Detection of a possible limit leading to a violation causality for low-mass particles at very high energies.

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I. SUMMARY

This study examines the implications of Planck scales on the causality of low-mass particles at very high energies. Using a fractal approach to space-time, we propose new dynamics for the fabric of space-time and its effect on special relativity. Our results suggest a convergence of physical values at the Planck scale and possible implications for quantum gravity and theories of everything.

Key words: Planck scales, causality, low-mass particles, special relativity, quantum gravity.

II. INTRODUCTION

As part of the quest for a unified theory of quantum gravitation, the examination of the fundamental properties of space-time on the Planck scale has gained crucial importance. Planck time (tp) and Planck length (lp) considered as the minimum scales of measurement, provide a discrete and invariant framework for examining the structure of space-time. This approach postulates the existence of elementary temporal intervals tp and spatial lp where e_i represent the basis vectors in a four-dimensional vector metric.

Efforts to reconcile the two main branches of physics, namely quantum physics and general relativity, have mainly taken two paths: the quantization of gravity, i.e., the attempt to apply the principles of quantum physics to gravity, and the geometrization of quantum physics, i.e., the attempt to formulate quantum physics in a geometric framework similar to general relativity. These approaches tend to explore the properties of one theory through the prism of the other without achieving total unification. In other words, a gap remains between the foundations of the two theories, making it difficult to create a unified framework that integrates them coherently.

III. METHODS

We explore an iterative, self-similar approach to Planck-scale spacetime, using fundamental axioms and assumptions. The fundamental spacetime metric and zero-point energy Ω are considered as a single constituent element in a Planck space-2 :

$$lp^2=\frac{\hbar G}{c^3} \quad \textbf{(1)}$$

IV. RESULTS

Analysis of the properties on the Planck scale revealed a number of significant results:

1. **Self-similarity and Fractals :** The expression of the fundamental physical constants $\hbar$, G, and c in an iterative framework shows that the sequence S_n tends asymptotically towards zero as n increases. This sequence, characterised by fractal self-similarity, is bounded and compact, ensuring convergence to a limit value.

2. **Vacuum permittivity on the Planck scale :** By applying a fractal approach to vacuum permittivity ε_0 the results show that for $n=0$ the fine structure constant α is found in a theoretical framework. For $n=1$ a combination of the electromagnetic coupling constant and the reduced gravitational coupling factor is obtained, which could have implications for quantum gravity theories.

3. **Anomaly of the Cone of Light** : This requalification of the Planck length, from a simple limit of space-time to a granular fractal structure, leads to the observation of an anomaly in very high-energy low-mass particles. A hypothesis is then put forward suggesting the existence of a second fermionic causality limit, distinct from the constant c, which could explain this anomaly.

4. **Cosmological implications** : The results show that the properties of the vacuum at the Planck scale could offer new explanations for cosmic inflation and the Big Bang singularity, as well as implications for the nature of dark matter and dark energy. A fractal approach to lp as a compact object with iterative properties could revolutionise our understanding of space-time at these scales.

These results show that the study of Planck-scale properties is not only consistent with known physics, but also opens up promising avenues for future research into quantum gravity and theories of everything.

V. AXIOMS and FUNDAMENTAL ASSUMPTION

The axiom put forward here, potentially at odds with current research trends, postulates that the fundamental metric of space-time on the one hand and the zero-point energy Ω on the other, form one and the same constituent element, unified at the base in a Planck space-2 $lp^2=\frac{\hbar G}{c^3}$

on which an iterative process such as : $S_0 : \hbar G \frac{1}{c^3}$ **(2)**

as $c^{-3}=\frac{tp^3}{lp^3} \Rightarrow c^{-3}=tp^3\frac{1}{lp}\frac{1}{lp^2}$ and thus $S_0 : (\hbar G)\, tp^3\frac{1}{lp}\frac{1}{lp^2}$

if we assume that $S_1 : (\hbar G)\, tp^3\frac{1}{lp}$ then $S_0 : S_1\frac{1}{lp^2}$

S_1 becomes $S_1 : (\hbar G)\, tp^2\frac{1}{c}$ with $tp^2=\frac{\hbar G}{c^5}$

S_1 is simplified, which implies that $S_1 : (\hbar G)^2\frac{1}{c^5}\frac{1}{c}$ then

$$S_1 : (\hbar G)^2\frac{1}{c^6} \quad \textbf{(3)}$$

Following the same process, we find :

$$S_2 : (\hbar G)^3 \frac{1}{c^9} \quad \textbf{(4)}$$

$$S_3 : (\hbar G)^4 \frac{1}{c^{12}} \quad \textbf{(5)}$$

Generalising, with

$$S_n : \left(\frac{\hbar G}{c^3}\right)^{n+1} \quad \textbf{(6)}$$ which tends to 0 when $n \to +\infty$.

We are therefore in the presence of a geometric process that is recursive (since we take as its basis lp^2) and exhibits self-similarity $\frac{\hbar G}{c^3}$, and therefore have the characteristics of Self-Similar fractal (IFS).

Examination of this expression reveals that the sequence $\{S_n\}$ tends asymptotically towards zero as the index n increases indefinitely, signalling convergence of the sequence. This behaviour results from the specific values of the physical constants $\hbar, G, et\, c$, where the ratio $\frac{\hbar G}{c^3}$ represents an extremely small positive number in the physical context. As a result, the sequence is $\{S_n\}$ bounded and, moreover, closed because it contains its own limit point, namely zero.

By virtue of these properties, $\{S_n\}$ is therefore compact.

