A DARK MATTER THEORY BY QUANTUM GRAVITATION FOR GALAXIES AND CLUSTERS

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1. ABSTRACT

This paper develops an original theory of dark matter in the current ΛCDM framework, whose main hypothesis is that DM is generated by the own gravitational field, according an unknown quantum gravitational phenomenon. This work is the best version of the theory, which I have been developing and publishing since 2013.

The hypothesis of DM by quantum gravitation, DMbQG hereafter, has two main consequences: the first one is that the law of DM generation has to be the same, in the halo region, for all the galaxies and the second one is that the haloes are unbounded, so the total DM goes up without limit as the gravitational field is unbounded as well. The first one consequence is backed by the fact that M31 and MW has a fitted function with the same power exponent for the rotation curve at the halo region and both giant galaxies are the only ones whose rotation curves at the halo region may be studied with accuracy.

This work has a newness regarding previous papers: it is demonstrated that the Bernoulli mass formula becomes a more simple formula, called direct mass, when it is considered as initial point, a point belonging to the function fitted associated to the rotation curve at the halo region.

In this paper is firstly developed all the theory with M31 rotation curve data up to the chapter 10. The chapter 11 is dedicated to apply the theory to Milky Way and precisely its direct mass formula is tested successfully using the data published at different radius by two researcher teams.

In the chapter 12 is calculated the direct mass for the Local Group, and it is shown how the DMbQG theory is the only one able to calculate the total mass at 770 kpc, that the dynamical methods estimate to be $5\cdot10^{12}\ M_\Theta$.

In the chapter 13 it is shown a method to estimate the Direct mass formula for a cluster of galaxies, using only its virial mass and radius. By this method it is estimated the parameter a^2 of the L.G. which match with the one calculated in previous chapter by a different method. Also are calculated the parameters a^2 associated to Virgo and Coma clusters.

In the chapter 14 it is demonstrated how the DE is able to counterbalance the DM at cluster scale, as the Direct mass grows up with the square root of radius whereas the DE grows up with the cubic power. The chapter is an introduction to the DMbQG theory for cluster of galaxies, which has been developed by the author in other previous works.

This theory aims to be a powerful method to study DM in the halo region of galaxies and cluster of galaxies and conversely the measures in galaxies and clusters offer the possibility to validate the theory.

2. INTRODUCTION

Since 2013 up to 2019 I have published several papers studying DM in galactic halos, especially in M31 and Milky Way although also I have published some papers studying other galaxies and cluster, see bibliography.

As reader knows M31 is the twin galaxy of Milky Way in the Local Group of galaxies. According [5] Sofue, Y. 2015 its baryonic masses are $M_{BARYONIC\text{-}M31} = 1.61\cdot10^{11}\ M_\Theta$ and $M_{BARYONIC\text{-}MILKY\ WAY} = 1.4\cdot10^{11}\ M_\Theta$ where M_Θ is the Solar mass equal to 1.99E30 kg .

The DM by Quantum Gravitation, DMbQG hereafter, theory was introduced in [1] Abarca, M.2014. *Dark matter model by quantum vacuum*. It considers that DM is generated by the own gravitational field. In order to study purely the phenomenon it is needed to consider a radius dominion where it is supposed that baryonic matter is negligible. i.e. radius bigger than 30 kpc for MW and 40 kpc for M31, as it will be shown in chapter 6.

This hypothesis has two main consequences: the first one is that the law of dark matter generation, in the halo region, has to be the same for all the galaxies and the second one is that the haloes are unlimited so the total dark matter goes up without limit, in the chapter 14 will be solved this apparent divergence of the total mass, thanks to the DE.

The first consequence before mentioned, dark matter generated by a Universal law, has been studied by all my papers, especially inside M31 and Milky Way thanks the remarkable data of rotation curves published in papers [5] Sofue, Y.2015 and [6] Sofue, Y.2020.

