The MM Theory:

# A Fundamental Revision of the Laws of Motion and Introducing Centrial Motion and Centrial Momentum 

Part (2)

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Summary: In this paper, classical concepts of motion and momentum are revisited and a novel vision of these concepts is analyzed at a fundamental level of classical physics, whereby a new form of motion is presented and defined as "centrial motion." Additionally, a practical theoretical concept of "centrial momentum" is determined and justified.


#### Abstract

In classical physics, linear and angular motion as well as linear and angular momentum have long been defined. In this paper it becomes apparent through analysis that there is much need for the presence and denotation for a new type of motion. As such, centrial motion is introduced and described as another form of motion not previously presented. Furthermore, a new form of momentum called centrial momentum is defined and elaborated. As a result, the motion of complex bodies can be analyzed and studied with much more simplicity and ease than previously done via classical physics. Along with the discussion of centrial motion and momentum, the concepts of linear motion based on the motion of momentum is also studied and analyzed and the law of motion of momentum is defined. Additionally, complex scenarios are introduced where the discussions assist in the much simpler understanding of the classical scenarios of the motions presented. It becomes readily apparent that the use of centrial motion equations and relationships derived are the best suited for the purposes of the study of these types of motions.

In addition, in this paper, motion scenarios that cannot be explained by classical physics are discussed and adequately explained by presenting new concepts. Through deeper analyses, it is found that momentum is not conserved. However, the kinetic energy of an isolated system, if not transformed to other forms of energy, remains conserved.


## Key words

MM Theory, Linear motion, Linear momentum, Centrial motion, Centrial momentum

## 1. Introduction

As it has been shown classically, the motions of objects are focused on two forms, linear motion and angular motion. Linear motion is a motion of a body on a straight line, and angular motion is the motion of a body about a fixed point or fixed axis. These types of motions are classically analyzed and studied by Newtonian Mechanics [1], Lagrangian Mechanics [2], and Hamiltonian Mechanics [3]. In this paper, it will be shown that these motions are more precisely defined via the Law of Motion of Momentum which was introduced by the author in "The MM Theory: The Theory of Everything Part (1)." [4]

Additionally, here, another form of motion being denoted as "Centrial Motion" which has not been studied in classical physics thus far is introduced and defined. To introduce and understand this form of motion, a stationary object that does not have any motion is considered. If an explosion takes place right in the center of that object and the object turns into smaller pieces, then all the pieces will be thrown apart from the center of the object. After the explosion, each of the pieces will have its speed and therefore will have certain linear momentum and possible angular momentum. In classical physics, this motion is not defined as a standard form of motion because the resultant sum of the linear momentums and also resultant sum of the angular momentums of all pieces as a whole is equal to zero. In considering the fact that all the pieces or parts are moving,
it is impractical to model or consider them as a stationary body. Therefore, it is necessary to define this type of motion.

To demonstrate this in better clarity, the above centrial explosion scenario can be studied in another manner. [Fig. 1] shows a number of objects that are initially at rest next to one another. Now, if an explosion occurs in the center, all the objects will be thrown outwardly from the center at certain speeds. If, for example, the intensity or power of the explosion is doubled, then the speed of the objects increases and their speeds will be greater than the first explosion. The speed of motions of the objects after the explosion depends on the energy released from explosion. In all cases, the sum of the linear and the sum of angular momentums of the objects with respect to the center of the explosion is equal to zero, both before and after the explosion. For the sake of simplicity, we assume that all objects for this case are rigid and equal in mass and equally distributed about the center of explosion and can only gain linear momentum upon the explosion. As such, then the sum of the magnitudes of linear momentums and the speeds of all these objects relative to the center can be considered as characteristics of the explosion. As such, the value of the sum of the magnitudes of linear momentums along with their speeds are differentiating characteristics that depend on the energy of each explosion. It is here deemed practical to denote a new form of motion as each explosion scenario of a certain energy will have a corresponding sum of the magnitudes of linear momentums from that explosion. Therefore, "Centrial Motion" can accordingly be defined as this new form of motion. And to study this new type of motion, "Centrial Momentum" is defined as the momentum manifestation of "Centrial Motion."


Fig. 1. Number of objects initially at rest, and subsequently thrown outwardly with certain equal speeds after an explosion at the central point of reference.

In addition to the above example, the following example case study is considered as another compelling reason for the need to introduce centrial motion. This example may seem overly-simple and unnecessary, but it will later be seen that it will be essential and required as a stepping stone to studying such motions.

Assuming that bodies (1), (2), (3), and (4), all of which have equal mass ( $m$ ), are moving with equal speed and in the directions shown in [Fig. 2]. The entire set of bodies can be considered as an isolated system. As such, the linear momentum of this system is equal to zero, while the kinetic energy of the entire system is non-zero. That such a system has kinetic energy while it does not have momentum is a result based on the classical laws of physics. However, it does not make sense to accept that a system without momentum has kinetic energy. When we think about motion, we are actually thinking about momentum. Any object or system that has motion and accordingly kinetic energy has momentum; and vice-versa, any object or system that has momentum has motion and kinetic energy. It is meaningless to say that a system does not have momentum while it has motion and kinetic energy. Therefore, the definitions and laws of fundamental physics should be revised.


Fig. 2. An isolated system with total summation of linear momentum at zero, where the kinetic energy of the system is non-zero.

While these flaws with classical physics as described are presented here, there are also other cases where classical physics falls short to thoroughly explain the scenarios presented. Thus, for a broader and more general case to illustrate the need for a fundamental revision in the laws of physics, the scenario of the two bodies as shown in [Fig. 3] are considered.

We assume an isolated system where two bodies are rigid, in a gravity-free, frictionless environment and are freely moving in space. As is shown in [Fig. 3(a)], we denote them as the two bodies (1) and (2) in 3-D space, where each is moving towards one another with the same momentum, $(p)$. Additionally, we next assume that a spring system is placed between the two bodies as shown in [Fig. 3(b)].


Fig. 3. (a) two rigid bodies (1) and (2) moving towards one another each with equal momentum, ( $p$ ), (b) a spring system is placed between the two bodies, (c) the time at which the energy of the momentums have in totality been transferred to the spring system, (d) a torque causes change in the axis AB to CD of the two bodies, (e) ultimately the spring system is released, ( f ) each body moves with momentum $(p)$ along the CD axis.

Then, in such a scenario, it can be depicted as per [Fig. 3(c)] the instant in time where the linear momentums reduce to zero due to their associated energies being transferred to the spring system. At such an instant, we assume that for the system at the $A B$ axis we freeze it and employ energy via an applied torque such that the two bodies are reoriented and will be stopped along the CD axis as shown in [Fig. 3(d)]. At CD, we gain back the energy that we applied to the system resulting in a net zero energy change in the system and thus no energy is expended to achieve this. If, then the spring system is released, the end result of this case is that the two bodies move along the CD axis and the initial momentums are restored as shown in [Fig. 3(e)]. In the end, the bodies will each continue to move opposite to one another with momentums $(p)$ along the CD axis as shown in [Fig. 3(f)].

Now in the above scenario, two items of note cannot be adequately explained by classical physics. First, it is unexplained how all the momentums from [Fig. 3(a)] reduce to zero in [Fig. 3(c)]; in other words, it is observed that all the initial momentums in the system vanishes in [Fig. 3(c)]. Second, it can be noted here in this scenario, as the orientation of linear momentums is altered, it becomes unfit to be analyzed as compared to the basic linear scenarios presented in classical physics. In the application of classical physics, it is unclear how linear momentums can be
readopted along a new axis that is considered and solved for, where such a new alignment falls outside the confines of classical theory itself. Classical physics cannot be used to solve for such a system as the axis of the momentums are now in a new orientation where it contradicts the classical principal of momentum being conserved in a given direction.

As a result of the above scenario in [Fig. 3], it can clearly be noted that new concepts and revisions are required in classical physics to aid us in the studying and solving of such cases and systems.

From the overall discussions presented for centrial motion in [Fig. 1], it may be pondered and argued that it is not needed to define such motion as the movement of each individual object can be independently studied and examined to derive the overall output using concepts from classical physics. However, it should be noted that first of all, such a task is very difficult and, in some cases, impossible. Secondly, in classical physics, the angular motion can theoretically be analyzed based on the laws of linear motion. However, since the study of angular motion based on the laws of linear motion was practically deemed by physicists as very difficult and sometimes impossible, it was needed to study and present the laws of angular motion. Similarly, then, here in the same line of reasoning, it can be deemed necessary to define and study centrial motion. Finally, as per the scenarios presented in [Fig. 2] and [Fig. 3], we also need to study the concepts of momentum and revise the law of conservation of momentum.

## 2. Theory

### 2.1. Concepts of Momentum

For studying the concepts of momentum, we begin with the following scenario,


Fig. 4. Body (1) with (a) initial momentum $\vec{p}_{1 i}$ and with (b) initial velocity $\vec{v}_{1 i}$ colliding with body (2), resulting in the horizontal momentums $\vec{p}_{1 f x}, \vec{p}_{2 f x}$, and momentum velocity vectors $\vec{v}_{1 f x}, \vec{v}_{2 f x}$. Notably there are generated centrial momentums in the form of $\vec{p}_{1 f y}$ and $\vec{p}_{2 f y}$. The corresponding velocity vectors $\vec{v}_{1 f y}$ and $\vec{v}_{2 f y}$ are also shown in the figure.

In [Fig. 4], we envision a fundamental scenario where momentum is created and where we witness how momentum is not conserved. In addition to momentums being shown in the [Fig. 4(a)], there are also corresponding velocities that are being depicted in [Fig. 4(b)]. In such a scenario, an initial momentum $\vec{p}_{1 i}$ with corresponding velocity $\vec{v}_{1 i}$ is moving via body (1). Body (1) moves along the illustrated path of motion until it collides with the second body (2). Following the impact, body (1) then continues to travel along at an angle $\alpha$ from its original path of motion. In turn, body (2) then travels along at an angle $\beta$. Following this, based on classical physics it can be derived that body (1)'s final momentum when projected along the original path of motion is $\vec{p}_{1 f x}$. Similarly, it can be noted that the final momentum of body (2) when decomposed utilizing angle $\beta$ yields $\vec{p}_{2 f x}$. As a result, the sum of $\vec{p}_{1 f x}$ and $\vec{p}_{2 f x}$ is equal to the initial incoming momentum from body (1), $\vec{p}_{1 i}$. However, what is also apparent is that there are new momentum components perpendicular to these that are created as seen in [Fig. 4] and which are denoted as $\vec{p}_{1 f y}$ and $\vec{p}_{2 f y}$. Uniquely, these components are created and recognized as momentums from a simple impact scenario. In other words, new momentums perpendicular to the initial momentum direction were created in this collision scenario and thus, momentum is not conserved. They are considered, understood, and explained via the concept of "Centrial Momentum" as was discussed earlier. In classical physics these new components are dealt with via momentum vector cancellations.