More specifically, if we subject the permittivity of the vacuum to this fractality, such as :

$$\varepsilon_0 = \left(\frac{e^2 c}{4\pi\alpha\hbar}\right)\frac{1}{c^2} \text{ with } \mu_0 = \frac{4\pi\alpha\hbar}{e^2 c}$$

by replacing c^2 by $\frac{lp^2}{tp^2}$

$$\varepsilon_0 = \left(\frac{e^2 c}{4\pi\alpha\hbar}\right)\frac{tp^2}{lp^2} \Rightarrow \varepsilon_0 = \left(\frac{e^2 c}{4\pi\alpha\hbar^2 G}\right) tp^2 c^3$$

we obtain $\varepsilon_0 = \left(\frac{e^2 c^2}{4\pi\alpha\hbar^2 G}\right) lp^2$

or as $S_0 : lp^2 = \frac{\hbar G}{c^3}$ or $S_n : \left(\frac{\hbar G}{c^3}\right)^{n+1}$

$$\Rightarrow \varepsilon_n = \left(\frac{e^2 c^2}{4\pi\alpha\hbar^2 G}\right)\left(\frac{\hbar G}{c^3}\right)^{n+1}$$

$$\Rightarrow \varepsilon_n = \left(\frac{e^2}{4\pi\alpha}\right)\left(\frac{\hbar^{n-1} G^n}{c^{3n+1}}\right) \quad \textbf{(7)}$$

- Thus at iteration $n=0$ we find the fine structure constant posed in a theoretical framework $\alpha=\left(\frac{e^2}{4\pi\varepsilon_n}\right)\left(\frac{1}{\hbar c}\right)$. **(8)**

- For $n=1$ we obtain $\alpha_{k_e k_G}=\left(\frac{e^2}{4\pi\varepsilon_n}\right)\left(\frac{G}{c^4}\right)$ **(9)**

  which is the product of an electromagnetic coupling constant $\frac{e^2}{4\pi\varepsilon_0}$ multiplied by the gravitational coupling factor $\frac{G}{c^4}$ reduced.

This combination may be particularly relevant in theories that attempt to unify fundamental forces, such as quantum gravity or theories of everything. It could give clues as to how electromagnetic energy scales compare to energy scales where gravity becomes significant, especially when looking at extreme astrophysical phenomena or conditions in the primordial universe.

- Finally, when $n \to +\infty$: $\varepsilon_n=\left(\frac{e^2}{4\pi\alpha}\right)\left(\frac{\hbar^{n-1}G^n}{c^{3n+1}}\right)$ tends to zero, the fine structure constant can be defined as an emergent value on the Planck scale.

A . Discussion

1) ***Quantum gravity**:*

If the Planck scale is no longer a limit but a gateway to new physics, this could imply new dynamics for the fabric of space-time itself. This could affect the formulation of quantum gravity, because the quantum fluctuations of the vacuum and the fundamental interactions at these scales could be radically different from what is envisaged in current models.

2) ***String theory and theories of everything**:*

These theories, which attempt to reconcile gravity with quantum mechanics, could be directly affected. The extra dimensions of string theory, for example, could interact in unexpected ways with these new properties of the vacuum.

B. Implications for Cosmology

4) ***Big Bang and Inflation**:*

The extreme conditions at the beginning of the universe could be better understood if the properties of the vacuum could vary beyond the Planck scale. This could offer new explanations or mechanisms for cosmic inflation or the Big Bang singularity.

5) ***Dark Matter and Dark Energy**:*

If the modified Planck scale affects the structure of the vacuum, it could also affect theories on the nature of dark matter and dark energy, which are deeply linked to the structure of space-time.

To conclude this chapter, we will therefore pose lp and tp no longer as a limit on the metric of space-time, but rather as a compact fractal object with intrinsic iterative properties.

This approach could be interpreted as an extension of scale relativity, where it could open a door to the sub-Planckian scale.[1]

VI. QUANTIFICATION and PHYSICAL IMPLICATIONS

- The fractal object defined by s_n will be considered unbreakable if any attempt to subdivide it into smaller parts (in particular a subdivision involving $\left(\frac{l_p^2}{k}\right)$ with $(k \in N)$) destroys the fundamental fractal properties of (S_n). Since lp^2 is the smallest definable metric value, the operation is impossible. This can be interpreted as a restriction preventing any subdivision from maintaining the fundamental property of the system.

Consequently, lp^2 is therefore unitary and non-divisible. *(condition 1)*

- In order to reconcile this perspective with conventional physics, and to re-establish a continuous description of space-time compatible with the foundations of differential geometry, the sum $\sum lp^2$ must be envisaged with characteristics of **overlap, dissociation and connection** that allow the transition to a structure of continuous varieties on a larger scale. These properties must ensure coherence with adjacent space-time elements at the Planck scale, while respecting the constraints imposed by quantization at this scale.

- Similarly, if lp^2 can be considered as a fundamental compact object, any measure of magnitude, expressed **as $k \cdot lp^2 > lp^2$ with $k \in \mathbb{R}$ will have to be reduced to the k fundamental structural element of dimension** lp^2.

This implies that the Planck-scale structure, although discrete, behaves like a variety in terms of its metric and topological properties on a larger scale, making it possible to integrate the concepts of quantization with the geometric continuity required by classical differential geometry.

This proposal suggests that space, defined by lp^2 at this scale would have tiling properties, meaning that two lengths lp^2 and $lp^{2\prime}$ could, by sliding, overlap locally and thus increase the energy level by accumulation. (2)

In this context, the idea of "overlap" can be interpreted as the phenomenon whereby the physical properties or phenomena observed in one region of space-time, defined by a Planck length, are identical or consistent with those in another adjacent region of similar size.

This implies a certain uniformity or homogeneity of the properties of space-time on the scale of the Planck length, thus calling into question the uncertainty principle. We shall see that this is not the case.

This property of overlap or "tiling" could define a limit of curvature for space-time, making it possible to specify a state of maximum overlap tending towards lp the **sum of the unified energies of which would correspond to the Planck energy** E_{Planck}. *(condition 2)*

This postulate introduces an invariant tensor of order 2, $\Lambda^i_j = lp\, e_i$ which is consistent with our axiom.