In fact I could develop rigorously the theory because the rotation curve of M31 at halo region decreased with a power regression fitted curve whose exponent is -1/4. However with data published for Milky Way at the same paper (2015) it was not possible to fit rigorously the rotation curve with such exponent.

Fortunately, in a new paper [6] Sofue,Y.2020, the author gives a new rotation curve data for Milky Way at halo region whose fitted curve has an exponent -1/4. Such result was good news for DM by gravitation theory, because the theory states a universal law of DM generation in the halo region of galaxies or clusters.

In this paper it is firstly developed all the theory with M31 rotation curve data up to the chapter 10 and the chapter 11 is dedicated to apply the theory to Milky Way and also is validated successfully its Direct mass formula with published results of masses at different radiuses.

In the chapter 12 is estimated the total mass of L.G. by the Direct mass, considering MW, M31 and its main galaxy satellites: the Large Magellan Cloud and M33 respectively. The total mass calculated for MW, LMC, M31 and M33 is $4{,}97\cdot10^{12}\ M_\Theta$. The mass at 770 kpc for MW and M31 is accepted to be $5\cdot10^{12} M_\Theta$. See [16] Azadeh Fattahi, Julio F. Navarro.2020. So it is possible to state that both results match perfectly. The importance of these findings is high because there is not any other theory able to explain such amount of mass offering a physic nature of dark matter.

In the chapter 13 is shown a method to calculate the Direct mass in clusters using the virial mass and radius.

Finally in the chapter 14 is introduced the DMbQG theory in clusters, specifically it is shown how the Dark Energy is able to counter balance the DM at cluster scale. In the paper [3] Abarca,M.2024 it is developed the DMbQG theory in clusters with remarkable theoretical findings validated with published measures in clusters.

As I have mentioned before, this theory has been developed assuming the hypothesis that DM is a quantum gravitational effect. However, it is possible to remain into the Newtonian framework to develop the theory. In my opinion there are two factors to manage the DM conundrum with a quite simple theory.
The first one, that it is developed into the halo region, where baryonic matter is negligible. The second one, that the mechanics movements of celestial bodies are very slow regarding velocity of light, which is supposed to be the speed of gravitational bosons.

It is known that community of physics is researching a quantum gravitation theory since many years ago, but does not exist yet, however I think that my works in this area support strongly that DM is a quantum gravitation phenomenon.

Use a more simple theory instead the general theory is a typical procedure in physics.
For example the Kirchhoff ´s laws are the consequence of Maxwell theory for direct current and remain valid for alternating current, introducing complex impedances, on condition that signals must have low frequency. However these laws do not work for electromagnetic microwaves because of its high frequency.

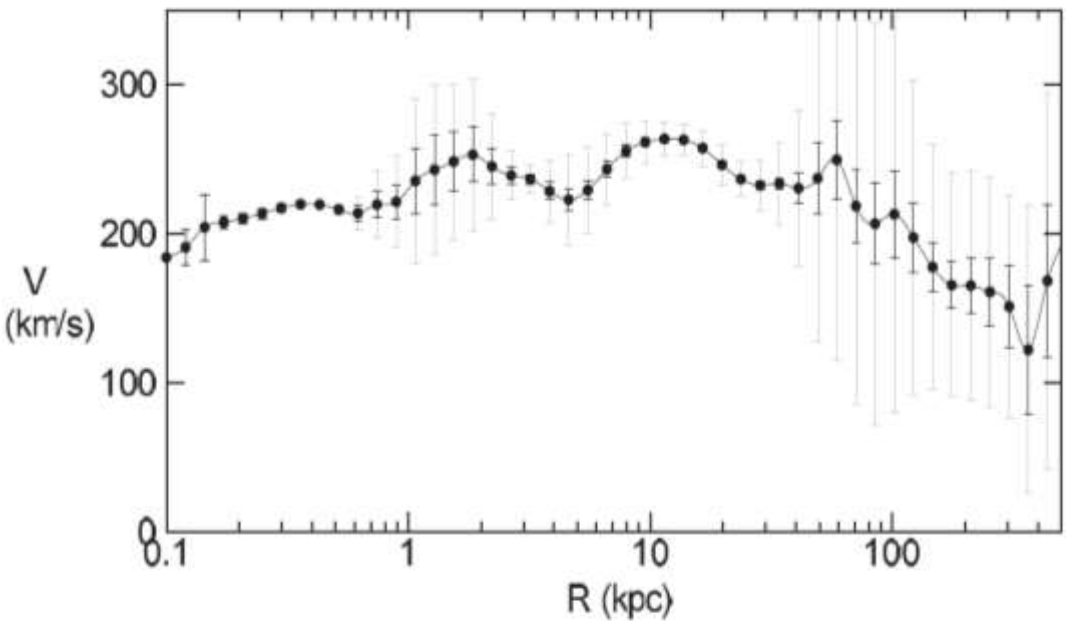
Thanks the possibility to study the gravitational effect of DM pure, in halo regions of M31 and MW, it have been possible to develop a theory mathematically simple. When baryonic mass is mixed with dark matter as it happens inside the galactic disc the mathematical treatment is by far more complex.