It should be noted that in applying classical physics to [Fig. 4], only the sum of the two final momentum components in the direction of the initial momentum is considered and mathematically shown as equal to the initial momentum. While classical physics calculations hold that momentums in a given direction add up to the same value, they are intrinsically not in fact the same as to the initial momentum. That is, if they were the same, their energies must also be equal to each other's and this is clearly not the case.

Now, based on the new concepts of momentum introduced, we can more precisely solve for scenario [Fig. 4] as follows,

We can state that for any considered initial momentum, the magnitude of the momentum in its initial direction is always constant, and therefore,

$$
\begin{equation*}
p_{1 i}=p_{1 f x}+p_{2 f x} \tag{2.1}
\end{equation*}
$$

For the newly created centrial momentum,

$$
\begin{equation*}
p_{1 f y}=p_{2 f y} \tag{2.2}
\end{equation*}
$$

Energy of the system before collision is,

$$
\begin{equation*}
E_{i}=\frac{1}{2} p_{1 i} v_{1 i} \tag{2.3}
\end{equation*}
$$

Energy of the two final momentums in the direction of the initial momentum is,

$$
\begin{equation*}
E_{f x}=E_{1 f x}+E_{2 f x}=\frac{1}{2} p_{1 f x} \cdot v_{1 f x}+\frac{1}{2} p_{2 f x} \cdot v_{2 f x} \tag{2.4}
\end{equation*}
$$

Energy of the centrial momentum, $\left(E_{c . m}\right)$, is calculated as,

$$
\begin{equation*}
E_{c . m}=E_{1 f y}+E_{2 f y}=\frac{1}{2} p_{1 f y} \cdot v_{1 f y}+\frac{1}{2} p_{2 f y} \cdot v_{2 f y} \tag{2.5}
\end{equation*}
$$

Based on the above, it can be shown that the addition of the energy of centrial momentum to that of the two final momentum components in the direction of the initial momentum equates to the total energy of the two bodies (1) and (2) combined. This can be illustrated as follows:

$$
\left.\left.\left.\begin{array}{rl}
E_{f}= & E_{f x}+E_{c . m} \\
E_{f}= & \frac{1}{2}\left(p_{1 f} \cos \alpha \cdot v_{1 f} \cos \alpha+p_{2 f} \cos \beta \cdot v_{2 f} \cos \beta+p_{1 f} \sin \alpha\right. \\
& \left.\cdot v_{1 f} \sin \alpha+p_{2 f} \sin \beta \cdot v_{2 f} \sin \beta\right)
\end{array}\right] \begin{array}{rl}
E_{f}= & \frac{1}{2}\left(p_{1 f} v_{1 f} \cos ^{2} \alpha+p_{2 f} v_{2 f} \cos ^{2} \beta+p_{1 f} v_{1 f} \sin ^{2} \alpha\right. \\
& \left.\quad+p_{2 f} v_{2 f} \sin ^{2} \beta\right)
\end{array}\right] \begin{array}{rl}
E_{f}= & \frac{1}{2}\left(p_{1 f} v_{1 f}\left(\cos ^{2} \alpha+\sin ^{2} \alpha\right)+p_{2 f} v_{2 f}\left(\cos ^{2} \beta+\sin ^{2} \beta\right)\right)
\end{array}\right\}
$$

As such, Eq. (2.10) shows the energy equivalency as has been stated. This result shows the perpendicular momentums cannot be ignored in our analysis. In doing so, we would merely be considering the momentum in its initial direction - conservation of energy of the system would then not be satisfied.

Now if we go back and reverse the whole scenario such as the incoming bodies are the final two bodies moving in the reverse directions, then we see that body (2) will stop at the center of the Cartesian axes. In this case, the initial magnitude of momentum of the system is the sum of the magnitudes of momentums of bodies (1) and (2) which is ( $p_{1 f}+p_{2 f}$ ), and the final momentum of the system is $p_{1 i}$. As a result, for such a scenario, the final momentum is less than the initial (i.e., momentum is destroyed) and again we note that the momentum is not conserved.

Despite this, the energy of a system is always conserved. If the kinetic energy of an isolated system is not transformed to other forms of energy or vice versa, then the kinetic energy remains conserved; since the kinetic energy is dependent on momentum and also the speed of momentum, ( $K_{E}=\frac{1}{2} p v=$ constant ), then if the momentum of a system is varied then the speed of the momentum changes as well and vis-versa.

Next, we will perform a deeper analysis of the concept of momentum. It will be shown that rather than considering the momentum components only, if we consider the momentum velocities'
vectors as well, then we will find that the sum of the energies of the final momentum(s) is equal to that of the initial.

To examine this in detail, [Fig. 5] is considered. In [Fig. 5(a)] vector, $\overrightarrow{O A}$, shows the momentum, $(\vec{p})$, on a body at origin, O , in a 2 -axis Cartesian coordinate system. In this Figure, a separate vector, $\overrightarrow{O B}$, is shown representing the velocity, $(\vec{v})$, of this momentum.


Fig. 5. The decomposition of a momentum vector in (a) 2D space passing through origin, (b) 3D space passing through origin, and (c) 3D space where the momentum vector does not pass through origin, demonstrate that energy of the momentum is equal to the sum of its components' energies.

First, from its basic conceptualization, momentum always has an intrinsically velocity at any point in time that is in the same direction as that of itself. As a result, if the momentum of the body is
denoted as $\vec{p}$ and its velocity $\vec{v}$, then we deduce and consider a momentum that moves with the speed of $v$. The energy of this momentum $(p)$ with the speed of $v$ is calculated as per the below relationship:

$$
\begin{equation*}
E=\frac{1}{2} p v \tag{2.11}
\end{equation*}
$$

Here, angle $\alpha$ as shown in [Fig. 5(a)], represents the angle between the momentum or velocity vector to the x -axis. It is noted that the momentum vector and velocity vector can be decomposed into X and Y vector components in the Cartesian coordinate system. To be able to conduct a more detailed analysis of the momentum vector's energy, we must determine the components of the velocity in addition to the components of the momentum. To do so, the Cartesian components of the momentum and velocity can be derived from the below geometric relationships:

$$
\begin{array}{ll}
p_{x}=p \cos \alpha, & p_{y}=p \sin \alpha \\
v_{x}=v \cos \alpha, & v_{y}=v \sin \alpha \tag{2.13}
\end{array}
$$

Next, let's assume for a moment that kinetic energy is directional. If so, then in the case of [Fig. 5(a)], we will consider this energy being directed in the same orientation as the momentum $\vec{p}$ and velocity $\vec{v}$. We can then decompose it in the Cartesian coordinate system of X and Y axes. Notably, based on this assumption, we find that the sum of the energies in the two directions of X and Y will be equivalent to that of the whole based on the below mathematical calculations:

$$
\begin{align*}
& E_{x}+E_{y}=\frac{1}{2} p \cos \alpha \cdot v \cos \alpha+\frac{1}{2} p \sin \alpha \cdot v \sin \alpha  \tag{2.14}\\
& E_{x}+E_{y}=\frac{1}{2}\left(p v \cos ^{2} \alpha+p v \sin ^{2} \alpha\right)  \tag{2.15}\\
& E_{x}+E_{y}=\frac{1}{2} p v\left(\cos ^{2} \alpha+\sin ^{2} \alpha\right)=\frac{1}{2} p v \tag{2.16}
\end{align*}
$$

Based on the above result, it is concluded that given any considered momentum, its vector and its velocity vector can always be decomposed into Cartesian components. Such decomposition components can subsequently be utilized in energy calculations as per the above, and their combination shown to be equivalent in energy as to that of the considered momentum's.

Similarly, this analysis and argument can be made for a 3-dimensional system. To begin, we assume here the Cartesian coordinate axes of X, Y, and Z. As shown in [Fig. 5(b)], an initial vector $\overrightarrow{O A}$ is utilized to represent the vector of momentum $\vec{p}$, and can also be utilized for the analysis of the velocity vector $\vec{v}$. Our analysis is a geometric one and it is noted that the angle created between vector $\overrightarrow{O A}$ and the plane XZ is represented in [Fig. 5(b)] by $\alpha$. Similarly, the angle between vector $\overrightarrow{O A}$ and plane YZ is denoted by $\beta$. If we draw a line from point A perpendicular to the plane XZ (i.e., $\overrightarrow{A C}$ ), then the plane OAC would be perpendicular to the XZ plane. Therefore, the angle $\angle \mathrm{AOC}$
as described earlier would be $\alpha$. In addition, if we draw a line from point A perpendicular to plane YZ (i.e., AB ), then the plane OAB would be perpendicular to the YZ plane. Accordingly, the angle $\angle A O B$ as described prior would be $\beta$. Based on geometric theorems, the decomposition of $\overrightarrow{O A}$ can be then accomplished in the three Cartesian directions of $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ as $\overrightarrow{O F}, \overrightarrow{O E}$, and $\overrightarrow{O D}$. Hence, the following can be seen and deduced from [Fig. 5(b)]:

$$
\begin{align*}
& O F=D C=B A=O A \sin \beta  \tag{2.17}\\
& O E=D B=C A=O A \sin \alpha  \tag{2.18}\\
& O D=\sqrt{O C^{2}-D C^{2}}=\sqrt{(O A \cos \alpha)^{2}-(O A \sin \beta)^{2}}  \tag{2.19}\\
& O D=O A \sqrt{\cos ^{2} \alpha-\sin ^{2} \beta} \tag{2.20}
\end{align*}
$$

Based on the geometric analysis above, then the similar decomposition of the vectors for momentum and velocity can be completed as per the below:

$$
\begin{align*}
& p_{X}=p \sin \beta, \quad v_{X}=v \sin \beta  \tag{2.21}\\
& p_{y}=p \sin \alpha, \quad v_{y}=v \sin \alpha  \tag{2.22}\\
& p_{z}=p \sqrt{\cos ^{2} \alpha-\sin ^{2} \beta}, \quad v_{z}=v \sqrt{\cos ^{2} \alpha-\sin ^{2} \beta} \tag{2.23}
\end{align*}
$$

Then, following the above, we can readily calculate for energy in each Cartesian direction as follows:

$$
\begin{align*}
& E_{x}=\frac{1}{2} p_{x} v_{x}=\frac{1}{2} p v \sin ^{2} \beta  \tag{2.24}\\
& E_{y}=\frac{1}{2} p_{y} v_{y}=\frac{1}{2} p v \sin ^{2} \alpha  \tag{2.25}\\
& E_{z}=\frac{1}{2} p_{z} v_{z}=\frac{1}{2} p v\left(\cos ^{2} \alpha-\sin ^{2} \beta\right) \tag{2.26}
\end{align*}
$$

And the total energy, $E_{t}$, can then be calculated as below:

$$
\begin{align*}
& E_{t}=E_{x}+E_{y}+E_{z}  \tag{2.27}\\
& E_{t}=\frac{1}{2} p v \sin ^{2} \beta+\frac{1}{2} p v \sin ^{2} \alpha+\frac{1}{2} p v \cos ^{2} \alpha-\frac{1}{2} p v \sin ^{2} \beta  \tag{2.28}\\
& E_{t}=\frac{1}{2} p v\left(\sin ^{2} \alpha+\cos ^{2} \alpha\right)=\frac{1}{2} p v \tag{2.29}
\end{align*}
$$

Thus, it is shown that the total energy of the system calculated as per the above by decomposition of the momentum and velocity vectors into the Cartesian coordinate system will still be exactly equivalent as to the total energy of the system when not decomposed.