Let Δt be any time interval and a speed $\vec{\varepsilon}$ defined by : $\vec{\varepsilon}\,\Delta t = lp\vec{e}$ we want to study the variation of $\vec{\varepsilon}$ as a function of Δt to begin with.

In this context, the value $\Delta t = 1$ appears to be an increased limit, because for all $\Delta t > 1$, this would result in a value less than $lp . s^{-1}$ which, under the conditions set out above, is impossible.

On the other hand, $\Delta t = tp$ is a minor limit since, by definition, $c = \frac{lp}{tp}$.

If we now propose to define the norm of $\vec{\varepsilon}$ as a function of lp on the major boundary $\Delta t = 1$ given that lp is of granular type, an integer summation n is necessary, such that :

$$\vec{\varepsilon} . \Delta t = \sum_n n lp\, \vec{e}$$

With $ct = \frac{\Delta t}{tp} \cdot lp$ we are interested in the boundary of $n = \frac{\Delta t}{tp}$.

If we round off to the nearest inverse of the Planck time : $\lfloor \frac{1}{tp} \rfloor$

a quick dimensional analysis reveals that $tp * \lfloor \frac{1}{tp} \rfloor \approx 1$, *which* implies a relative deviation $\sigma_{er} = 1 - \lfloor \frac{1}{tp} \rfloor \frac{lp}{c}$ very close to zero. Consequently, we can postulate that $n = \frac{\Delta t}{tp}$ is comparable to the set of natural integers $\mathbb{N}$ when $\Delta t \to 1s$.

This particular case $n = \frac{\Delta t}{tp}$ raises the possibility of "quantifying" the speed c by $n.lp$ since :

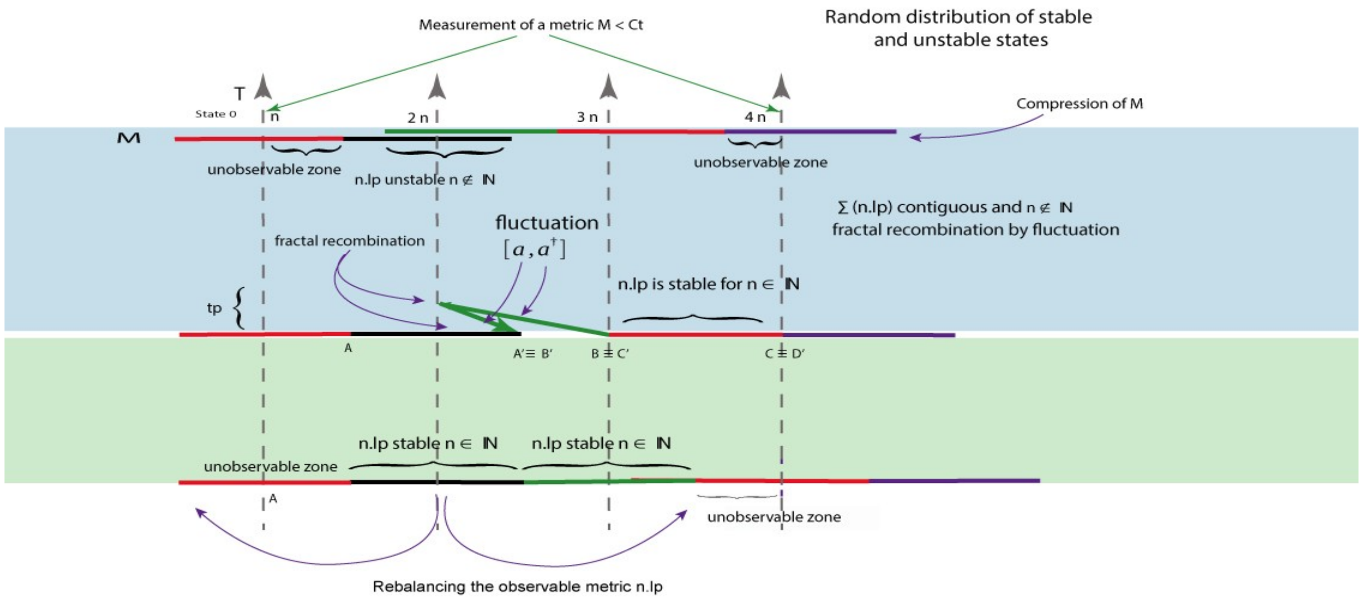
$c.\Delta t = n\,lp$ **(10)**

Thus the length ct can be defined as n elements lp contiguous

Conversely, when Δt deviates from 1 and tends towards the minor limit tp the relative deviation tends towards 1.

n can therefore no longer be integrable to the set of natural integers and any metric defined by $n.lp$ **leads to fractional values of lp which by definition is impossible.** *(condition 1)*

We therefore propose a mechanism for tiling space-time at the Planck scale in order to satisfy *condition 1*, ***i.e., an unstable equilibrium position where it could, transiently and completely, be contained within the metric observed by overlap. This model thus proposes a chaotic small-scale space-time with large-scale smoothing consistent with the continuum principle.***



- If the metric is an open, i.e. an edgeless subspace X in E, the measurement performed by an observer to satisfy the above condition must randomly lead to the emergence of creation and annihilation operators, satisfying the commutation relations.

- Assume lp^2 as the unit tile. Given a closed metric with a perimeter X <ct, where $n \cdot lp$ is always less than X and lp is indivisible, this tiling process transitions from being transient to becoming permanent. This permanent tiling process induces a local energy mass, represented by a tensor, which generates a secondary tiling effect on the contiguous space, analogous to gravity.

