
\]

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In the next step, the velocity \( \vec{v} \) is solved from equation 2.4.9 and equation 2.4.10 is created. The velocity vector \( \vec{v} \) can also be expressed as \( \frac{\delta \vec{s}}{\delta t} \) and therefore also as \( \left| \begin{array}{c} \frac{\delta x}{\delta t} \\ \frac{\delta y}{\delta t} \\ \frac{\delta z}{\delta t} \end{array} \right| \).

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\[
\text{rot } \vec{E} = - \left( \begin{array}{c} \frac{\delta B_x}{\delta t} \cdot \frac{\delta x}{\delta x} + \frac{\delta B_z}{\delta t} \cdot \frac{\delta y}{\delta y} + \frac{\delta B_x}{\delta t} \cdot \frac{\delta z}{\delta z} \\ \frac{\delta B_y}{\delta t} \cdot \frac{\delta x}{\delta x} + \frac{\delta B_z}{\delta t} \cdot \frac{\delta y}{\delta y} + \frac{\delta B_y}{\delta t} \cdot \frac{\delta z}{\delta z} \\ \frac{\delta B_z}{\delta t} \cdot \frac{\delta x}{\delta x} + \frac{\delta B_y}{\delta t} \cdot \frac{\delta y}{\delta y} + \frac{\delta B_z}{\delta t} \cdot \frac{\delta z}{\delta z} \end{array} \right) \quad (2.4.10)
\]

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In a final step, the velocity vector \( \vec{v} \) in equation 2.4.10 is decoupled from the overall vector. This is shown in equation 2.4.11.

\[
\text{rot } \vec{E} = - \left( \begin{array}{c} \frac{\delta B_x}{\delta x} \frac{\delta B_y}{\delta y} \frac{\delta B_z}{\delta z} \\ \frac{\delta B_y}{\delta x} \frac{\delta B_z}{\delta y} \frac{\delta B_x}{\delta z} \\ \frac{\delta B_z}{\delta x} \frac{\delta B_x}{\delta y} \frac{\delta B_y}{\delta z} \end{array} \right) \left| \begin{array}{c} \frac{\delta x}{\delta t} \\ \frac{\delta y}{\delta t} \\ \frac{\delta z}{\delta t} \end{array} \right| \quad (2.4.11)
\]

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The velocity vector \( \vec{v} \) and the gradient of the magnetic flux density \( (\text{grad } \vec{B}) \) are created in equation 2.4.11. The simplified notation is shown in equation 2.4.12.

\[
\text{rot } \vec{E} = -(\text{grad } \vec{B}) \vec{v} \quad (2.4.12)
\]
Equation 2.4.12 describes the unsimplified form of Heaviside's induction law. If this formulation is now compared with equation 2.2.4, it is noticeable that equation 2.4.12 is mathematically incomplete.

\[
\text{rot} \vec{E} = -(\text{grad} \vec{B}) \vec{v} \tag{2.4.12}
\]

\[
\text{rot} \vec{E} = (\text{grad} \vec{v}) \vec{B} - (\text{grad} \vec{B}) \vec{v} + \vec{v} \text{ div} \vec{B} - \vec{B} \text{ div} \vec{v} \tag{2.2.4}
\]

Apparently, three of the five terms in equation 2.2.4 must be interpreted with the value 0 in order to fulfill the requirements from equation 2.4.12, Heaviside's induction law. Due to the mathematical formulation from equations 2.2.7 and 2.2.8, it must be stated at this point that it is not mathematically possible to interpret these three terms with the value 0. At least three terms from equation 2.2.4 must therefore have a value that is not equal to 0 if \( \text{rot} \vec{E} \) is to deliver a value that is not equal to 0.

\[
(S_p)(\text{grad} \vec{B}) = \text{div} \vec{B} \tag{2.2.7}
\]

\[
(S_p)(\text{grad} \vec{v}) = \text{div} \vec{v} \tag{2.2.8}
\]

If equations 2.2.7 and 2.2.8 are considered, it must be noted that two terms in equation 2.2.4 are connected to each other. On the one hand the term \( (\text{grad} \vec{B}) \vec{v} \) with the term \( \vec{v} \text{ div} \vec{B} \) and on the other hand the term \( (\text{grad} \vec{v}) \vec{B} \) with the term \( \vec{B} \text{ div} \vec{v} \). The second pair of terms around the velocity gradient \( (\text{grad} \vec{v}) \) describes a formulation for the change in spatial content, for example material deformation. The first pair of terms around the gradient of the magnetic flux density \( (\text{grad} \vec{B}) \), on the other hand, describes, for example, a distortion or density states in the magnetic flux density \( \vec{B} \).