The previous analyses above were for the cases where the momentum vector passes through the center of a Cartesian coordinate system. Next, we consider a case where a momentum vector does not pass through the center. In other words, we would like to study a momentum vector when it is based off of any given point in the Cartesian coordinate system.

Now in [Fig. 5(c)], it is shown that momentum or velocity vector $\overrightarrow{A B}$ does not necessarily pass through the center $O$. As such, it is assumed that it can be at any location in space. Here, as is shown, the angle between momentum or velocity vector with the plane XZ is denoted as $\alpha$, and with the plane YZ is denoted as $\beta$. As can be seen, the plane GEF (i.e., the plane that vector $\overrightarrow{A B}$ is placed in) is perpendicular to plane XZ as line EF is perpendicular to plane XZ . Therefore, angle $\angle E G F$ is the angle that vector $\overrightarrow{A B}$ makes with plane XZ that here we are denoting as $\alpha$. Similarly, $\overrightarrow{A B}$ vector is in the plane GEH and this plane is perpendicular to plane YZ as line GH is perpendicular to plane YZ. As a result, angle $\angle \mathrm{GEH}$ is the angle that vector $\overrightarrow{A B}$ makes with the YZ plane which, here, we denoted as $\beta$. In accordance with [Fig. 5(c)], the decompositions of the vector $\overrightarrow{A B}$ are conducted via vectors parallel to the coordinate axes. As can be seen, the decomposition of $\overrightarrow{A B}$ vector parallel to the X axis is $\overrightarrow{A D}$. The decomposition of the $\overrightarrow{A B}$ vector parallel to the Y axis is $\overrightarrow{C B}$. And the decomposition of $\overrightarrow{A B}$ vector parallel to the Z axis is $\overrightarrow{D C}$. For calculating these vectors, geometrical relationship as per the before discussions can be utilized. For calculation of vector $\overrightarrow{A D}$, we consider the orthogonal triangle $\triangle \mathrm{ADB}$ and derive that,

$$
\begin{equation*}
A D=A B \sin \beta \tag{2.30}
\end{equation*}
$$

Similarly, for the calculation of vector $\overrightarrow{C B}$, we consider the orthogonal triangle $\triangle \mathrm{ACB}$ and derive that,

$$
\begin{equation*}
C B=A B \sin \alpha \tag{2.31}
\end{equation*}
$$

And, for the calculation of vector $\overrightarrow{D C}$, we consider the orthogonal triangle $\triangle \mathrm{ADC}$ and derive that,

$$
\begin{align*}
& D C=\sqrt{A C^{2}-A D^{2}}=\sqrt{(A B \cos \alpha)^{2}-(A B \sin \beta)^{2}}  \tag{2.32}\\
& D C=A B \sqrt{\cos ^{2} \alpha-\sin ^{2} \beta} \tag{2.33}
\end{align*}
$$

Based on the geometric analysis above, then the similar decomposition of the vectors for momentum and velocity can be completed as per the below:

$$
\begin{align*}
& p_{X}=p \sin \beta, \quad v_{X}=v \sin \beta  \tag{2.34}\\
& p_{y}=p \sin \alpha, \quad v_{y}=v \sin \alpha  \tag{2.35}\\
& p_{z}=p \sqrt{\cos ^{2} \alpha-\sin ^{2} \beta}, \quad v_{z}=v \sqrt{\cos ^{2} \alpha-\sin ^{2} \beta} \tag{2.36}
\end{align*}
$$

Then, the same mathematical formulations as per the above for [Fig. 5(b)] can be utilized to confirm the same result in [Fig. 5(c)]. Doing so reveals that the total energy of the system calculated as per the above by decomposition into a 3-dimensional system will still be exactly equivalent as to the original energy of the system when not decomposed.

To expand on the above discussions, it is essential to note any given kinetic energy can be attributed a direction. For the above cases, we found that when we consider decompositions of momentum and find the energy in those directions, the total calculated energy is exactly equivalent as to the original. In addition, it can be noted that these components can be added together as scalars and not as vectors. In other words, we can say for example that while energies in the direction of X , Y , or Z may be added together as scalars, they cannot be added together in the same manner as vectors.

Next, the aim will be to study the summation of momentums in a system. In [Fig. 6(a)], considering two momentums $\vec{p}_{1}$ and $\vec{p}_{2}$, the classical representation of the summation of these two vectors is shown and denoted by vector $\vec{p}$. Based on classical physics, the addition of $\vec{p}_{1 x}$ to $\vec{p}_{2 x}$ results in the summation momentum vector $\vec{p}$, while the orthogonal components $\vec{p}_{1 y}$ and $\vec{p}_{2 y}$ are classically stated to "cancel" one another. However, this is, in fact, an incorrect assumption in classical physics. In deriving the classical summation vector $\vec{p}$, the exact velocity of $\vec{p}$ develops as an unknown and thus no energy calculation can be made utilizing it. Therefore, momentum vector $\vec{p}$ is not an accurate representation for the summation of vectors $\vec{p}_{1}$ and $\vec{p}_{2}$.


Fig. 6. (a) The decomposition of momentum vectors $\vec{p}_{1}$ and $\vec{p}_{2}$ into their Cartesian coordinates, and their incorrect classical summation as $\vec{p}$, and (b) the similar decomposition of velocity vectors $\vec{v}_{1}$ and $\vec{v}_{2}$.

The correct summation of the two momentum vectors must be a momentum vector or momentum vectors where their combined energies are equal to the total energy of the two initial momentums. To calculate the energy of momentum (1), $\vec{p}_{1}$, this vector is decomposed into $\vec{p}_{1 x}$ and $\vec{p}_{1 y}$ and its velocity components are depicted as per [Fig. 6(b)]. Similarly, for momentum (2), $\vec{p}_{2}$, this
vector is decomposed into $\vec{p}_{2 x}$ and $\vec{p}_{2 y}$ and its velocity components are utilized for the calculation of its energy. The momentum combination of $\vec{p}_{1}$ plus $\vec{p}_{2}$ can thus correctly be represented by the addition of the linear components $\vec{p}_{1 x}, \vec{p}_{2 x}, \vec{p}_{1 y}$ and $\vec{p}_{2 y}$. These latter two momentums are additional vectors which are here together considered as a centrial momentum as each equal to one another but with the opposite directions. From this decomposition, the momentum and energy of the system in [Fig. 6] can correctly be calculated and understood as follows:

$$
\begin{align*}
& E_{\text {total }}=E_{t}=E_{p 1 x}+E_{p 1 y}+E_{p 2 x}+E_{p 2 y}  \tag{2.37}\\
& \begin{array}{l}
E_{t}=\frac{1}{2} p_{1} \cos \alpha \cdot v_{1} \cos \alpha+\frac{1}{2} p_{1} \sin \alpha \cdot v_{1} \sin \alpha+\frac{1}{2} p_{2} \cos \beta \cdot v_{2} \cos \beta \\
\\
\quad+\frac{1}{2} p_{2} \sin \beta \cdot v_{2} \sin \beta
\end{array}  \tag{2.38}\\
& \begin{array}{l}
E_{t}=\frac{1}{2} p_{1} v_{1}\left(\cos ^{2} \alpha+\sin ^{2} \alpha\right)+\frac{1}{2} p_{2} v_{2}\left(\cos ^{2} \beta+\sin ^{2} \beta\right)
\end{array} \\
& E_{t}=\frac{1}{2} p_{1} v_{1}+\frac{1}{2} p_{2} v_{2} \tag{2.39}
\end{align*}
$$

Therefore, as demonstrated, the combined energies of all components are equivalent to the total energy of the two momentums.

### 2.1.1. Law of motion of momentum

This law states that for an isolated system in the absence of an external force, and where there is no internal released or stored energy on the system, for any considered initial momentum, the sum of the magnitudes of all subsequent momentum(s) decomposed in the direction of the initial momentum will always remain constant and equal to the initial magnitude.

When momentum is conveyed by a body without any action being applied to the body, the direction and the speed of momentum always remain constant. When the initial momentum is partially or totally transferred to or through another body or bodies and assuming that the other body or bodies was/were at rest, the magnitude and direction of summation vector of the final momentum vector(s) is always the same as that of the initial momentum, the total energy of the momentum(s) is conserved, and the speed of the subsequent momentum(s) varies and is dependent on,
a) the speed and the direction of the initial momentum, and
b) the directions of motions of all momentums after the collision, and
c) the mass and rigidity of all bodies.

In other words, the law states that the sum of the magnitude of the transferred momenta plus the magnitude of momentum which remains in the initial body, all with respect to the initial direction remains constant. As such, we can imagine any considered momentum as an entity that always
moves in space either being conveyed by a body(s) or transferred from one body(s) to other(s). This is such that the sum of its constituent vector(s) in the initial direction remains identical to the initial momentum's vector, its energy is conserved, and where its constituent speed(s) may not always remain constant.

Even though momentum is a property of moving body(s) and is dependent on the body(s), we can imagine any momentum as an energy entity. If this energy entity is not partially or totally stored, it can be considered as an independent entity that is always in motion or is being transferred from one or more bodies to another while its magnitude remains constant.

As the magnitude and direction of the mathematically summation of momentum's vector(s) at any point in time are always constant, then,

$$
\begin{equation*}
\sum \vec{p}_{\text {intital }}=\sum \vec{p}_{\text {final }}=\sum \overrightarrow{m v}=\text { Constant } \tag{2.41}
\end{equation*}
$$

Here, $m$ is the mass of body(s) that convey the momentum(s). The mass, $m$, and the speed of momentum, $v$, can be variable while momentum itself in its initial direction is always constant.