For a given metric, if its maximum overlap state corresponds to the Planck energy, then, when the overlap state is minimal, this energy, which must be conserved, is distributed over the entire length $C \cdot \Delta t$. It is then possible to calculate the value of the fundamental energy on the Planck scale by summing the constituent units.

Thus, if we assume $\frac{E_{Planck}}{n}$ *the* ***minimum energy value in the ground state is the reduced Planck constant :***

$$E_{Planck} \cdot \left(\frac{\Delta t}{tp}\right)^{-1} = \Delta t^{-1} \sqrt{\frac{c^5 \hbar}{G} \cdot \frac{\hbar G}{c^5}} \Rightarrow E_{Planck} \cdot \left(\frac{\Delta t}{tp}\right)^{-1} = \frac{\hbar}{\Delta t}$$

Simplifying by Δt under the boundary conditions :

$$E_{Planck} \cdot (tp) = \hbar \quad \textbf{(11)}$$

With $\Delta E < E_{Planck}$ and $\Delta t > tp$ for $\forall\, n < \frac{\Delta t}{tp}$ the two integers framing n play the role of standard deviation, which brings us back to Heisenberg's principle : $\Delta E \cdot \Delta t \geqslant \frac{\hbar}{2}$

Finally, to conclude on a fundamental note, this quantitative approach leads, when n tends towards 1, to a speed limit extremely close to c:

$$V_{limit} = c - n.\varepsilon \text{ with } lp = \varepsilon \Delta t \quad \textbf{(12)}$$

VII. ANGULAR ANOMALY in the LIGHT CONE LOW-MASS PARTICLES at VERY HIGH ENERGIES

In this work, we do not address the problem of renormalization of particle braking as defined by quantum electrodynamics (QED). Renormalization, as a crucial technique in QED, allows for the treatment of infinite divergences that appear in the calculations of interactions between charged particles. Although this method is fundamental for a deep understanding of radiative braking phenomena and associated corrections, our study focuses on other specific aspects of particle physics. Therefore, we deliberately omit the application of renormalization concepts in the context of our current analyses. The primary objective of our research is to explore the possible breaking of the equivalence principle at speeds approaching c. This approach allows us to simplify our model without the additional complications introduced by renormalization.

Replacing V_{limit} in the angular parameter of the Lorentz factor with $c - n.\varepsilon$ gives us

$$\frac{c - n\varepsilon}{c} = \tanh(\theta)$$

using trigonometric transformations, we obtain the logarithmic expression for θ:

$$\theta=\frac{1}{2}\ln\left(\frac{1+\frac{c-n\varepsilon}{c}}{1-\frac{c-n\varepsilon}{c}}\right)$$ which implies, after simplification $$\theta=\frac{1}{2}\ln\left(\frac{2C}{n\varepsilon}-1\right)$$

or as $\frac{2c}{n\varepsilon}\gg 1$ we obtain

$$\theta\approx\frac{1}{2}\ln\left(\frac{2c}{n\varepsilon}\right) \quad \textbf{(13)}$$

This simplification is justified by the fact that, in the high-energy limits, the term $\left(\frac{2c}{n\varepsilon}\right)$ becomes very large compared with unity.

Table I

	m_0 (kg)	m_0 (eV)	n	nε	θ	
e	9,22E-31	5,11E+05	1	1,62E-35	**50,161°**	M0 < 3.9606 MeV
m-	1,91E-28	1,06E+08	712	1,15E-32	**46,877°**	
π-	2,52E-28	1,40E+08	1242	2,01E-32	**46,599°**	$M_{0\geq}$ 3.9606 MeV *speed limit at Planck mass*
p	1,69E-27	9,38E+08	56123	9,07E-31	44,692°	
n	1,70E-27	9,40E+08	56274	9,10E-31	44,693°	

n: maximum summation operator for an accelerated particle

In the framework of the Standard Model of Special Relativity, the 45° limits delimiting the cone of light represent the causal boundary for all events, designated by "e". One notable observation, as illustrated in the above table, concerns an anomaly relating to low-mass particles. Although these particles adhere to the principles of relativity by maintaining speeds below the speed of light, they nevertheless appear to violate this limit.

There are two possible solutions to this anomaly:

- This quantification of c by lp may not accurately reflect the phenomena observed at very high energies. To verify this hypothesis, we will use a heuristic approach by examining whether, as a function of $\vec{p}$ and λ it is possible to reconstruct the known or expected physics at high energy levels. We start with the following basic equation :

$$V_{limit}=c-n.\varepsilon$$

- If this quantification is consistent with known or expected physics for $\vec{p}$ and λ then a bold hypothesis can be considered: a violation of causality as an edge effect when $V_{limit}\rightarrow c$.

A strong convergent index that can validate this angular anomaly on the light cone is one of the conclusions of Charles Marteau, in his 2020 thesis awarded by the SFP. He demonstrates that when the curvature of anti-de Sitter space tends towards zero, a flattening of the light cone implies a probable loss of causality. [2]

A. General solution for a theoretical limit velocity very close to c, non-asymptotic, for all particles of non-zero mass.

So let $V_{limit}=c-n.\varepsilon$ then the inverse Lorentz factor :

$$\gamma^{-1}=\sqrt{\left(1-\frac{(c-n\varepsilon)^2}{c^2}\right)} \quad \Rightarrow \gamma^{-1}=\sqrt{\frac{2n\varepsilon c-(n\varepsilon)^2}{c^2}}$$

we can neglect $(n\varepsilon)^2$ since $2n\varepsilon c \gg (n\varepsilon)^2$ and $(n\varepsilon)^2 \to 0$

The Lorentz factor becomes : $\gamma \approx \frac{c}{\sqrt{2n\varepsilon c}}$ either :

$$\gamma \approx \sqrt{\frac{c}{2n\varepsilon}} \qquad \textbf{(14)}$$

B. Special solution

In general relativity, the Planck mass is frequently linked to a mass scale beyond which the effects of quantum gravity become manifest. It thus marks a boundary where, for a relativistic particle reaching this energy threshold, it would constitute a particular solution to this theoretical speed limit.