If the volume is not subject to such influences, for example there is no material deformation in possible tests, the influence of the velocity gradient \( (\text{grad} \vec{v}) \) and the velocity divergence \( \text{div} \vec{v} \) can be assumed to be 0. This results in equation 2.4.13. However, it must be expressly pointed out at this point that these two terms must not generally be assumed to have the value 0.

\[
\text{rot} \vec{E} = 0 - (\text{grad} \vec{B}) \vec{v} + \vec{v} \text{ div} \vec{B} - 0 \tag{2.4.13}
\]
\[ (Sp)(\text{grad}\ \vec{B}) = \text{div}\ \vec{B} \quad (2.2.7) \]

The two remaining terms, i.e. \((\text{grad}\ \vec{B})\vec{v}\) and \(\vec{v} \quad \text{div}\ \vec{B}\), are directly connected to each other by the mathematical requirement from equation 2.2.7. This was sufficiently explained in the paper "The reinterpretation of the 'Maxwell equations'[1]". The elements of the trace \((Sp)\) of the gradient of the magnetic flux density \((\text{grad}\ \vec{B})\) are the elements that form the basis for the expression \(-\frac{\delta \vec{B}}{\delta t}\) and, according to equation 2.2.7, also describe the divergence of the magnetic flux density \(\text{div}\ \vec{B}\). To illustrate this, the term \((\text{grad}\ \vec{B})\vec{v}\) from equation 2.4.13 is presented in column notation. This is shown in equation 2.4.14.

\[
\begin{align*}
\text{rot}\ \vec{E} &= 0 - \begin{vmatrix}
\frac{\delta B_x}{\delta x} & \frac{\delta B_y}{\delta y} & \frac{\delta B_z}{\delta z} \\
\frac{\delta B_x}{\delta x} & \frac{\delta B_y}{\delta y} & \frac{\delta B_z}{\delta z} \\
\frac{\delta B_x}{\delta x} & \frac{\delta B_y}{\delta y} & \frac{\delta B_z}{\delta z}
\end{vmatrix}
\begin{vmatrix}
\frac{\delta x}{\delta t} \\
\frac{\delta y}{\delta t} \\
\frac{\delta z}{\delta t}
\end{vmatrix} + \vec{v} \quad \text{div}\ \vec{B} - 0 \\
&= (2.4.14)
\end{align*}
\]

The components marked in Equation 2.4.14, i.e. \(\frac{\delta B_x}{\delta x}\), \(\frac{\delta B_y}{\delta y}\) and \(\frac{\delta B_z}{\delta z}\), when added together, form the trace \((Sp)\) of the gradient of the magnetic flux density \((\text{grad}\ \vec{B})\) and thus also its field density \(\text{div}\ \vec{B}\). Looking back at Equation 2.4.11, these are also the components that in Heaviside's induction law, define the value \(-\frac{\delta \vec{B}}{\delta t}\). This specifically means that if the trace of the gradient of the magnetic flux density \((Sp)(\text{grad}\ \vec{B})\) has a value that is not equal to 0, then mathematically the divergence of the magnetic flux density \(\text{div}\ \vec{B}\) must also have a value that is not equal to 0.

\[
\begin{align*}
\text{rot}\ \vec{E} &= - \begin{vmatrix}
\frac{\delta B_x}{\delta x} & \frac{\delta B_y}{\delta y} & \frac{\delta B_z}{\delta z} \\
\frac{\delta B_x}{\delta x} & \frac{\delta B_y}{\delta y} & \frac{\delta B_z}{\delta z} \\
\frac{\delta B_x}{\delta x} & \frac{\delta B_y}{\delta y} & \frac{\delta B_z}{\delta z}
\end{vmatrix}
\begin{vmatrix}
\frac{\delta x}{\delta t} \\
\frac{\delta y}{\delta t} \\
\frac{\delta z}{\delta t}
\end{vmatrix} \\
&= (2.4.11)
\end{align*}
\]
If the term \( \vec{v} \ \text{div} \ \vec{B} \) is now represented in equation 2.4.14 in its column notation or component notation, this results in equation 2.4.15.