### 2.1.2. Law of conservation of momentum with respect to any given direction

For an isolated system in the absence of an external force, and where there is no internal released or stored energy on the system, the sum of the magnitudes of all momentum(s) decomposed in any considered direction remains constant. This is while the sum of the total magnitudes of all subsequent momentum(s) may or may not be conserved. Even though there are no internal released or stored energy on the system, momentum may be created or destroyed.

Having revised the concept of motion in physics, returning to our previous discussions it is found that we need to introduce a new type of motion. This is denoted as "Centrial Motion." The new ideas and concepts are introduced in this paper and are shown to be capable of being thorough in all systems, and being broadly and generally applied than the limited cases studied thus far in classical physics.

### 2.2. Centrial Motion

To begin studying centrial motion, it is helpful to discuss a simple scenario so that we can better determine its basic properties, and derive solutions and answers to which can later be broadly and easily be applied to complex cases. To this end, we begin by presenting a simple scenario as per [Fig. 7(a)]. Here, it is assumed that two bodies (1) and (2) are at rest and positioned together in contact side by side. To make matters simple, these two bodies are considered as two points for the purpose of our analysis. Now if there is a sudden explosion in-between them or say a sudden burst of energy that is solely transferred to both bodies as kinetic energy and none else, it can be concluded that the total gained kinetic energy of both bodies is equal to the energy of the system prior to the explosion. Such an occurrence is henceforth defined and denoted as an "energy blast."

During the energy blast, the speeds of the bodies increased from zero to their final speeds (gained acceleration). Therefore, per Newton's Second Law of Motion, there must have been forces acting on the bodies to achieve this. Following per Newton's Third Law of Motion, it can also be concluded that the two forces acting on these two bodies are equal but in opposite directions. To then determine the initial energy of the system (i.e., energy of the energy blast), we need to find the total kinetic energy of both bodies based on their gained momentums and speeds of momentums.


Fig. 7. (a) Two bodies (1) and (2) in an energy blast scenario and their corresponding force and momentum vectors, (b) three bodies (1), (2) and (3) in an energy blast scenario where $\vec{F}_{2}$ and $\vec{F}_{3}$ are classically considered together as a reaction force to $\vec{F}_{1}$, (c) and the detailed simultaneously action and reaction by $\vec{F}_{2}$ and $\overrightarrow{F^{\prime \prime}}{ }_{3}$ in the three-body energy blast scenario.
In [Fig. 7(a)], if the amount of gained linear momentums of body (1) and (2) are denoted as $p_{1}$ and $p_{2}$ and the amount of their final speeds are denoted as $v_{1}$ and $v_{2}$ (i.e., the speeds of motion of momentums), then the total energy of the system is calculated as follow:

$$
\begin{equation*}
E=E_{1}+E_{2}=\frac{1}{2}\left(p_{1} v_{1}+p_{2} v_{2}\right) \tag{2.42}
\end{equation*}
$$

Next, we consider a scenario where rather than two bodies, there are three bodies in the system. In that case, for analysis of the energy of the whole system in the event of an energy blast, we initially determine the forces being applied to the bodies and the corresponding momentums. Based on classical physics, if we analyze the forces acting on the bodies as the energy blast is being applied to a three-body system, then we can denote and analyze the forces as $\vec{F}_{1}, \vec{F}_{2}$, and $\vec{F}_{3}$ as shown in [Fig. 7(b)]. Thereupon, based on classical physics, we consider and denote $\overrightarrow{F^{\prime}}$ as an opposite and equal reaction of the force to $\vec{F}_{1}$, as per Newton's Third Law of Motion. However, this brings about a conundrum as considering such a force disregards the fact that bodies (2) and (3) do not, in fact, move in the exact direction of $\overrightarrow{F^{\prime}}$. There must also be other forces acting upon these two bodies simultaneously in addition to $\overrightarrow{F^{\prime}}$ to push them apart. In other words, these two bodies are not only acted upon by $\overrightarrow{F^{\prime}}$ but also, they are acted on by two other forces of $\overrightarrow{F "}_{2}$ and $\vec{F}_{3}$ as shown in [Fig. 7(c)]. These two latter forces are equal and in opposite directions to one another and can be in any direction.

For further analysis, it shall be noted that $\overrightarrow{F^{\prime}}$ is acting on bodies (2) and (3) simultaneously. Here, at this point, since the accelerations of the two bodies in direction of $\overrightarrow{F^{\prime}}$ are the same, then, the forces acting on each body, denoted as ${\overrightarrow{F^{\prime}}}_{2}$ and $\overrightarrow{F^{\prime}}{ }_{3}$, are proportional to their masses and the total of these two forces are equivalent to $\overrightarrow{F^{\prime}}$. Again, it must be noted that, in addition to $\overrightarrow{F^{\prime}}$, these two forces, $\vec{F}_{2}$ and $\vec{F}_{3}$, are also simultaneously acting on the two bodies (2) and (3) to push them apart.

Based on this analysis, we note that this breakdown and view of the forces is in-line with reality and what actually takes place. This view of the forces is also well-suited with the Third Law of Motion, assuring us to conduct all studies and analyses correctly. In contrast, only considering the three forces as shown in [Fig. 7(b)] as per classical physics to study these cases would not be the correct view of the reality and it is not compatible with the Third Law of Motion.

Thus, the correct view of the analyze would be as per [Fig. 7(c)], whereby we denote and draw the vectors as shown. In this Figure, we may consider $\vec{F}_{2}$ the vector combination of $\vec{F}_{2}$ and $\vec{F}^{\prime \prime}$, as the total force acting upon body (2) as per the classical physics view. Similarly, we may consider $\vec{F}_{3}$, the vector combination of ${\overrightarrow{F^{\prime}}}_{3}$ and $\vec{F}_{3}$, as the total force acting upon body (3). With the force vectors analyzed as such, it is clear then how Newton's Third Law of Motion is satisfied. While $\vec{F}_{2}$ and $\vec{F}_{3}$ obey Newton's Third Law of Motion and cancel each other out, $\vec{F}_{2}$ and $\vec{F}_{3}$ accumulate as equal and opposite to $\vec{F}_{1}$, appropriately highlighting force cancellation.

By applying a Cartesian coordinate system to the center of the energy blast as show per [Fig. 7(c)], we can decompose $\vec{F}_{2}$ into the two components of $\vec{F}_{2 x}$ and $\vec{F}_{2 y}$. Similarly, we can decompose
$\vec{F}_{3}$ into the two components of $\vec{F}_{3 x}$ and $\vec{F}_{3 y}$. Again, we can analyze the system in this Cartesian coordinate system and derive the same force cancellation results as discussed prior.
[Fig. 7(b)] and [Fig. 7(c)] show that our analysis is derived based on reference to body (1)'s primary axis of movement but it can be shown that the same argument and analysis can be made for bodies (2) and (3) if their axes of movements is considered as the primary axis instead.

To extend our analyses in the cases of more than three bodies, it can be accomplished via one of two methods. The first method is to consider one body and solve for the rest of the bodies modelled as a 3-body system. This modelling technique is repeated for the new group of bodies until there are the final 3-bodies. The second method is to group the bodies into three groups and solve for each group individually. Subsequently, the results can be aggregated and solved for using the same techniques as was presented for the 3-body system.

Next, our focus will be to analyze momentums and their corresponding velocities as per the energy blast scenario presented above. As such, we can now consider [Fig. 8] where the bodies are in motion as shown. It is important to note that with reference to the force decomposition in [Fig. 7 (b)] where the net force of $\vec{F}_{2}$ and $\vec{F}_{3}$ are represented by $\overrightarrow{F^{\prime}}$, it is incorrect to consider the illustrated $\overrightarrow{p^{\prime}}$ opposite to $\vec{p}_{1}$ as the true total manifestation of the momentums of bodies (2) and (3) ([Fig. 8(a)]). Doing so would violate the conservation of energy principles as explained earlier. In other words, the total energy of bodies (2) and (3) cannot be correctly represented and calculated via $\overrightarrow{p^{\prime}}$. Even if velocity was decomposed from known values of $\vec{v}_{2}$ and $\vec{v}_{3}$ onto the $\overrightarrow{p^{\prime}}$ axis, it would still result in an incorrect value for the total energy as the orthogonal values would be missing.

The correct methodology to solve the system is to assign a coordinate system as per [Fig. 8(b)] and [Fig. 8(c)]. Then, in doing so, the velocities and the momentums of bodies (2) and (3) would correspond to our force analysis in [Figure 7(c)]. First, we will focus on the velocities [Fig. 8(b)]. The equal velocities in the direction of $\overrightarrow{F^{\prime}}$, are: ${\overrightarrow{v^{\prime}}}_{2}$ and ${\overrightarrow{v^{\prime}}}_{3}$. As such, bodies (2) and (3) gain velocities simultaneously by a force reaction to $\vec{F}_{1}$. The velocities ${\overrightarrow{v^{\prime}}}_{2}$ and ${\overrightarrow{v^{\prime}}}_{3}$ are found to be equal because as it was noted on discussions of force analyses in [Figure 7(c)], these two bodies are such that $\overrightarrow{F^{\prime}}$ is distributed to the bodies in such a way that they react together and are proportional to their masses, resulting in their equal velocities. In addition, two additional velocity vectors $\overrightarrow{v "}_{2}$ and $\overrightarrow{v "}_{3}$ are simultaneously created at an assumed angle $(\angle \theta)$ with respect to the X axis by forces $\overrightarrow{F "}_{2}$ and $\overrightarrow{F "}_{3}$. The velocity vectors $\overrightarrow{v "}_{2}$ and $\overrightarrow{v "}_{3}$ vary in their magnitudes and are inversely proportional to the masses of the two bodies. It can geometrically be observed that $\vec{v}_{2}$ is the combination of velocity vectors of ${\overrightarrow{v^{\prime}}}_{2}$ and $\overrightarrow{v^{\prime \prime}}{ }_{2}$. Velocity vector $\overrightarrow{v^{\prime \prime}} 2$ is further decomposable into a $\overrightarrow{v^{\prime \prime}}{ }_{2 x}$ component and $\vec{v}_{2 y}$. Then, the addition of ${\overrightarrow{v^{\prime}}}^{\prime}$ and ${\overrightarrow{v^{\prime \prime}}}_{2 x}$ results in the value for $\vec{v}_{2 x}$
as per [Fig. 8(b)]. This can also geometrically be verified as the line segment OB being equal to DE. Similarly, the same arguments can be made for $\vec{v}_{3}$ and its components.