The coincidence of the same exponent to the fitted function for the rotation curves at the halo region for both galaxies is crucial in order to state that dark matter is generated according an Universal law because MW and M31 are the only giant galaxies whose rotation curve at the halo region are feasible to be studied with accuracy.

3. OBSERVATIONAL DATA FOR M31 GALAXY FROM SOFUE. 2015 DATA

Graphic comes from [5] Sofue,Y. 2015. The axis for radius has logarithmic scale. Although Sofue rotation curve ranges from 0.1 kpc up to 352 kpc the range of dominion considered for this work is only the halo region where ratio baryonic matter is negligible. In chapter 6, will be shown that this happens for radius bigger than 40 kpc, despite the fact that disc radius for M31 is accepted to be 35 kpc.

Table 1	
kpc	km/s
40,5	229,9
49,1	237,4
58,4	250,5
70,1	219,2
84,2	206,9
101,1	213,5
121,4	197,8
145,7	178,8
175	165,6
210,1	165,6
252,3	160,7
302,9	150,8



The measure at 352 kpc has been rejected because has a velocity too high, so does not match with the other measures. May be an celestial object captivated by the gravitational field of M31 afterwards to M31 formation and it is right to think that it is not in dynamical equilibrium with M31.So it is a good criteria to consider a data set with high statistical correlation.

3.1 POWER REGRESSION TO THE ROTATION CURVE

The measures of rotation curve have a very good fitted curve by power regression.

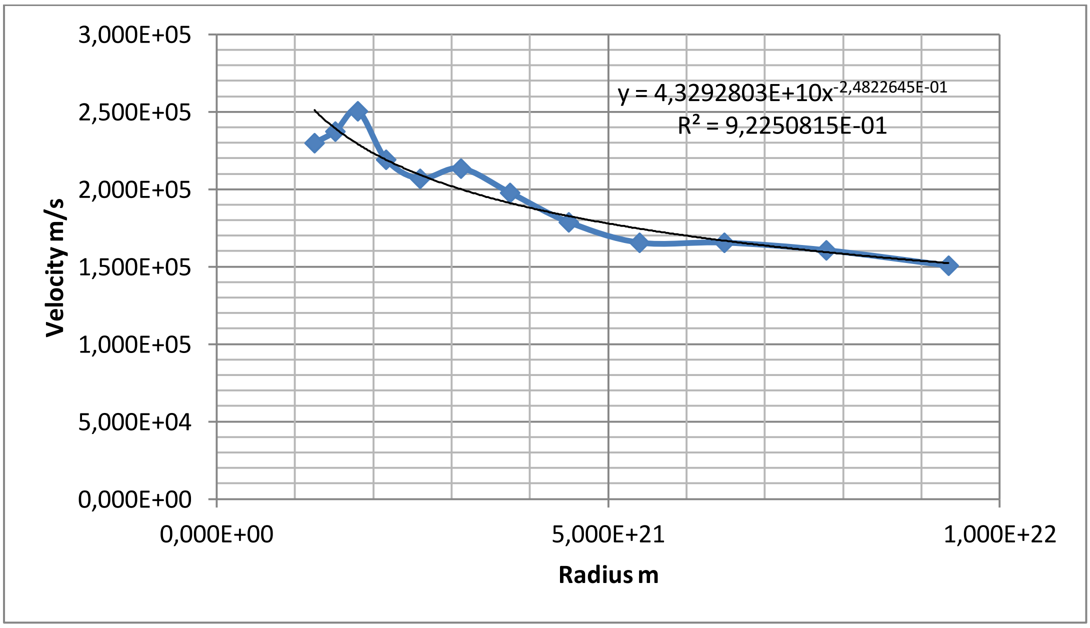
Table 2 -Power regression for M31 rot. curve	
$V = a \cdot r^b$	
a	$4.32928 \cdot 10^{10}$
b	-0.24822645
Correlation coeff.	0.96