\[
\text{rot} \ \vec{E} = 0 - \begin{bmatrix}
\frac{\delta B_x}{\delta x} & \frac{\delta B_x}{\delta y} & \frac{\delta B_x}{\delta z} \\
\frac{\delta B_y}{\delta x} & \frac{\delta B_y}{\delta y} & \frac{\delta B_y}{\delta z} \\
\frac{\delta B_z}{\delta x} & \frac{\delta B_z}{\delta y} & \frac{\delta B_z}{\delta z}
\end{bmatrix} \begin{bmatrix}
\frac{\delta x}{\delta t} \\
\frac{\delta y}{\delta t} \\
\frac{\delta z}{\delta t}
\end{bmatrix} + \begin{bmatrix}
\frac{\delta x}{\delta t} \\
\frac{\delta y}{\delta t} \\
\frac{\delta z}{\delta t}
\end{bmatrix} \left( \frac{\delta B_x}{\delta x} + \frac{\delta B_y}{\delta y} + \frac{\delta B_z}{\delta z} \right) - 0 \quad (2.4.15)
\]

Since the divergence of the magnetic flux density \( \text{div} \ \vec{B} \) is a single numerical value consisting of an addition of the components \( \frac{\delta B_x}{\delta x} \), \( \frac{\delta B_y}{\delta y} \) and \( \frac{\delta B_z}{\delta z} \), it must be multiplied by each element of the velocity vector \( \vec{v} \). This circumstance is shown in equation 2.4.16.

\[
\text{rot} \ \vec{E} = 0 - \begin{bmatrix}
\frac{\delta B_x}{\delta x} & \frac{\delta B_x}{\delta y} & \frac{\delta B_x}{\delta z} \\
\frac{\delta B_y}{\delta x} & \frac{\delta B_y}{\delta y} & \frac{\delta B_y}{\delta z} \\
\frac{\delta B_z}{\delta x} & \frac{\delta B_z}{\delta y} & \frac{\delta B_z}{\delta z}
\end{bmatrix} \begin{bmatrix}
\frac{\delta x}{\delta t} \\
\frac{\delta y}{\delta t} \\
\frac{\delta z}{\delta t}
\end{bmatrix} + \begin{bmatrix}
\frac{\delta x}{\delta t} \\
\frac{\delta y}{\delta t} \\
\frac{\delta z}{\delta t}
\end{bmatrix} \left( \frac{\delta B_x}{\delta x} + \frac{\delta B_y}{\delta y} + \frac{\delta B_z}{\delta z} \right) - 0 \quad (2.4.16)
\]

If equation 2.4.16 is now considered under the assumption that the magnetic flux density \( \vec{B} \) is not subject to deformation, distortion or torsion, equation 2.4.16 can be simplified to equation 2.4.17. It must also be made clear at this point that this assumption cannot be made in principle, since there are definitely circumstances under which a deformation, distortion or torsion can arise in the magnetic flux density \( \vec{B} \).

\[
\text{rot} \ \vec{E} = 0 - \begin{bmatrix}
\frac{\delta B_x}{\delta x} & 0 & 0 \\
0 & \frac{\delta B_y}{\delta y} & 0 \\
0 & 0 & \frac{\delta B_z}{\delta z}
\end{bmatrix} \begin{bmatrix}
\frac{\delta x}{\delta t} \\
\frac{\delta y}{\delta t} \\
\frac{\delta z}{\delta t}
\end{bmatrix} + \begin{bmatrix}
\frac{\delta x}{\delta t} \\
\frac{\delta y}{\delta t} \\
\frac{\delta z}{\delta t}
\end{bmatrix} \left( \frac{\delta B_x}{\delta x} + \frac{\delta B_y}{\delta y} + \frac{\delta B_z}{\delta z} \right) - 0 \quad (2.4.17)
\]
If equation 2.4.17 now calculates the elements of the velocity vectors, i.e. \( \frac{\delta x}{\delta t} \), \( \frac{\delta y}{\delta t} \) and \( \frac{\delta z}{\delta t} \), with the elements of the magnetic flux density, i.e. \( \frac{\delta B_x}{\delta x} \), \( \frac{\delta B_y}{\delta y} \) and \( \frac{\delta B_z}{\delta z} \), mathematically correctly, equation 2.4.18 is created.