Consider the particular solution, when a moving particle of mass m_0 reaches

Planck's mass : $Mplanck=\frac{m_0 c}{\sqrt{2n\varepsilon c}} \qquad \Rightarrow \left(\frac{m_0 c}{Mplanck}\right)^2=2n\varepsilon c \qquad \frac{m_0{}^2 C^2}{\frac{\hbar C}{G}}=2n\varepsilon c$

finally $\quad n\varepsilon=\frac{m_0{}^2 G}{2\hbar} \Rightarrow \quad V_{limit}=c-\frac{m_0{}^2 G}{2\hbar}$ **(15)**

Note that $\frac{m_0{}^2 G}{2\hbar\varepsilon} \geqslant 1$ since $n \geq 1$

$\Rightarrow$ For a rest mass $m_0 < \sqrt{\frac{2\hbar\varepsilon}{G}}$ (3.9606 MeV; 7.15E-30 kg) this limit becomes invalid,

Any particle with an initial mass < 3.9606 MeV will never reach Planck mass.
If we take an electron with mass 5.11 KeV < 3.9606 MeV, the speed limit should show $c-2{,}7.10^{-37} m.s^{-1}$ at Planck mass, which is strictly impossible because it is less than $\frac{lp}{t}$. this limit reduced to n = 1 due to (1) becomes $v_{limit}=c-\varepsilon$ and the Lorentz factor can then be written as $\gamma \approx \frac{1}{\sqrt{2t_p t^{-1}}}$ where t = 1 s **(16)**

VIII. Heuristic consistency with known physics

Here, we will investigate, in a heuristic way, whether it is possible to reformulate the momentum and wavelength of a particle using this quantification of C. The aim is to validate this reformulation of the RR by checking the concordance of the results or by detecting any anomalies.

A. Quantity of motion

by replacing in the relativistic expression of the momentum : $\vec{p}=\gamma m_0 \vec{v}$

γm_0 by Planck mass $\sqrt{\dfrac{\hbar c}{G}}$

and $\vec{v}$ by $\left(c - \dfrac{m_0{}^2 G}{2\hbar}\right)\vec{i}$ we find

$$\sqrt{\frac{\hbar c}{G}}\left(c - \frac{m_0{}^2 G}{2\hbar}\right) = p$$

$$C\sqrt{\frac{\hbar c}{G}}\left(1 - \frac{m_0{}^2 G}{2\hbar c}\right) = p \quad \text{posing} \quad X = -\frac{m_0{}^2 G}{2\hbar c}$$

as $X \to 0$ by development limited of $c\sqrt{\dfrac{\hbar C}{G}}(1+X) = p$ we obtain

$\dfrac{\hbar c^3}{G}(1+2X) = p^2$ and replacing X again gives

$$\frac{\hbar C^3}{G} - \left(m_0^2 c^2\right) = p^2 \qquad \textbf{(17)}$$

which once homogenised by c^2

gives us the energy-impulse quadrivector $E^2 - c^2 p^2 = m_0^2 c^4$

or $\dfrac{\hbar C^5}{G} - \left(c^2 p^2\right) = m_0^2 c^4$

There is therefore agreement with known physics and no edge effect for $\vec{p}$

$E^2_{c_{maximale}} = \dfrac{\hbar c^5}{G} - m_0^2 c^4$ **(18)** for $m_0 > 3.9606$ MeV	$E_{c_{Maximun}} = \left(\dfrac{1}{\sqrt{2 t_p}} - 1\right) m_0 c^2$ **(19)** for $m_0 \leq 3.9606$ MeV

<u>By applying this non-asymptotic speed limit close to c, we do away with the idea of infinity associated with special relativity.</u>

B. Verification via Wavelength

Consider a particle whose wavelength λ is defined on the axis λ_z of motion. De Broglie's relation gives us :

$$\lambda_Z = \frac{h}{p}$$

The momentum previously defined by $p^2 = \dfrac{\hbar c^3}{G} - \left(m_0^2 c^2\right)$ can be reduced to

$$p^2 = \frac{\hbar c^3}{G}$$

Since $\frac{\hbar c^3}{G} \gg \left(m_0^2 c^2\right)$ this implies that $m_0^2 c^2$ can be ignored and λ_z becomes :

$$\lambda_Z = 2\pi\sqrt{\frac{\hbar G}{c^3}} = 2\pi lp$$

Considering the uncertainty on λ_z and using Heisenberg's uncertainty principle $\Delta\lambda_Z \geqslant \frac{\hbar}{2p}$ we can assume that, simplified by the wave vector 2π :

$$\lambda_Z = lp \pm \Delta\lambda_Z$$

and under the conditions of our study :

as $\Delta\lambda_Z \geqslant 8{,}08.10^{-36} m$ or $\Delta\lambda_Z < lp$, $\Delta\lambda_Z$ must be reduced to the value lp

Thus we model λ_z as a harmonic oscillator with two states |lp> and |2lp>, the |0> state being unobservable since it is less than lp.

We then look for the proper duration of the oscillation of λ_z <u>in observable space</u>, i.e. over the half base length $\lambda_Z = 2\pi lp$ becomes :

$$\lambda_Z = \pi\, lp \text{ , } \quad \textbf{(20)}$$

or as $\lambda_Z = \pi c . tp$ and $f = \frac{c}{\lambda_Z}$ we find

$$t_O = \pi\, tp.$$

This natural oscillation period can be compared with the Schwarzschild time t_s given by :

$$ts = \frac{\pi G M_P}{c^3}$$

with a strictly similar result:

$$t_O = ts = 1{,}6937.10^{-43} s$$

To continue, let's see if it's possible to go further by finding Hawking's entropy applied to a particle.