In particular coefficients of $v = a \cdot r^b$ are in table below. Units are into I.S.

Table 3 Relative difference Fitted velocity curve and measures vel.					
Radius kpc	Vel. km/s	Radius m	Vel. m/s	Vel. fitted	Relative Diff.
40,5	229,9	1,250E+21	2,299E+05	2,510E+05	8,397E-02
49,1	237,4	1,515E+21	2,374E+05	2,393E+05	7,777E-03
58,4	250,5	1,802E+21	2,505E+05	2,292E+05	-9,304E-02
70,1	219,2	2,163E+21	2,192E+05	2,190E+05	-8,154E-04
84,2	206,9	2,598E+21	2,069E+05	2,093E+05	1,138E-02
101,1	213,5	3,120E+21	2,135E+05	2,000E+05	-6,755E-02
121,4	197,8	3,746E+21	1,978E+05	1,911E+05	-3,500E-02
145,7	178,8	4,496E+21	1,788E+05	1,826E+05	2,107E-02
175	165,6	5,400E+21	1,656E+05	1,745E+05	5,115E-02
210,1	165,6	6,483E+21	1,656E+05	1,668E+05	7,100E-03
252,3	160,7	7,785E+21	1,607E+05	1,594E+05	-8,307E-03
302,9	150,8	9,347E+21	1,508E+05	1,523E+05	9,891E-03

Data fitted are in grey columns below. In fifth column is shown results of fitted velocity curve and sixth column shows relative difference between measures and fitted results.

Below is shown a graphic with measures data and power regression function.



The correlation coefficient is 0.96 which is a superb result especially when dominion measures are up to 303 kpc. There is not any other galaxy with a rotation curve data set so wide.

According the theory of DM generated by gravitational field, the galaxy halo is unbounded, but as the distance MW and M31 is 770 kpc, it is right to consider that up to 300 kpc from M31 it is this galaxy which dominates the gravitational field, because the gravitational influence of MW in that region is lower. Obviously is not negligible but in this paper the gravitational field is studied using the most simple model with spherical symmetry.

4. DIRECT FORMULA FOR DM DENSITY ON M31 HALO GOT FROM ROTATION CURVE

4.1 GETTING DIRECT DM DENSITY FROM NEWTONIAN DYNAMICS

Outside disk region, rotation curve it is fitted by power regression with a high correlation coefficient according formula $v = a \cdot r^{b}$. As $M_{DYNAMIC}(<r) = \frac{v^2 \cdot R}{G}$ represents total mass enclosed by a sphere with radius r, by substitution of velocity results $M = \frac{v^2 \cdot R}{G} = \frac{a^2 \cdot r^{2b+1}}{G}$. Hereafter this formula will be called Direct Mass $M_{DIRECT}(<r) = \frac{a^2 \cdot r^{2b+1}}{G}$ because it has been got rightly from rotation curve.