\[
\text{rot } \vec{E} = 0 - \begin{vmatrix}
\frac{\delta B_x}{\delta x} \frac{\delta x}{\delta t} + 0 \frac{\delta y}{\delta t} + 0 \frac{\delta z}{\delta t} & \frac{\delta B_x}{\delta x} \frac{\delta x}{\delta t} + \frac{\delta B_y}{\delta y} \frac{\delta x}{\delta t} + \frac{\delta B_z}{\delta z} \frac{\delta x}{\delta t} \\
0 + \frac{\delta B_x}{\delta y} \frac{\delta y}{\delta t} + 0 \frac{\delta z}{\delta t} & 0 + \frac{\delta B_y}{\delta y} \frac{\delta y}{\delta t} + \frac{\delta B_z}{\delta z} \frac{\delta y}{\delta t} \\
0 + 0 + \frac{\delta B_x}{\delta z} \frac{\delta z}{\delta t} & 0 + 0 + \frac{\delta B_y}{\delta z} \frac{\delta z}{\delta t} + \frac{\delta B_z}{\delta z} \frac{\delta z}{\delta t}
\end{vmatrix}
\]

If equation 2.4.18 now assumes that there are no spatial distortions, deformations or torsions, only the expressions that contain \( \frac{\delta x}{\delta x} \), \( \frac{\delta y}{\delta y} \) and \( \frac{\delta z}{\delta z} \) remain. This circumstance is shown in equation 2.4.19. Here, too, it must be made clear that this assumption cannot be made in principle, as there are circumstances under which spatial deformation, distortion or torsion can be assumed.

\[
\text{rot } \vec{E} = 0 - \begin{vmatrix}
\frac{\delta B_x}{\delta x} \frac{\delta x}{\delta t} + 0 + 0 & \frac{\delta B_x}{\delta x} \frac{\delta x}{\delta t} + \frac{\delta B_y}{\delta y} \frac{\delta x}{\delta t} + \frac{\delta B_z}{\delta z} \frac{\delta x}{\delta t} \\
0 + \frac{\delta B_x}{\delta y} \frac{\delta y}{\delta t} + 0 + 0 & 0 + \frac{\delta B_y}{\delta y} \frac{\delta y}{\delta t} + \frac{\delta B_z}{\delta z} \frac{\delta y}{\delta t} \\
0 + 0 + \frac{\delta B_x}{\delta z} \frac{\delta z}{\delta t} & 0 + 0 + \frac{\delta B_y}{\delta z} \frac{\delta z}{\delta t} + \frac{\delta B_z}{\delta z} \frac{\delta z}{\delta t}
\end{vmatrix}
\]

If equation 2.4.19 is further simplified, equation 2.4.20 is created.

\[
\text{rot } \vec{E} = 0 - \begin{vmatrix}
\frac{\delta B_x}{\delta x} \frac{\delta x}{\delta t} & \frac{\delta B_x}{\delta x} \frac{\delta x}{\delta t} \\
\frac{\delta B_y}{\delta y} \frac{\delta y}{\delta t} & \frac{\delta B_y}{\delta y} \frac{\delta y}{\delta t} \\
\frac{\delta B_z}{\delta z} \frac{\delta z}{\delta t} & \frac{\delta B_z}{\delta z} \frac{\delta z}{\delta t}
\end{vmatrix}
\]

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If the elements $\frac{\delta x}{\delta x}$, $\frac{\delta y}{\delta y}$ and $\frac{\delta z}{\delta z}$ are shortened in equation 2.4.20, this results in equation 2.4.21.

$$\text{rot} \vec{E} = 0 - \begin{vmatrix} \frac{\delta B_x}{\delta t} \\ \frac{\delta B_y}{\delta t} \\ \frac{\delta B_z}{\delta t} \end{vmatrix} + \begin{vmatrix} \frac{\delta B_x}{\delta t} \\ \frac{\delta B_y}{\delta t} \\ \frac{\delta B_z}{\delta t} \end{vmatrix} - 0 \quad (2.4.21)$$

Further simplifying equation 2.4.21 results in equation 2.4.22.

$$\text{rot} \vec{E} = -2 \cdot \begin{vmatrix} \frac{\delta B_x}{\delta t} \\ \frac{\delta B_y}{\delta t} \\ \frac{\delta B_z}{\delta t} \end{vmatrix} \quad (2.4.22)$$

In the last step, the column notation of the vector is transferred to the arrow notation. This results in equation 2.4.23.

$$\text{rot} \vec{E} = -2 \cdot \frac{\delta \vec{B}}{\delta t} \quad (2.4.23)$$

$$\text{rot} \vec{E} = -\frac{\delta \vec{B}}{\delta t} \quad (2.4.4)$$

A comparison of the result from equation 2.4.23 with the result from equation 2.4.4, which represents Heaviside's induction law, shows that under the stated conditions of a distortion-free magnetic flux density and a distortion-free volume, that same induction law increases by a factor of 2. It needs to be expanded if the assumptions made in deriving equation 2.4.23 hold. That is why the formulation for the time-dependent magnetic moment $\vec{m}_t$ must now be expanded by this factor; it must be reformulated.
2.5 THE REFORMULATION OF THE MAGNETIC MOMENT