The standard model gives us the Schwarzschild radius, for a Planck mass with no momentum,

$$R_S = M_P \cdot \frac{2G}{c^2} \Rightarrow \quad R_S^2 = 4\hbar \cdot \frac{G}{c^3}$$

i.e. a spherical model with radial value : $R_S = 2\, lp$.

Here, as we are dealing with the case of a moving particle close to C, we assume that λ_{xyz} is isotropic in $\mathbb{R}^3$ when the particle's momentum is small. For relativistic dynamics, since the contraction of lengths only takes place along the axis of displacement, Rs will take the form of a flattened ellipsoid of revolution, with half major axis λ_{xy} and half minor axis |2lp> = $\vec{R_S}$

The probability amplitude Ψ is that of a membrane of unit thickness lp and $|\Psi|^2$ becomes homeomorphic to an ellipsoid of revolution contracted along the minor z axis.

The interaction with the zero-point energy field will depend on the effective cross-section of the ellipsoid in the direction of motion: the effective cross-sectional area $A=\pi\lambda_{XY}^2$ using here $\lambda_{XY}=\frac{2\pi\hbar}{p}$ or the values not affected by movement,

i.e. $\frac{A}{\pi}=\frac{4\pi^2\hbar^2}{p^2}$

Now, as we have established, above, the form not homogenized by momentum $\frac{\hbar C^3}{G}-\left(m_0^2c^2\right)=p^2$

with $\frac{\hbar C^3}{G}\geqslant\left(m_0^2c^2\right)\Rightarrow\frac{\hbar C^3}{G}\approx p^2$ finally : $\frac{A c^3}{4\hbar G}=\pi^3$ **(21)**

What we're looking for here is entropy, so if we homogenise by Boltzmann's constant :

$$\frac{A c^3}{4\hbar G}k_B=\pi^3 k_B$$

we find the Bekenstein-Hawking entropy of a black hole given by $S_{BH}=\frac{k_B}{4}\frac{A c^3}{\hbar G}$

Finally, the particle acquires an entropy of :

$$S=\pi^3 k_B=4{,}281.10^{-22}\,J\,K^{-1}$$

Entropy here means the amount of Hawking radiation confined between the effective cross-section and the limiting velocity c.

This leads to the conclusion that, for any particle with a rest mass greater than 3.9606 MeV, evaporation can be anticipated in a probable time of 8.67160 × 10^{-40} s when its dynamics reaches Planck mass.

There is therefore concordance with known or expected physics and no edge effect concerning the momentum and wavelength of an observed particle.

This validates the possible use of $V_{limit}=c-n.\varepsilon$

C. Light cone anomaly

It therefore seems that the RR behaves as expected when $V_{limit}=c-n.\varepsilon$ except for this anomaly concerning the cone of light.

To resolve this paradox, and in the absence of other explanations, we assume that there is a second limit, **a fermionic causality limit, which is not confused with C** and can be redefined by :

if we extract $n\,\varepsilon$ by the inverse of $\theta\approx\frac{1}{2}\ln\left(\frac{2c}{n\varepsilon}\right)$ we obtain $\frac{2C}{e^{(2\theta)}}=n\varepsilon$

The existence of this hypothetical limiting velocity V_{limit} is less than C and does not alter the opening angle of the light cone, i.e. $\theta=45°$

we place $n\varepsilon = \frac{2c}{e^{90}}$ **or** $V_{limit} = C\left(1 - \frac{2}{e^{90}}\right) = c - 4{,}913.10^{-31}\, m.s^{-1}$ *(calculation by W.A.)* **(22)**

Since $V_{limit} < c$ **is a crossable limit**, this implies ∀ fermion of mass $m_0 \leq$ 3.9606 MeV can become **a non-observable with no possibility of interactions with the external universe** at this energy level.

VIII. SIMPLIFIED INVOLVEMENT IN EXTREME ENVIRONMENTS

Since V_ζ cannot be reached under normal conditions, we will assume that it could be reached under extreme circumstances, such as below the event horizon of a black hole. This is based on three assumptions:

1. The decrease in angular momentum $\vec{J}$ in favour of gravitational waves during the formation of a black hole is a gradual process. Consequently, the extraction of energy from angular momentum can never be complete; a residual velocity must remain, so that $\vec{J} \neq 0$ when the radial distance of the collapsing mass reaches the event horizon.

2. The expression of the energy gradient remains strictly positive beyond this horizon, since any emission would violate the constant c. This observation implies the conservation of angular momentum, so that it is respected again, **together with the Planck mass being exceeded for any particle with an initial mass greater than 3.9606 MeV.**

3. In accordance with the first part, it should be noted that the gravitational compression cannot exceed the Planck density. At this limit, the volume of the collapsing mass reaches a constant value, allowing us to determine the minimum value of the radius r_l.

In *condition 2*, we set a limit to the curvature of space-time, characterized by maximum tiling at Planck energy. The notion of singularity is therefore no longer relevant, and a more classical geometry could be envisaged, such as the calculation of three-dimensional volume in constant time slices, i.e. a 4 ellipsoid such as :

$$\frac{\partial V_{ol}}{\partial t} = \frac{1}{2}\pi^2 r^3(t). \quad \text{(postulates 1 \& 2)}$$

This gives, when the collapse stabilises at the Planck density. :

$$V_{ol} = \pi^2(\eta\, lp)^2 lp\, t \qquad \eta \in \mathbb{N}^+_*$$

The final result is an oblate ellipsoid of revolution with lp as the polar and $(\eta\, lp)^2$ equatorial, M the mass of the compressed core and $\frac{c^5}{\hbar G^2}$ the Planck density (postulate 3).

$$\frac{2M}{\pi^2(\eta\, lp)^2 lp} = \frac{c^5}{\hbar G^2}$$

$$\Rightarrow \frac{1}{\pi\sqrt{lp}}\sqrt{M\frac{2G}{c^2}} = \eta$$

$$\Rightarrow \frac{\sqrt{lp\, R_s}}{\pi} = r_l \quad \text{with} \quad r_l = \eta\, lp \quad \textbf{(23)}$$

According to this model, by integrating Planck's limits as simple properties inherent to space-time, M r_l then acquires a quantifiable spatial dimension, whatever M : **product of the Schwarzschild radius and the Planck length.**

A . Comparison of collapse times

It is obviously difficult to estimate the duration of the collapse below the event horizon, as it cannot be measured by an outside observer.