Fig. 8. (a) Classically incorrect view of $\overrightarrow{p^{\prime}}$ as the total manifestation of the momentums of bodies (2) and (3), (b) correct method to solve the velocities by assigning a coordinate system and detailing the components, and (c) the correct method to solve the momentums by applying vector decomposition.

For momentum analyses of this case, we can move on to [Fig. 8(c)] and note that momentums $\overrightarrow{p^{\prime}}{ }_{2}$ and ${\overrightarrow{p^{\prime}}}_{3}$ are created upon an energy blast in opposite to $\vec{p}_{1}$ by forces ${\overrightarrow{F^{\prime}}}_{2}$ and $\vec{F}_{3}$ acting on the bodies proportional to their masses. As a result, ${\overrightarrow{p^{\prime}}}_{2}$ and ${\overrightarrow{p^{\prime}}}_{3}$ are also respectively proportional to the masses of bodies (2) and (3). Furthermore, two additional momentum vectors ${\overrightarrow{p^{\prime \prime}}}_{2}$ and $\overrightarrow{p^{\prime \prime}}{ }_{3}$ are simultaneously created at an assumed angle $(\angle \theta)$ with respect to the X axis by forces $\vec{F}_{2}$ and $\overrightarrow{F^{\prime \prime}}{ }_{3}$.

At the moment of the energy blast, and as these two equal forces act on these bodies simultaneously, they gain identical magnitudes of momentums as per [Fig. 8(c)]. Now the two momentum vectors ${\overrightarrow{p^{\prime \prime}}}_{2}$ and ${\overrightarrow{p^{\prime \prime}}}_{3}$ are in fact components of a centrial momentum. It can geometrically be shown that $\vec{p}_{2}$ is the combination of momentum vectors $\overrightarrow{p^{\prime}}{ }_{2}$ and $\overrightarrow{p^{\prime \prime}}{ }_{2}$. Momentum vector ${\overrightarrow{p^{\prime \prime}}}_{2}$ is further decomposable into a ${\overrightarrow{p^{\prime \prime}}}_{2 x}$ component and $\vec{p}_{2 y}$. Then, the addition of $\overrightarrow{p^{\prime}}{ }_{2}$ and $\overrightarrow{p^{\prime \prime}}{ }_{2 x}$ results in the value for $\vec{p}_{2 x}$ as shown in [Fig. 8(c)]. This can geometrically be seen as line segment OG being equal to HI. Now with $\vec{p}_{2 x}, \vec{v}_{2 x}, \vec{p}_{2 y}, \vec{v}_{2 y}$ known, it can be shown that the total energy of body (2) as per the previous methods used will be the same as if done by $\vec{p}_{2}$ alone. This is because if $\vec{p}_{2}$ is decomposed into the Cartesian X and Y axis, the results will be identical as was demonstrated earlier. Similarly, the same arguments can be made for $\vec{p}_{3}$ and its components. The analysis made above confirms that we can correctly determine the total energy of the system by either a) the magnitudes and the speeds of momentums $\vec{p}_{1}, \vec{p}_{2}$, and $\vec{p}_{3}$ or b) their corresponding components as was just done so prior.

Although we showed from a geometric viewpoint that our analyses thus far are correct, we would like to additionally confirm our results by calculating the total energy of the energy blast based on their component momentums and their corresponding velocities. As is shown in [Fig. 8(c)] the realistic momentums of this scenario are $\vec{p}_{1},{\overrightarrow{p^{\prime}}}_{2},{\overrightarrow{p^{\prime \prime}}}_{2},{\overrightarrow{p^{\prime}}}_{3},{\overrightarrow{p^{\prime \prime}}}_{3}$, and their correspondent velocities are $\vec{v}_{1},{\overrightarrow{v^{\prime}}}_{2},{\overrightarrow{v^{\prime \prime}}}_{2},{\overrightarrow{v^{\prime}}}_{3},{\overrightarrow{v^{\prime \prime}}}_{3}$, as per [Fig. 8(b)]. Therefore, the total energy of the system can be calculated as follows:

$$
\begin{equation*}
E_{t}=\frac{1}{2}\left(p_{1} v_{1}+p_{2}^{\prime} v^{\prime}{ }_{2}+p^{\prime \prime}{ }_{2} v^{\prime \prime}{ }_{2}+p_{3}^{\prime}{ }_{3} v_{3}+p^{\prime \prime}{ }_{3} v^{\prime \prime}{ }_{3}\right) \tag{2.43}
\end{equation*}
$$

As we concluded, the energy of any momentum is always equal to the total energy of its decomposed momentums in any Cartesian system. Now, then, the energy of $\overrightarrow{p^{\prime \prime}}{ }_{2}$ is:

$$
\begin{align*}
& \frac{1}{2} p^{\prime \prime} v v^{\prime \prime}{ }_{2}=\frac{1}{2}\left(p^{\prime \prime}{ }_{2 x} v^{\prime \prime}{ }_{2 x}+p_{2 y} v_{2 y}\right)  \tag{2.44}\\
& p^{\prime \prime}{ }_{2} v^{\prime \prime}{ }_{2}=p^{\prime \prime}{ }_{2 x} v^{\prime \prime}{ }_{2 x}+p_{2 y} v_{2 y} \tag{2.45}
\end{align*}
$$

And for $p^{\prime \prime}{ }_{3}$,

$$
\begin{equation*}
p^{\prime \prime}{ }_{3} v^{\prime \prime}{ }_{3}=p^{\prime \prime}{ }_{3 x} v^{\prime \prime}{ }_{3 x}+p_{3 y} v_{3 y} \tag{2.46}
\end{equation*}
$$

Therefore, the total energy of the system is equal to,

$$
\begin{gather*}
E_{t}=\frac{1}{2}\left(p_{1} v_{1}+p_{2}^{\prime} v_{2}^{\prime}+p^{\prime \prime}{ }_{2 x} v^{\prime \prime}{ }_{2 x}+p_{2 y} v_{2 y}+p_{3}^{\prime} v^{\prime}{ }_{3}\right.  \tag{2.47}\\
\left.+p^{\prime \prime}{ }_{3 x} v^{\prime \prime}{ }_{3 x}+p_{3 y} v_{3 y}\right)
\end{gather*}
$$

Now, based on the above discussions, it is noted that $p^{\prime \prime}{ }_{2 x}=p^{\prime \prime}{ }_{3 x}$ and $v^{\prime}{ }_{2}=v^{\prime}{ }_{3}$. As such, it can be deduced that,

$$
\begin{align*}
& p^{\prime \prime}{ }_{2 x}{v^{\prime}}_{2}=p^{\prime \prime}{ }_{3 x} v^{\prime}{ }_{3}  \tag{2.48}\\
& p^{\prime \prime}{ }_{2 x} v^{\prime}{ }_{2}-p^{\prime \prime}{ }_{3 x} v^{\prime}{ }_{3}=0 \tag{2.49}
\end{align*}
$$

If we denote the masses of bodies (2) and (3), as $m_{2}$ and $m_{3}$ and noting that $p^{\prime \prime}{ }_{2 x}=p^{\prime \prime}{ }_{3 x}$ then,

$$
\begin{equation*}
m_{2} v^{\prime \prime}{ }_{2 x}=m_{3} v^{\prime \prime}{ }_{3 x} \Rightarrow v^{\prime \prime}{ }_{2 x}=\frac{m_{3}}{m_{2}} v^{\prime \prime}{ }_{3 x} \tag{2.50}
\end{equation*}
$$

And also,

$$
\begin{equation*}
p_{2}^{\prime}=m_{2} v^{\prime}{ }_{2} \text { and } p_{3}^{\prime}=m_{3} v^{\prime}{ }_{3} \tag{2.51}
\end{equation*}
$$

And as $v^{\prime}{ }_{2}=v^{\prime}{ }_{3}$ then,

$$
\begin{equation*}
p_{2}^{\prime}=\frac{m_{2}}{m_{3}} p_{3}^{\prime} \tag{2.52}
\end{equation*}
$$

Then, it can be derived that,

$$
\begin{align*}
& {p_{2}^{\prime}}_{2} v^{\prime \prime}{ }_{2 x}=p_{3}^{\prime} v^{\prime \prime}{ }_{3 x}  \tag{2.53}\\
& {p^{\prime}}_{2} v^{\prime \prime}{ }_{2 x}-p^{\prime}{ }_{3} v^{\prime \prime}{ }_{3 x}=0 \tag{2.54}
\end{align*}
$$

Now, we add left-hand side of equations (2.49) and (2.54), (as both are zero-valued), to the righthand side of the of energy equation (2.47) that we obtained earlier. Therefore, we have,

$$
\begin{align*}
E_{t}=\frac{1}{2}\left(p_{1} v_{1}+\right. & p^{\prime}{ }_{2} v^{\prime}{ }_{2}+p^{\prime}{ }_{2} v^{\prime \prime}{ }_{2 x}+p^{\prime \prime}{ }_{2 x}{v^{\prime}}_{2} \\
& +p^{\prime \prime}{ }_{2 x} v^{\prime \prime}{ }_{2 x}+p_{2 y} v_{2 y}+p^{\prime}{ }_{3} v^{\prime}{ }_{3}-p^{\prime}{ }_{3} v^{\prime \prime}{ }_{3 x}-p^{\prime \prime}{ }_{3 x} v^{\prime}{ }_{3}  \tag{2.55}\\
& \left.+p^{\prime \prime}{ }_{3 x} v^{\prime \prime}{ }_{3 x}+p_{3 y} v_{3 y}\right) \\
E_{t}=\frac{1}{2}\left(p_{1} v_{1}+\right. & \left(p_{2}^{\prime}+p^{\prime \prime}{ }_{2 x}\right)\left(v^{\prime}{ }_{2}+v^{\prime \prime}{ }_{2 x}\right)+p_{2 y} v_{2 y}  \tag{2.56}\\
& \left.+\left(p_{3}^{\prime}-p^{\prime \prime}{ }_{3 x}\right)\left(v^{\prime}{ }_{3}-v^{\prime \prime}{ }_{3 x}\right)+p_{3 y} v_{3 y}\right)
\end{align*}
$$

Now referring to the [Fig. 8(b)] and [Fig. 8(c)],

$$
\begin{align*}
& E_{t}=\frac{1}{2}\left(p_{1} v_{1}+\left(p_{2 x}\right)\left(v_{2 x}\right)+p_{2 y} v_{2 y}+\left(p_{3 x}\right)\left(v_{3 x}\right)+p_{3 y} v_{3 y}\right)  \tag{2.57}\\
& E_{t}=\frac{1}{2}\left(p_{1} v_{1}+p_{2 x} v_{2 x}+p_{2 y} v_{2 y}+p_{3 x} v_{3 x}+p_{3 y} v_{3 y}\right)  \tag{2.58}\\
& E_{t}=\frac{1}{2}\left(p_{1} v_{1}+p_{2} v_{2}+p_{3} v_{3}\right) \tag{2.59}
\end{align*}
$$

And this result is what was expected as we obtained the exact same value for the total energy of the system as we would have if it were not decomposed. Ultimately, decomposing the momentum and velocity vectors of the bodies resulted in the same total energy.