If it is considered outside region of disk where baryonic matter is negligible regarding dark matter it is possible to calculate DM density by a simple derivative. In the next chapter will be show that for r > 40 kpc baryonic matter is negligible.

As density of D.M. is $D_{DM} = \frac{dm}{dV}$ where $dm = \frac{a^2 \cdot (2b+1) \cdot r^{2b} dr}{G}$ and $dV = 4\pi r^2 dr$ results $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$. Writing $L = \frac{a^2 \cdot (2b+1)}{4\pi G}$ results $D_{DM}(r) = L \cdot r^{2b-2}$. If b = -1/2 then DM density is cero which is the Keplerian rotation.

4.2 DIRECT DM DENSITY FOR M31 HALO

Parameters a & b from power regression of M31 rotation curve allow calculate easily direct DM density

Direct DM density for M31 halo 40 < r < 300 kpc
$D_{DM}(r) = L \cdot r^{2b-2}$ kg/m^3 being L= 1.1255E+30 and 2b-2= -2.4964529

It is important to highlight that at this moment this formula is only a statistical approximation of DM density able to explain the rotation curve, without any physic meaning.

5. DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD E

As independent variable for this function is E, gravitational field, previously will be studied formula for E in the following paragraph.

5.1 GRAVITATIONAL FIELD E BY THE VIRIAL THEOREM

As it is known total gravitational field may be calculated through Virial theorem, formula $E = v^2/R$ whose I.S. unit is m/s^2 is well known. Hereafter, Virial gravitational field, E, got through this formula will be called E.

The key to state the Virial theorem is the dynamical equilibrium. It is supposed that celestial bodies are quite close to dynamical equilibrium, because the most of celestial bodies belong to a specific galactic system from its formation times.

By substitution of $v = a{\cdot}r^b$ in formula $E = \frac{v^2}{r}$ it is right to get $E = \frac{a^2{\cdot}r^{2b}}{r} = a^2{\cdot}r^{2b-1}$ briefly $E = a^2{\cdot}r^{2b-1}$

5.2 DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

According hypothesis dark matter by quantum vacuum $D_{DM} = A{\cdot}E^B$.Where A & B are parameters to be calculated. This hypothesis has been widely studied by the author in previous papers. [1] Abarca,M. [2] Abarca,M. [4] Abarca,M. y [8] Abarca,M. This hypothesis fulfils the physic meaning of D.M. Density formula in the halo region because it is supposed that such D.M. density is generated as a consequence of gravitational field propagation in the framework of a quantum gravitation theory.

As it is known direct DM density $D_{DM} = \frac{a^2\cdot(2b+1)}{4\pi G}{\cdot}r^{2b-2}$ depend on a & b parameters which come from power regression formula for velocity. In previous paragraph has been shown formula for gravitational field $E = \frac{a^2{\cdot}r^{2b}}{r} = a^2{\cdot}r^{2b-1}$ which depend on a & b as well. Through a simple mathematical treatment it is possible to get A & B to find function of DM density depending on E. Specifically formulas are:

$$A = \frac{a^{\frac{2}{2b-1}}\cdot(2b+1)}{4\pi G} \ \& \ B = \frac{2b-2}{2b-1}.$$

Table 4 M31 galaxy	$D_{DM} = A{\cdot}E^B$
A	$3.6559956 \cdot 10^{-6}$
B	1.6682469

According parameters a & b shown in table 2, got in previous chapter, A & B parameters are written beside.

Conversely $b = \frac{B-2}{2B-2}$ and $a = \left[\frac{4\pi GA(B-1)}{2B-3}\right]^{\frac{2b-1}{2}}$ being $B \neq 1$ *and* $B \neq 3/2$.