However, by a quick and naive calculation, it is possible to compare the duration $t_{\zeta\, radiale}$ (necessary for the collapse speed to reach $V_{\zeta\, radiale}=c\left(1-\frac{2}{e^{90}}\right)$) to the time it would take t_l compression would take to reach Planck density : $r_l=\frac{\sqrt{lp\, R_s}}{\pi}$.

If $t_l \geqslant t_{\zeta radiale}$ then the causal limit V_ζ would be considered accessible.

then for $t_{\zeta\, radiale}$: $c\left(1-\frac{2}{e^{90}}\right)=\frac{g\,t}{\sqrt{1+\frac{(g\,t)^2}{c^2}}}$

$$\Rightarrow \quad \frac{g^2}{c^2}\left(\left(\frac{e^{90}}{e^{90}-2}\right)^2-1\right)=\frac{1}{t^2}$$

by replacing $k=\left(\frac{e^{90}}{e^{90}-2}\right)^2-1=3{,}27761\text{E-}39$ *(W.A. calculation*)

we find $\frac{1}{\sqrt{k}}\frac{c}{g}=t_{\zeta radiale}$

for t_l : $\frac{r_l}{R_S}=\sqrt{1-\frac{v^2}{c^2}}$

$$\Rightarrow \; v^2=c^2\left(1-\frac{r_l^2}{R_S^2}\right) \text{ with } v=g\,t$$

which finally gives $t=\frac{c}{g}\sqrt{\left(1-\frac{r_l^2}{R_S^2}\right)}$

by replacing $r_l\, et\, R_S$ t_l becomes $t_l=\frac{c}{g}\sqrt{\left(1-\frac{lp}{\pi^2 R_s}\right)}$

or as $\frac{lp}{\pi^2 R_s}\ll 1 \quad \Rightarrow t_l\approx\frac{c}{g}$ **(24)**

Finally $t_l \ll t_{\zeta\, radiale}$ the causal limit can never be reached in radial velocity, whatever the mass of the black hole.

We now check $\underline{V_{\zeta\, tangentielle} \propto V_{\zeta radiale}}$

As a basis, when $r=\frac{\sqrt{lp\, R_s}}{\pi}$ then $V_{\zeta\, tangentielle}=c\left(1-\frac{2}{e^{90}}\right)m.s^{-1}$

For $\vec{J}=\vec{r}\wedge\left(k\, M_{\odot} V_{\zeta}\right)\vec{i}$ from $r=\frac{\sqrt{lp\, R_s}}{\pi}$ to $r=R_s$ the conservation of angular momentum, as shown in the following table for 3 typical TN masses, leads us to extremely low initial velocities V_s , probably below the reality compatible with postulates 1 & 2.

Table II

k	$k\, M_{\odot}$	R_s	r_l	V_s
1,00E-13	1,98850E+17	2,95338E-10	2,199198E-23	2,23E-05
10	1,98850E+31	2,95338E+04	2,199198E-16	2,23E-12
1,00E+09	1,98850E+39	2,95338E+12	2,199198E-12	2,23E-16

This makes $V_{\zeta\, tangentielle}$ for $r \geqslant \frac{\sqrt{lp\, R_s}}{\pi}$ theoretically accessible to the 3 categories of black holes.

In order to rigorously respect the principle of equivalence, it is crucial to go beyond the traditional conception of acceleration, which is usually limited to a single frame of reference. It is proposed to consider acceleration as having a dualistic nature, manifesting itself in both inertial and gravitational states. In a context where the space-time metric has a weak curvature, these two states are practically indistinguishable. **However, passing through this hypothetical causal limit should lead to a break in the equivalence principle, with only the gravitational state becoming observable.**

It is important to stress that this violation of causality mainly affects the frame of reference associated with inertial mass. In such a context, in the absence of energy dissipation, the conservation of potential energy remains a primary consideration. This raises a pertinent question about the possibility of energy dispersion in this process.

As far as dispersion is concerned, if we calculate the collapse time of a black hole in a non-exhaustive way :

by taking $dg=\int \frac{GM}{r^2}dr \;\Rightarrow\; g=\left[-\frac{GM}{r}\right]_{R_s}^{\frac{\sqrt{lp\, R_s}}{\pi}}$

as $\frac{1}{R_s} \ll \frac{\pi}{\sqrt{lp\, R_s}} \;\Rightarrow\; g=\frac{\pi\, G\, M}{\sqrt{lp\, R_s}}$

we find for $\Rightarrow t_l \approx \frac{c}{g} \rightarrow t_p$ when $g \rightarrow \frac{c}{t_p}$ *(Planck acceleration)*. **(25)**

B. GTS assumption

It is conceivable that the process of quantum degeneracy could slow down the collapse time, as calculated above, to make it compatible with the observation time of accretion disks. However, according to our current understanding of physics, once a mass collapses to form a black hole, with the creation of an event horizon, it appears that quantum degeneracy, as we know it in less extreme contexts such as neutron stars, is probably not sufficient to counter the effect of gravitational attraction.