In addition, in abiding by the conservation of energy principal, for the purposes of analysis of momentums and velocities, their vectors may be decomposed into their components onto any chosen coordinated system and will always yield the same results regardless of the coordinate system chosen.

As per the analyses above, it has now been shown how we need various approaches to solving the different variable quantities and aspects of motion. Based on these approaches and the in-depth analyses of forces, momenta, and energies completed, we can now begin to define and formulate the basic properties of centrial motion.

### 2.2.1. Centrial Motion Definition

Centrial motion is only defined for cases with two or greater number of bodies. Centrial motion is a motion when the following conditions are met or satisfied:

1. The axis of the linear momentum vector of each body and/or the axis of the vector of summation of linear momentum vectors of each group of bodies under consideration must pass through a point of reference (central point).
2. The summation of considered linear momentums vectors of all bodies and/or the summation of considered linear momentums vectors of group(s) of bodies must be equal to zero.

Centrial motion is defined either "outward" or "inward." If the directions of the vectors of momentums such as described for the conditions of the centrial motion are outward when referenced with respect to the central point, then centrial motion is considered "outward." In contrast, when the directions of the vectors are inward, then centrial motion is considered "inward."

From the above definition and considering the linear momentum vectors in a stationary Cartesian coordinate system, it can be concluded that the sum of all the linear momentums vectors of bodies in a system that is in centrial motion will be equal to zero.

$$
\begin{equation*}
\sum \vec{p}=0 \tag{2.60}
\end{equation*}
$$

If we consider the components of the linear momentum vectors in a Cartesian coordinate system in directions of $\mathrm{X}, \mathrm{Y}$ and Z axes, then,

$$
\begin{equation*}
\Sigma_{X} \vec{p}=0, \Sigma_{Y} \vec{p}=0, \Sigma_{Z} \vec{p}=0 \tag{2.61}
\end{equation*}
$$

However, not every system in which $\sum \vec{p}=0$ can be considered a system in centrial motion. In other words, $\sum \vec{p}=0$ is a resultant of a system in centrial motion. This is a necessary but not a
sufficient condition for a system to be in centrial motion. To be in centrial motion, in addition to $\sum \vec{p}=0$, the system must be verified to satisfy the conditions of centrial motion as defined earlier. For the illustration of what is meant by centrial motion, [Fig. 9] and [Fig. 10] are presented below.


Fig. 9. Centrial motion analysis with the axes of considered linear momentum vectors satisfying the condition of passing through a central point.

It can be seen that [Fig. 9] illustrates centrial motion in a 2D plane, while [Fig. 10] shows centrial motion in 3D space.

Now in [Fig. 9], we can firstly note that the axes of linear momentum vectors and the axes of summations of groups of linear momentum vectors all pass through the central point ( $O$ ). Clearly $\vec{p}_{1}, \vec{p}_{2}, \vec{p}_{5}$ and $\vec{p}_{6}$, all pass through $O \cdot \vec{p}_{3}$ and $\vec{p}_{4}$ are considered together as $\vec{p}_{(3+4)}$ in a vector summation, while $\vec{p}_{7}$ and $\vec{p}_{8}$ are taken as $\vec{p}_{(7+8)}$ and both of these summations are seen as passing through $(0)$. This satisfies the first condition for Centrial Motion. It can also be shown that [Fig. 9] satisfies the second condition for Centrial Motion as the summation of considered linear momentums vectors of all bodies and/or the summation of considered linear momentums vectors of group(s) of bodies is also equal to zero. As per the below;

$$
\begin{equation*}
\vec{p}_{1}+\vec{p}_{2}=0, \quad \vec{p}_{(3+4)}+\vec{p}_{5}=0, \quad \vec{p}_{6}+\vec{p}_{(7+8)}=0 \tag{2.62}
\end{equation*}
$$

Then, $\sum \vec{p}=0$

As a side note, alternatively $\vec{p}_{6}$ along with $\vec{p}_{7}$ and $\vec{p}_{8}$ may be viewed as passing through $O$ and with their linear momentums summating to zero, meeting the condition for centrial motion.

Therefore, this motion can be described as a centrial motion. The directions of all momentum vectors with respect to the central point are outward. Therefore, this is an outward centrial motion. Similarly, in [Fig. 10], we consider a Cartesian coordinate system. Here, it is assumed that all axes of momentum vectors under consideration pass through the coordinate origin. Each momentum vector can be a momentum vector of a body or summation of momentum vectors of a set of bodies under consideration. Here they are denoted as $\vec{p}_{1}, \vec{p}_{2} \ldots$ and $\vec{p}_{n}$ and all momentum vectors are outward.


Fig. 10. Momentum vectors in a centrial motion in 3D space.
[Fig. 10] shows the components of the linear momentum vectors on the $\mathrm{X}, \mathrm{Y}$ and Z axes. In this scenario, if the following conditions are met, then the motion of this system is a centrial motion, and the center of the centrial motion is located at the coordinate origin $O$ :

$$
\begin{gather*}
\sum \vec{p}_{x}=0 \Rightarrow \vec{p}_{1 x}+\vec{p}_{2 x}+\cdots \ldots+\vec{p}_{n x}=0  \tag{2.63}\\
\sum \vec{p}_{y}=0 \Rightarrow \vec{p}_{1 y}+\vec{p}_{2 y}+\cdots \ldots+\vec{p}_{n y}=0  \tag{2.64}\\
\sum \vec{p}_{z}=0 \Rightarrow \vec{p}_{1 z}+\vec{p}_{2 z}+\cdots \ldots+\vec{p}_{n z}=0  \tag{2.65}\\
\sum \vec{p}=0 \tag{2.66}
\end{gather*}
$$

### 2.2.2. Centrial Momentum Definition

Centrial Momentum is defined for a centrial motion, and it has three defining components: a magnitude, a central point, and directionally inward or outward.

The magnitude of Centrial Momentum is hereby denoted by " $Q$." $Q$, the magnitude of a centrial momentum, is the total magnitude of all momentum vectors of bodies and/or group(s) of bodies that are referenced with respect to a central point with the condition described for centrial motion.

If the directions of the vectors of momentums such as described for the conditions of the centrial motion are outward when referenced with respect to the central point, then $Q$ 's directionality is considered outward. In contrast, when the directions of the vectors are inward, then Q's directionality is considered inward.

For the case shown in [Fig. 9], the magnitude of centrial momentum is derived as follows,

$$
\begin{align*}
& Q=p_{1}+p_{2}+p_{(3+4)}+p_{5}+p_{6}+p_{(7+8)}  \tag{2.67}\\
& Q=2 p_{1}+2 p_{5}+2 p_{6}=2\left(p_{1}+p_{5}+p_{6}\right) \tag{2.68}
\end{align*}
$$

And the magnitude of centrial momentum of the case for [Fig. 10] will be:

$$
\begin{equation*}
Q=\sum p=p_{1}+p_{2}+\cdots+p_{n} \tag{2.69}
\end{equation*}
$$

### 2.2.3. Centrial Motions types

### 2.2.3.1. Concentric Centrial Motion

Concentric Centrial Motion is where the start or end of the motions of all considered bodies or the start or end of the motion of their considered effective momentums is a center point (reference point). As such, all momentum vectors are either outward or inward.

### 2.2.3.2. Concentric Bodies Centrial Motion

Concentric Bodies Centrial Motion is the case where, simply, the start or end of the motion of all considered bodies is a centre point (reference point).

### 2.2.3.3. Uniform Centrial Motion

Uniform centrial motion is defined in threefold. Firstly, as a motion where the magnitude of the linear momentum vectors of all bodies or sets of bodies considered for the condition of centrial motion as described above, are equal to each other. Secondly, that those vectors are equally distributed about the center of reference (i.e., at equal angles from one another). And finally, that the direction of all momentum vectors when referenced with respect to the central point are either inward or outward. [Fig. 11] shows an example of a case of uniform centrial motion.


Fig. 11. An example scenario demonstrating a case of Uniform Centrial Motion, where the bodies are distributed uniformly from one another with angle $\alpha$, equal momentums $p$, and where all vectors are directionally outwards.

Here the linear momentum vectors are equally distributed with angle $(\alpha)$ and,

$$
\begin{equation*}
p_{1}=p_{2}=p_{3}=p_{4}=p_{(5+6)}=p_{7}=p_{8}=p_{(9+10+11)}=p \tag{2.70}
\end{equation*}
$$

The magnitude of centrial momentum of this set is derived from the general formula,

$$
\begin{equation*}
Q=\sum_{i=1}^{n} p_{i}=n p \tag{2.71}
\end{equation*}
$$

For [Fig. 11],

$$
\begin{equation*}
Q=8 p \tag{2.72}
\end{equation*}
$$

[Fig. 11] illustrates a uniform centrial motion in a 2D plane. However, a similar system can be extrapolated to a 3D space and studied in a similar manner.

### 2.2.3.4. Symmetrical Centrial Motion:

Symmetrical centrial motion is a motion where there are a number of bodies with equal and uniform mass that are equidistant from one another, with each body having the same distance to the central point of reference, and having the same speeds with respect to that central point. In addition, the directions of all their motions are either inward or outward per [Fig. 12] and [Fig. 13]. By this definition, symmetrical centrial motion is always a concentric and uniform centrial motion.


Fig. 12. A model typifying symmetrical centrial motion. It can be noted that each body is equidistant from each other and has the same speed with respect to the central point of reference.
[Fig. 12] shows symmetrical centrial motion in a 2D plane. However, this Figure can also be extrapolated and utilized for the case of bodies in symmetrical centrial motion in 3D space as shown in [Fig. 13].


Fig. 13. Number of bodies with their axes of motion passing through a central point of reference, and momentum vector axes satisfying the conditions for a symmetrical centrial motion in 3D space.

Here in the case of symmetrical centrial motion, if we represent the number of bodies as $n$, and each body's mass as $m$, and the total masses of all bodies as $M$, and each body's velocity as $\vec{v}$, and that all bodies are moving outward or inward, it can be derived that $\sum \vec{p}=0$. All the conditions for centrial motion as described earlier are met and therefore the system can be
described to be in symmetrical centrial motion. The magnitude of centrial momentum of the system will thus be,

$$
\begin{equation*}
Q=\sum_{i=1}^{n} p_{i}=n m v=M v \tag{2.73}
\end{equation*}
$$

### 2.2.4. Comparison between Outward and Inward Centrial Momentums

For the purpose of comparison of the two types of Centrial Momentums, [Fig. 14] is considered and studied.