The fact explained in Chapter 2.4 means that the formulation for the time-dependent magnetic moment $\vec{m}(t)$ from equation 2.4.3 is also influenced.

\[
\vec{m}(t) = \left( -\frac{\delta B}{\delta t} \cdot \vec{A} \right) \cdot \vec{A}
\]  \hspace{1cm} (2.4.3)

In equation 2.4.3, the formulation from equation 2.4.4 can now be replaced by the formulation from equation 2.4.23. This turns equation 2.4.3 into equation 2.5.1.

\[
\vec{m}(t) = \left( -2 \cdot \frac{d\vec{B}}{dt} \cdot \vec{A} \right) \cdot \vec{A}
\]  \hspace{1cm} (2.5.1)

This also results in an adjustment for equation 2.3.1. This is shown in equation 2.5.2.

\[
\vec{m}(t) = i(t) \cdot \vec{A}
\]  \hspace{1cm} (2.3.1)

\[
\vec{m}(t) = 2 \cdot i(t) \cdot \vec{A}
\]  \hspace{1cm} (2.5.2)

The comparison between equation 2.3.1 and equation 2.5.2 shows why there is a difference of a factor of 2 between the measured value and the calculation for the time-dependent magnetic moment in the experiments described in chapter 2.1.

3. Discussion

1. Apart from the situation presented in this paper, are there other possibilities of calculation errors in the Einstein de Haas experiment with regard to factor 2 in equation 2.4.23?

2. What impact does the situation presented in this paper have on the Landé G factor?

3. What effects does the facts presented in this paper have on the gyromagnetic factor $g$?
4. What effects does the facts presented in this paper have on the physical area of quantum mechanics? Theories regarding spin and intrinsic angular momentum of the electron may be affected.

5. What effects does the facts presented in this paper have on the physical subfield of electrodynamics? The "Maxwell equations" and the Lorenz force are affected here.

6. What effects does the facts presented in this paper have on the physical subfield of solid state physics? Ferromagnetism and superconductivity can be affected.

7. Are there other areas of physics that are influenced by the facts presented in this paper and if so, which ones and how?

4. CONCLUSION

Under the mathematical requirement from equation 2.2.7, based on the magnetic flux density $\vec{B}$, to be exact $(\text{Sp})(\text{grad} \vec{B}) = \text{div}(\vec{B})$, the physical requirement based on the assumption that the divergence of the magnetic flux density $\vec{B}$ is fundamentally assigned the value 0 ($\text{div}(\vec{B}) = 0$) is only valid under the assumption, that the value of the trace of the magnetic flux density gradient is also 0 ($\text{Sp}(\text{grad} \vec{B}) = 0$). However, since the trace of the gradient of the magnetic flux density $\text{Sp}(\text{grad} \vec{B})$ and the divergence of the magnetic flux density $\text{div}(\vec{B})$ contain the elements that, in combination with the velocity vector $\vec{v}$, describe the expression $-\frac{\delta \vec{B}}{\delta t}$, to be exact $\frac{\delta B_x}{\delta x}$, $\frac{\delta B_y}{\delta y}$, and $\frac{\delta B_z}{\delta z}$, these two expressions are mathematically inseparable from each other. This leads to either the physical concept of the magnetic field having to be reinterpreted or the assumption that the divergence of the magnetic flux density basically has the value 0 ($\text{div}(\vec{B}) = 0$) is wrong. This was sufficiently explained in the paper "The reinterpretation of the 'Maxwell equations'[1]". For the Einstein de Haas experiment, the consequence is that equation 2.3.1 for the time-dependent magnetic moment $\vec{m}_{(t)}$ must be expanded by a factor of 2. This results in a new equation for the time-dependent magnetic moment $\vec{m}_{(t)}$, to be exact equation 2.5.2. Other areas
of physics are also affected, including quantum mechanics. The task now is to identify these sub-areas and then correct them based on the facts presented here.

Due to the discrepancy between the measured value and the calculated value, the Einstein de Haas experiment can also be assumed to be experimental evidence that the facts from the elaboration "The reinterpretation of the 'Maxwell equations'[1]" are correct.

5. CONFLICTS OF INTEREST

The author(s) declare that there is no conflict of interest regarding the publication of this article.

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