As conclusion, in this chapter has been demonstrated that a power law for velocity $v = a{\cdot}r^b$ *is mathematically equivalent to a power law for DM density depending on E.*
$D_{DM} = A{\cdot}E^B$ *if it is considered as physic hypothesis that D.M. is generated by the gravitational field.*

5.3 THE IMPORTANCE OF $D_{DM} = A{\cdot}E^B$

This formula is vital for theory of dark matter generated by gravitational field because it is supposed that DM is generated locally according an unknown quantum gravity mechanism. In other words, the propagation of gravitational field has this additional effect on the space as the gravitational wave goes by.

The formulas $D_{DM} = \frac{a^2\cdot(2b+1)}{4\pi G}{\cdot}r^{2b-2}$ and $E = a^2{\cdot}r^{2b-1}$ have been got rightly from rotation curve. Therefore it can be considered more specific for each galaxy. However the formula $D_{DM} = A{\cdot}E^B$ is much more essential.

The basis of this theory is that such formula is right for different gravitational systems. Therefore A & B parameters have to be similar for different galaxies on condition that the galaxies are similar. In further chapters will be got that power B is exactly the same for M31 and Milky Way although parameter A will be a bit different.

However, there is an important fact to highlight. It is clear that A depend on a and b, both parameters are global parameters. As the gravitational interaction time between masses is proportional to distance, it is right to think that DM generated by a gravitational field has a bigger proportion as the system increase its size. For example inside the Solar system it is clear that Newton and General Relativity Theory is able to explain with total accuracy every gravitational phenomenon without DM hypothesis. Therefore it is right to conclude that DM arises when gravitational interaction takes a longer time to link the matter. Namely, at galactic scale or cluster of galaxies.

6. RATIO BARYONIC MASS VERSUS DARK MATTER MASS DEPENDING ON RADIUS FOR M31

In this paragraph will be estimated radius which is needed to consider negligible baryonic density regarding DM density in M31 galaxy. [5] According Sofue, Y. data for M31 disk are

Table 5 M31 Galaxy	Baryonic Mass at disk	a_d	Σ_0
	$M_d = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$		
	$M_d = 1.26 \cdot 10^{11}\ M_\Theta$	5.28 kpc	1.5 kg/m^2

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents superficial density at disk. Total mass disk is given by integration of superficial density from cero to infinite. $M_d = \int_0^\infty 2\pi \cdot r \Sigma(r) \cdot dr = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$

To convert superficial baryonic density to volume density it is right to get the formula

$D_{BARYONIC}^{VOLUME} = \dfrac{\Sigma(r)}{2r}$ so $D_{BARYONIC}^{VOLUME}(40kpc) = 3.1 \cdot 10^{-25}\ kg/m^3$.

The formula of Direct Dark matter density is got afterwards, in page 17. $D_{DM}(r) = L \cdot r^{\frac{-5}{2}}$ being L = 1.33E+30. For example D_{DM} (40 kpc) = 2.5E-23 kg/m^3 . So the ratio of both volume density is 0.0124 .In conclusion it is right to consider negligible the baryonic density at 40 kpc, therefore it is possible to estate that halo dominion begins at 40 kpc in M31.

7. A BERNOULLI DIFFERENTIAL EQUATION FOR THE GRAVITATIONAL FIELD E

7.1 INTRODUCTION

This formula $D_{DM} = \dfrac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ is a local formula because it has been got by differentiation. However E, which represents a local magnitude $E = \dfrac{G \cdot M(<r)}{r^2} = \dfrac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ has been got through $v = a \cdot r^b$ whose parameters a & b were got by a regression process on the whole dominion of rotation speed curve. Therefore, D_{DM} formula has a character more local than E formula because the former was got by a differentiation process whereas the later involves M(<r) which is the mass enclosed by the sphere of radius r.

In other words, the process of getting D_{DM} involves a derivative whereas the process to get E(r) involves M (<r) which is a global magnitude. This is a not suitable situation because the formula $D_{DM} = A \cdot E^B$ involves two local magnitudes. Therefore it is needed to develop a new process with a more local nature or character.