Nevertheless, the tiling process, presented as a fundamental axiom, predicts that lp^2 superposition at this scale remains consistent with quantum gravity models, thus delineating a locus where quantum fluctuations in spacetime become significant. Consequently, the effects of quantum gravity could generate a region of repulsion increasing the collapse time. This significantly modifies the naive result obtained for the calculation of the collapse time of a black hole core.

However, in the absence of data, we will speculate here on a time close to the formation of the event horizon, as obtained by simulation of GW170817 observations, i.e. of the order of a second.[6]

The timescale is incompatible with the observation of accretion discs over long periods, which runs counter to the idea of dispersion.

In accordance with the model shown here , we need to put forward a new hypothesis:

We then postulate the existence of a new class of topological soliton structured around a gradient of potential energy and zero angular momentum : $\overrightarrow{Rot}\,\vec{J}=\vec{0}$, and for which, for an incoming particle $\vec{p}\neq 0$, there is no longer any possible radial acceleration but only tangential acceleration, in other words a simple curvature of its trajectory. [4]

The Schwarzschild metric must therefore be adapted.

Using the heuristic method, for a test particle of mass m falling towards a black hole of mass M we need to consider that the gravitational potential energy at a distance of r from the black hole is equal to the maximum kinetic energy that the particle could have just before falling into the black hole.
In Newtonian mechanics, gravitational potential energy is given by $\frac{-GMm}{r}$ and the kinetic energy is given by $\frac{1}{2}mv^2$. At the event horizon, the speed of liberation is equal to the speed of light C so we can postulate $\frac{1}{2}mc^2=\frac{GMm}{r}$ which gives the Schwarzschild radius $\frac{2GM}{c^2}$

It is no longer possible to find an equivalence between $\frac{1}{2}mc^2$ and $\frac{GMm}{r}$however, it should be noted that this soliton would not be sensitive to any of the other 3 forces of the Standard Model, so its energy balance could be summarised as $E_p=Mc^2$.

The same heuristic method can then be applied to find the Schwarzschild metric such that :

$$ds^2=-\left(1-\frac{GM}{c^2 r}\right)c^2dt^2+\left(1-\frac{GM}{c^2 r}\right)^{-1}dr^2+r\left(d\theta^2+\sin^2\theta\, d\varphi^2\right) \quad \textbf{(26)}$$

The radial value of the event horizon would therefore be reduced by a factor of 2, and there is no other way to distinguish a classical black hole from a topological gravitational soliton (GTS). The accretion disk, with its non-zero momentum, could become the inertial mass of the topological object. In the absence of an accretion disk, the topological gravitational soliton acts as a gravitational lens.

IX. CONCLUSION

A. Falsification

In the context of falsifying this study, we examine the potential formation of molecular-sized gravitational solitons by hypothetical primordial micro black holes. These solitons, characterized by non-inertial mass, absence of electric charge, and magnetic spin, would have no immediate effect on baryonic matter on a local scale. However, they are expected to exhibit notable dynamics in the presence of this matter, influenced by the curvature of spacetime, showing oscillations around mass centers.

An interaction with matter would then be conceivable, particularly with the fundamental rotational state of the para water molecule $|000\rangle$, where gravitational coupling could occur at this minimal energy level.

It would then be possible to anticipate a deviation of photons above the predicted refractive index, observed during the ortho-para water isomeric transition at ultra-low temperatures down to its fundamental rotational state. This phenomenon could be attributed to a high concentration of gravitational topological solitons (GTS) around the para water molecule compared to its immediate surroundings.

By extension, the distribution of dark matter in galaxies that can be correlated with the distribution of cold hydrogen. Specifically, studies have suggested that the radial distribution of dark matter density correlates with cold atomic hydrogen of extremely low density in its lowest fundamental state (i.e., the ground state HI with antiparallel electron spin). This correlation is often interpreted as an artifact explained by the proportionality between the surface density of the gas and the critical density for gravitational stability. A coupling between HI and GTS would bring a new perspective to this debate, invalidating this idea of artifact.[3]

B. Research avenues

The results of this study suggest that exploring the properties of the Planck scale, through a fractal approach to space-time, opens up new and promising perspectives for understanding quantum gravity and theories of everything. In particular, the redefinition of the fundamental constants and metrics of space-time in an iterative and self-similar framework proposes new dynamics that could have profound implications for special relativity, light-cone anomalies, and the fundamental structure of the universe.

1) Compatibility with the Holographic Principle :

   To explore how the fractal properties of space-time at the Planck scale can be compatible with the holographic principle. In particular, to study whether information encoded fractally on a two-dimensional surface (lp^2) can provide a complete description of the universe, as proposed by the holographic principle. [5]

2. Extension of AdS/CFT :

   Investigate the application of the results of this study to the AdS/CFT correspondence. To examine whether higher dimensional space (HDS) can be seen as emerging from the fractal properties of conformal field theory (CFT), and what the implications of this perspective would be for string theory.

3. Quantum Gravity :

   To study in more detail the implications of the fractal properties of the Planck scale for quantum gravity. In particular, to analyse how these new dynamics can influence the fundamental interactions and the structure of space-time at very high energies.

4. Fermionic causality limit :

   Continue research into the light-cone anomaly and the possible existence of a second fermionic causality limit. Assess the implications of this limit for low-mass particles at very high energies and for the formulation of special relativity at these scales.

5. Cosmology and Particle Physics :

   To explore the cosmological implications of the properties of the Planck scale, in particular with regard to the soliton hypothesis (GTS) and the link they could have with dark matter. To study how a fractal approach to space-time could offer new explanations and mechanisms for these phenomena.

In conclusion, this study proposes an innovative redefinition of the fundamental concepts of theoretical physics, opening up multiple and diversified avenues of research. The results obtained, although preliminary, provide a solid basis for future investigations aimed at unifying quantum gravity and theories of everything, while remaining compatible with the established principles of modern physics.D

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