Fig. 14. A model depicting motion of four bodies, with two bodies (1) and (2) in outward centrial motion and the other two (3) and (4) in inward centrial motion. The latter two bodies begin with inward centrial motion at time $T=t_{1}$ and transfer their momentums at time $T=t_{2}$ to end in an outward centrial motion at time $T=t_{3}$.

It is assumed that four bodies (1), (2), (3), and (4), whose masses are equal, have the same speed, and also that they are all perfectly elastic (rigid) and in motion in the directions shown. At the time $T=t_{1}$ the speed of bodies (1) and (2) with respect to point $O$ are equal to each other with opposite directions and are denoted by $v$ in [Fig. 14(a)]. Concurrently, the speed of bodies (3) and (4) with respect to point $O^{\prime}$ are also equal to each other with opposite directions and are also denoted by $v$. According to the definitions presented thus far, it can be said that bodies (1) and (2), they have a centrial motion with reference to central point $O$, and similarly for bodies (3) and (4) having centrial motion relative to point $O^{\prime}$. The magnitude of the centrial momentums for both sets is equal to,

$$
\begin{equation*}
Q_{(1,2)}=Q_{(3,4)}=2 m v \tag{2.74}
\end{equation*}
$$

The only difference between these two motions at the time $T=t_{1}$ is that the centrial momentum of the set of bodies (1) and (2) is outward while the centrial momentum of (3) and (4) is inward.

At the time $T=t_{2}$, as shown in [Fig. 14(b)], bodies (3) and (4) collide and at the moment of collision the momentums of the two bodies are transferred. Following this at time $T=t_{3}$, as shown in [Fig. 14(c)], bodies (3) and (4) are moving apart from the center of collision. The above scenario thus demonstrates that after collision, an inward centrial momentum is converted into an outward one. From the model depicted and analyzed in [Fig. 14], it can be concluded that an outward centrial momentum would always remain outward. Eventually, an inward centrial momentum will always convert to an outward one.

### 2.2.5. Study of centrial motion and centrial momentum for the case where two bodies are moving along an axis

We assume that two bodies with masses $m_{1}$ and $m_{2}$ are moving along an axis such that their velocities with respect to a point $O$ are denoted as $\vec{v}_{10}$ and $\vec{v}_{20}$ respectively [Fig. 15].


Fig. 15. Centrial motion of two bodies can be derived with respect to a Point $O^{\prime}$, were the bodies are moving in reference to Point $O$, and where the local reference system at $O$ is in motion with respect to the one at $O^{\prime}$ with velocity $\vec{v}_{O^{\prime}}$.

For this case to be in centrial motion with the point of reference of $O$, it must satisfy that $\sum \vec{p}_{o}=0$. However, if this condition is not met with reference to point $O$, then we can always find a point in which this condition is fulfilled (i.e., that the sum of momentum vectors of all bodies is equal to zero with respect to that point of reference.) Point $O$ can then be referenced with respect to such a new point of reference. In fact, if we consider any set of bodies is moving along an axis the same as that of its initial motion and with respect to a new coordinate system with velocity $\vec{v}_{O^{\prime}}$ as per [Fig. 15], then its center of centrial motion can be found. It does not matter if we consider the motion of the set of bodies with respect to the new coordinate center $O^{\prime}$, or whether we consider the motion of the new coordinate center with respect to point $O$. Here we assume that the new coordinate system is fixed and we consider the motion of the set of bodies with respect to the new coordinate center, $O^{\prime}$. Therefore, the motions of bodies with respect to this new coordinate center are henceforth considered. As such, per [Fig. 15] the velocities of the bodies are considered with respect to point $O^{\prime}$. In order for any point like point $O^{\prime}$ to be the center of the centrial motion, in
addition to the condition that the axes of all momentum's vectors must pass through that point, one other condition must also be met, $\sum \vec{p}_{o^{\prime}}=0$

If the velocity of point $O$ with respect to $O^{\prime}$ is denoted as $\vec{v}_{O^{\prime}}$, then,

$$
\begin{align*}
& \sum \vec{p}_{o^{\prime}}=m_{1}\left(\vec{v}_{1 o}+\vec{v}_{O^{\prime}}\right)+m_{2}\left(\vec{v}_{2 o}+\vec{v}_{O^{\prime}}\right)=0  \tag{2.75}\\
& m_{1} \vec{v}_{1 o}+m_{2} \vec{v}_{2 o}+\left(m_{1}+m_{2}\right) \vec{v}_{O^{\prime}}=0 \tag{2.76}
\end{align*}
$$

$$
\begin{equation*}
\vec{v}_{o^{\prime}}=-\frac{m_{1} \vec{v}_{1 o}+m_{2} \vec{v}_{2 o}}{m_{1}+m_{2}} \tag{2.77}
\end{equation*}
$$

Now, the magnitude of the centrial momentum can be calculated. $Q$ is equal to the total magnitude of linear momentums of bodies with respect to the new reference point, i.e., $O^{\prime}$. Thus,
since, $\left|\vec{v}_{1 o}-\vec{v}_{2 o}\right|=\left|\vec{v}_{2 o}-\vec{v}_{1 o}\right|$, then,

$$
\begin{equation*}
Q=\frac{2 m_{1} m_{2}}{m_{1}+m_{2}}\left|\vec{v}_{1 o}-\vec{v}_{2 o}\right| \tag{2.82}
\end{equation*}
$$

If two bodies have the same speed and are moving in the same direction, then $\left|\vec{v}_{1 o}-\vec{v}_{2 o}\right|=0$, and therefore $Q=0$. In other words, there is no centrial motion for this special condition.

Eq. (2.82) shows that the magnitude of the centrial momentum of two bodies that are moving along a straight axis can be calculated without the need to find the center of the centrial motion. This implies that simply using the bodies' velocities with respect to the reference point is sufficient.

In Eq. (2.79) If $m_{1} \vec{v}_{1 o}+m_{2} \vec{v}_{2 o}=0$ then,

$$
\begin{align*}
& Q=m_{1}\left|\vec{v}_{1 o}-0\right|+m_{2}\left|\vec{v}_{2 o}-0\right|=m_{1} v_{1 o}+m_{2} v_{2 o}  \tag{2.83}\\
& Q=2 m_{1} v_{1 o}=2 m_{2} v_{2 o} \tag{2.84}
\end{align*}
$$

$$
\begin{align*}
& Q=m_{1}\left|\vec{v}_{1 o}+\vec{v}_{O^{\prime}}\right|+m_{2}\left|\vec{v}_{2 o}+\vec{v}_{O^{\prime}}\right|  \tag{2.78}\\
& Q=m_{1}\left|\vec{v}_{1 o}-\frac{m_{1} \vec{v}_{1 o}+m_{2} \vec{v}_{2 o}}{m_{1}+m_{2}}\right|+m_{2}\left|\vec{v}_{2 o}-\frac{m_{1} \vec{v}_{1 o}+m_{2} \vec{v}_{2 o}}{m_{1}+m_{2}}\right|  \tag{2.79}\\
& Q=m_{1}\left|\frac{m_{1} \vec{v}_{1 o}+m_{2} \vec{v}_{1 o}-m_{1} \vec{v}_{1 o}-m_{2} \vec{v}_{2 o}}{m_{1}+m_{2}}\right| \\
& +m_{2}\left|\frac{m_{1} \vec{v}_{2 o}+m_{2} \vec{v}_{2 o}-m_{1} \vec{v}_{1 o}-m_{2} \vec{v}_{2 o}}{m_{1}+m_{2}}\right|  \tag{2.80}\\
& Q=\frac{m_{1}}{m_{1}+m_{2}}\left|m_{2}\left(\vec{v}_{1 o}-\vec{v}_{2 o}\right)\right|+\frac{m_{2}}{m_{1}+m_{2}}\left|m_{1}\left(\vec{v}_{2 o}-\vec{v}_{1 o}\right)\right| \tag{2.81}
\end{align*}
$$

### 2.2.6. Study of the speed of the motion of momentum in centrial motion

A case of centrial motion is studied here where only two perfectly elastic (rigid) bodies (1) and (2) are considered [Fig. 16]. The Figure illustrates the motions of the bodies before and after a headon collision.


Fig. 16. Analyses of before and after a collision scenario with initial momentums $\overrightarrow{m_{1} v_{1 i}}$ and $\overrightarrow{m_{2} v_{2 i}}$, where the final momentums remain constant and are traveling in the same direction with the only difference being which of the two bodies is transferring each of the momentums.

The study here is intended to analyze the speed of motion of momentum (speed of momentum) in a centrial motion case before and after the collision. The momentums of bodies (1) and (2) before the collision is $\overrightarrow{m_{1} v_{1 i}}$ and $\overrightarrow{m_{2} v_{2 i}}$, respectively. According to the definition of the speed of momentum, the speed of momentum of body (1) before the collision is equal to $\left(v_{1 i}\right)$ and its direction is to the right. And the speed of momentum of body (2) is equal to $\left(v_{2 i}\right)$ and its direction is to the left. If this motion is a centrial motion, then, $\left(\sum \overrightarrow{m v}=0\right)$. Therefore,

$$
\begin{equation*}
\overrightarrow{m_{1} v_{1 i}}+\overrightarrow{m_{2} v_{2 i}}=0 \quad \Rightarrow \quad \vec{v}_{2 i}=-\frac{m_{1}}{m_{2}} \vec{v}_{1 i} \tag{2.85}
\end{equation*}
$$

And after collision,

$$
\begin{align*}
& \overrightarrow{m_{1} v_{1 f}}+\overrightarrow{m_{2} v_{2 f}}=0 \quad \Rightarrow \quad \vec{v}_{2 f}=-\frac{m_{1}}{m_{2}} \vec{v}_{1 f}  \tag{2.86}\\
& \frac{v_{2 i}}{v_{2 f}}=\frac{v_{1 i}}{v_{1 f}} \tag{2.87}
\end{align*}
$$