It is clear that a differential equation for E is the best method to study locally such magnitude.

7.2 A BERNOULLI DIFFERENTIAL EQUATION FOR GRAVITATIONAL FIELD IN THE GALACTIC HALO

As it is known in this formula $\vec{E} = -G\frac{M(r)}{r^2}\hat{r}$, M(r) represents mass enclosed by a sphere with radius r. If it is considered a region where does not exit any baryonic matter, such as any galactic halo, then the derivative of M(r) depend on dark matter density essentially and therefore $M'(r) = 4\pi r^2 \varphi_{DM}(r)$.

If $E = G\frac{M(r)}{r^2}$, vector modulus, is differentiated then it is got $E'(r) = G\frac{M'(r)\cdot r^2 - 2rM(r)}{r^4}$

If $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ is replaced above then it is got $E'(r) = 4\pi G\varphi_{DM}(r) - 2G\frac{M(r)}{r^3}$ As $\varphi_{DM}(r) = A\cdot E^B(r)$ it is right to get $E'(r) = 4\pi\cdot G\cdot A\cdot E^B(r) - 2\frac{E(r)}{r}$ which is a Bernoulli differential equation.

$E'(r) = K\cdot E^B(r) - 2\frac{E(r)}{r}$ being $K = 4\pi\cdot G\cdot A$ Calling y to E, the differential equation is written in this simple way $y` = K\cdot y^B - \frac{2\cdot y}{r}$ Bernoulli family equations $y` = K\cdot y^B - \frac{2\cdot y}{r}$ may be converted into a differential linear equation with this variable change $u = y^{1-B}$. This is $\frac{u`}{1-B} + \frac{2u}{r} = K$

The homogenous equation is $\frac{u`}{1-B} + \frac{2u}{r} = 0$ whose general solution is $u = C\cdot r^{2B-2}$ being C the integration constant.

If it is searched a particular solution for the complete differential equation with a simple linear function u = z·r then it is got that $z = \frac{K\cdot(1-B)}{3-2B}$. Therefore the general solution for u- equation is $u = C\cdot r^{2B-2} + z\cdot r$

When it is inverted the variable change it is got the general solution for field E.

General solution is $E(r) = \left(Cr^{2B-2} + \frac{K(1-B)\cdot r}{3-2B}\right)^{\frac{1}{1-B}}$ *with* $B \neq 1$ *and* $B \neq 3/2$ where C is the parameter of initial condition of gravitational field at a specific radius.

Calling $\alpha = 2B-2$ $\beta = \frac{1}{1-B}$ and parameter $D = \left(\frac{K(1-B)}{3-2B}\right)$ then $E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$

Calculus of parameter C through initial conditions Ro and Eo

Suppose R_0 *and* E_0 are the specific initial conditions for radius and gravitational field, then $C = \frac{E_0^{1/\beta} - D\cdot R_0}{R_0^{\alpha}}$

Final comment

It is clear that the Bernoulli solution contains implicitly the fact that it is supposed there is not any baryonic matter inside the radius dominion and the only DM matter is added by $\varphi_{DM}(r) = A\cdot E^B(r)$. Therefore this solution for field works only in the halo region and R_0 *and* E_0 could be the border radius of galactic disk where it is supposed begins the halo region and the baryonic density is negligible, also it is possible to select another point belonging to the halo.

8. DIMENSIONAL ANALYSIS FOR D.M. DENSITY AS POWER OF E FORMULA

8.1 POWER OF E USING ONLY ONE PI MONOMIAL

As it is supposed that DM density as power of E come from a quantum gravity theory, it is right to think that constant Plank h should be considered and universal constant of gravitation G as well.

So the elements for dimensional analysis are D, density of DM whose units are Kg/m^3, E gravitational field whose units are m/s^2, G and finally h.

In table below are developed dimensional expression for these four elements D, E, G and h.