From the conservation of energy, the initial kinetic energy is equal to the final, and therefore,

$$
\begin{align*}
& E_{k i}=E_{k f} \Rightarrow \frac{1}{2} m_{1} v_{1 i}^{2}+\frac{1}{2} m_{2} v_{2 i}^{2}=\frac{1}{2} m_{1} v_{1 f}^{2}+\frac{1}{2} m_{2} v_{2 f}^{2}  \tag{2.88}\\
& m_{1} v_{1 i}^{2}+m_{2}\left(-\frac{m_{1}}{m_{2}} v_{1 i}\right)^{2}=m_{1} v_{1 f}^{2}+m_{2}\left(-\frac{m_{1}}{m_{2}} v_{1 f}\right)^{2}  \tag{2.89}\\
& m_{1} v_{1 i}^{2}+\frac{m_{1}^{2}}{m_{2}} v_{1 i}^{2}=m_{1} v_{1 f}^{2}+\frac{m_{1}^{2}}{m_{2}} v_{1 f}^{2}  \tag{2.90}\\
& \left(m_{1}+\frac{m_{1}^{2}}{m_{2}}\right) v_{1 i}^{2}=\left(m_{1}+\frac{m_{1}^{2}}{m_{2}}\right) v_{1 f}^{2} \Rightarrow\left|v_{1 f}\right|=\left|v_{1 i}\right| \Rightarrow \tag{2.91}
\end{align*}
$$

As the vector direction of $\vec{v}_{1 f}$ and $\vec{v}_{1 i}$ are opposite to one another,

$$
\begin{array}{r}
v_{2 i} \\
v_{2 f} \\
\vec{v}_{1 f}=-\vec{v}_{1 i}  \tag{2.94}\\
v_{1 f}
\end{array} \frac{v_{2 i}}{v_{2 f}}=-1 \Rightarrow v_{2 f}=-v_{2 i} \Rightarrow
$$

This scenario can be viewed with a focus on momentum transfer. Analyzing it as so, it can be seen that the initial momentum of body (1) moves to the right prior to the collision. After the collision, it continues to move in the same direction and the only difference is that this momentum is embodied by body (2). The vice versa can be stated for the initial momentum of body (2). From a momentum perspective, it can be concluded that momentum as an entity itself always moves in the same initial direction.

From the equations derived earlier and with reference to [Fig. 16], it is thus determined that:

$$
\begin{equation*}
\overrightarrow{m_{2} v_{2 f}} \equiv \overrightarrow{m_{1} v_{1 i}} \tag{2.95}
\end{equation*}
$$

This equivalency is signifying that the initial momentum of body (1) which had the speed of $v_{1 i}$ before the collision, is in motion with the same magnitude after the collision but with the speed of $v_{2 f}$ via body (2).

Similarly,

$$
\begin{equation*}
\overrightarrow{m_{1} v_{1 f}} \equiv \overrightarrow{m_{2} v_{2 i}} \tag{2.96}
\end{equation*}
$$

The equivalency above is signifying that the initial momentum of body (2) which had the speed of $v_{2 i}$ before the collision, is in motion after the collision with the same magnitude but with the speed of $v_{1 f}$ via body (1).

In other words, it can be concluded here in a centrial motion that after the collision, the directions of motions of the bodies have been reversed, while the amounts and directions of each of the initial momentums ( $m_{1} v_{1 i}$ and $m_{2} v_{2 i}$ ) have not changed. Only the speeds of the motions of momentums have changed after collision.

Now per the results and conclusions of the above discussions, the head-on collisions of perfectly elastic bodies typically studied in classical physics books, is studied and solved here using centrial motion concepts and equations.


Fig. 17. Case study of a head-on collision of perfectly elastic bodies (1) and (2) using centrial motion concepts presented in this paper.

For the purposes of the study, [Fig. 17] illustrates two bodies (1) and (2) with masses $m_{1}$ and $m_{2}$ with initial velocities $\vec{v}_{1 i}$ and $\vec{v}_{2 i}$ with respect to $O$. The final velocities of bodies after collision are to be determined. To study such motions, a coordinate origin $O^{\prime}$ based on which the conditions of centrial motion are satisfied is considered. Then with reference to Eq. (2.77), the velocity of the set of bodies with respect to the new coordinate center, $O^{\prime}$, is:

$$
\begin{equation*}
\vec{v}_{O^{\prime}}=-\frac{m_{1} \vec{v}_{1 i}+m_{2} \vec{v}_{2 i}}{m_{1}+m_{2}} \tag{2.97}
\end{equation*}
$$

The velocity of body (1) with respect to the new origin before collision is,

$$
\begin{equation*}
\vec{v}_{1 i O^{\prime}}=\vec{v}_{1 i}+\vec{v}_{0^{\prime}} \tag{2.98}
\end{equation*}
$$

and the velocity of body (1) with respect to the new origin after collision is,

$$
\begin{equation*}
\vec{v}_{1 f O^{\prime}}=\vec{v}_{1 f}+\vec{v}_{O^{\prime}} \tag{2.99}
\end{equation*}
$$

Now, as a result of considering $O^{\prime}$ as a new coordinate origin that fulfils the requirements for a case of centrial motion, then it can be stated that,

$$
\begin{align*}
& \vec{v}_{1 f O^{\prime}}=-\vec{v}_{1 i O^{\prime}}  \tag{2.100}\\
& \vec{v}_{1 f}+\vec{v}_{O^{\prime}}=-\vec{v}_{1 i}-\vec{v}_{O^{\prime}} \quad \Rightarrow \quad \vec{v}_{1 f}=-\vec{v}_{1 i}-2 \vec{v}_{O^{\prime}}  \tag{2.101}\\
& \vec{v}_{1 f}=-\vec{v}_{1 i}+\frac{2 m_{1} \vec{v}_{1 i}+2 m_{2} \vec{v}_{2 i}}{m_{1}+m_{2}} \tag{2.102}
\end{align*}
$$

$$
\begin{array}{r}
\vec{v}_{1 f}=\frac{-m_{1} \vec{v}_{1 i}-m_{2} \vec{v}_{1 i}+2 m_{1} \vec{v}_{1 i}+2 m_{2} \vec{v}_{2 i}}{m_{1}+m_{2}} \\
=\frac{\left(m_{1}-m_{2}\right) \vec{v}_{1 i}+2 m_{2} \vec{v}_{2 i}}{m_{1}+m_{2}} \\
\quad \vec{v}_{1 f}=\frac{m_{1}-m_{2}}{m_{1}+m_{2}} \vec{v}_{1 i}+\frac{2 m_{2}}{m_{1}+m_{2}} \vec{v}_{2 i} \tag{2.104}
\end{array}
$$

To find the final velocity of body (2):

$$
\begin{gather*}
\vec{v}_{2 f O^{\prime}}=-\vec{v}_{2 i o^{\prime}}  \tag{2.105}\\
\vec{v}_{2 f}+\vec{v}_{O^{\prime}}=-\vec{v}_{2 i}-\vec{v}_{o^{\prime}} \quad \Rightarrow \quad \vec{v}_{2 f}=-\vec{v}_{2 i}-2 \vec{v}_{O^{\prime}}  \tag{2.106}\\
\vec{v}_{2 f}=-\vec{v}_{2 i}+\frac{2 m_{1} \vec{v}_{1 i}+2 m_{2} \vec{v}_{2 i}}{m_{1}+m_{2}}  \tag{2.107}\\
\vec{v}_{2 f}=\frac{-m_{1} \vec{v}_{2 i}-m_{2} \vec{v}_{2 i}+2 m_{1} \vec{v}_{1 i}+2 m_{2} \vec{v}_{2 i}}{m_{1}+m_{2}} \\
=\frac{2 m_{1} \vec{v}_{1 i}+\left(m_{2}-m_{1}\right) \vec{v}_{2 i}}{m_{1}+m_{2}}  \tag{2.108}\\
\quad \vec{v}_{2 f}=\frac{2 m_{1}}{m_{1}+m_{2}} \vec{v}_{1 i}+\frac{m_{2}-m_{1}}{m_{1}+m_{2}} \vec{v}_{2 i} \tag{2.109}
\end{gather*}
$$

Eq. (2.104) and Eq. (2.109) are equations that have been previously arduously obtained by classical physics. Here they have been simply obtained using centrial motion concepts in a much more efficient manner.

### 2.3. Law of conservation of centrial momentum

For an isolated system in the absence of an external force, and where there is no internal released or stored energy on the system, centrial momentum is always conserved.

For any centrial momentum, we can define a magnitude and an energy value where both are conserved.
$Q=C=$ Conserved
$E=C=$ Conserved
As such, we can view any centrial momentum as possessing a certain amount of energy. In addition, centrial momentum can be viewed as having its own center of reference and advancing concentrically outwards or inwards.

It is noted where centrial momentum is always conserved, linear momentum may or may not be conserved.

## 3. Conclusion

In this paper, along with the revisions of the concepts and definitions for the laws of motion, certain motions were identified and presented as being in "centrial motion." It was demonstrated that there is a need to derive and develop new concepts and equations for "centrial motion" and "centrial momentum."

With the analyses conducted, it is concluded that the magnitude of any linear momentum under consideration remains conserved along its initial direction, but only under certain pre-conditions. It was thus shown that linear momentum may not always be conserved. This is in contrast to energy, which always remains conserved and constant in an isolated system. While total linear momentum taken along its initial direction remains conserved, the speed of the momentum can vary from body to body that convey or transfer this momentum. As such, while linear momentum may have varying speeds, its energy is always constant. This concept is true for all momentum types (e.g., linear, angular, and centrial momentum). It was also shown that momentum as an independent entity can be viewed as a distinct concept or entity from mass.

Furthermore, as momentum always has an intrinsic speed, we can imagine any considered momentum as an entity that always moves in space. Even though momentum is a property of moving objects and is dependent on the objects, it be can imagined as an energy entity that is always in motion or is being transferred from one or more bodies to others while its energy magnitude remains constant.

It was also shown that while linear momentum may not always be conserved, centrial momentum does always remain conserved.

There are numerous types of motions and phenomena that can be analyzed and interpreted using these laws of centrial motion. For instance, in the universe, there are phenomena where the motions of constituents originate from a center and spread in a spherical form throughout space. As such, the concepts and ideas presented in this paper can be applied to cosmological and astrological phenomena and aid in the resolution of many questions. Explosions of small or massive objects in the universe can also be studied and analyzed using concepts of centrial motion and centrial momentum. As it has previously been mentioned in my last article [4], M particles are located throughout the universe. As described in that paper, the impacts and interactions of nuclear particles of atoms and molecules of any object cause the M particles to vibrate around that object. These vibrations cause the other $M$ particles in space to gain vibrations and consequently momentums in a spherical manner. The centrial motion concepts derived here in this current paper can then be used for analyzing such phenomena. Another application is the effect of spherical vibrations around massive objects causing variations in the densities of $M$ particles. This aids in
the study of the corresponding deflection of the angle of the light in the vicinity of such massive objects.

As a final point of note, it is worth acknowledging the relevance of the concepts of centrial motion and centrial momentum introduced in this paper to the engineering sciences as there are significant cases where these concepts are applicable.

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## Conflict of Interest

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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