Table 6	G	h	E	D
M	-1	1	0	1
L	3	2	1	-3
T	-2	-1	-2	0

According Buckingham theorem it is got the following formula for Density

$D = \dfrac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ Being K a dimensionless number which may be understood as a coupling constant between field E and DM density.

As it is shown in previous epigraph, parameters for M31 is B = 1.6682469

So the relative difference between B = 1. 6682469 and 10/7 ≈1.428 is 16. 7 % which is not excessive, however in the following epigraph will be demonstrated the impossibility of this formula in the framework of the theory.

8.2 IMPOSSIBILITY OF D.M. DENSITY FORMULA WITH ONLY ONE PI MONOMIAL THEOREM

As it was shown in paragraph **5.2** $A = \dfrac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ and $B = \dfrac{2b-2}{2b-1}$. Being a,b the parameters got to fit rotation curve of velocities $v = a \cdot r^b$

Conversely, it is right to clear up parameters a and b from above formulas.

Therefore $b = \dfrac{B-2}{2B-2}$ and $a = \left[\dfrac{4\pi GA(B-1)}{2B-3}\right]^{\frac{2b-1}{2}}$ being $B \neq 1$ *and* $B \neq 3/2$.

As A has to be a positive quantity then $2b+1>0$. As $2b+1 = \dfrac{2B-3}{B-1} > 0$ therefore $B \in (-\infty,1) \cup (3/2,\infty)$.

If B=3/2 then 2b+1=0 and A=0 so dark matter density is cero which is the Keplerian rotation curve.

In every galactic rotation curve studied, B parameter has been bigger than 3/2. See Abarca papers quoted in Bibliographic references. Therefore the experimental data got in several galaxies match perfectly with mathematical findings made in this paragraph, namely for $B \in (3/2,\infty)$.

The main consequence of this mathematical analysis is that formula $D = \frac{K}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}}$ got with only one pi monomial has to be rejected because B=10/7 < 3/2.

8.3 POWER E FORMULA FOR DM DENSITY WITH TWO PI MONOMIALS

As consequence of the previous theorem it is compulsory to explore a new formula with two pi monomials.

As this formula come from quantum gravitation theory it is right to include c, the velocity of light as the additional Universal constant required in this analysis. So the elements to make dimensional analysis are D, E, G, h and c.

Table 7	G	h	E	D	c
M	-1	1	0	1	0
L	3	2	1	-3	1
T	-2	-1	-2	0	-1

According Buckingham theorem, as matrix rank is three, there are two pi monomials. The first one was calculated in previous paragraph and the second one involves G, h, E and c.

These are both pi monomials $\pi_1 = D \cdot \sqrt[7]{G^9 \cdot h^2} \cdot E^{-\frac{10}{7}}$ and $\pi_2 = \frac{c}{\sqrt[7]{G \cdot h}} E^{-\frac{2}{7}}$. So according the Buckingham theorem the formula for DM density through two pi monomials will be $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ being J a dimensionless number and $f(\pi_2)$ an unknown function, which cannot be calculated by the dimensional analysis theory.

This formula is physically more acceptable because it is got considering G, h and c as universal constant involved in formula of density. As the hypothesis of the theory estates that DM is generated through a quantum gravitation mechanism it is right to consider not only **G** and **h** but also **c** as it is supposed that gravitons are virtual particles whose velocity is c.

8.4 LOOKING FOR A D.M. DENSITY FUNCTION COHERENT WITH DIMENSIONAL ANALYSIS

It is right to think that $f(\pi_2)$ should be a power of π_2, because it is supposed that density of D.M. is a power of E.

In other words, to select the function $f(\pi_2)$ it is used the razor`s Ockham principle.

Table 4 M31 galaxy	$D_{DM} = A \cdot E^B$
A	$3.6559956 \cdot 10^{-6}$
B	1.6682469

Taking in consideration the A &B parameters fitted to M31 halo galaxy it is right to consider 5/3 as the best simple number closer to the empirical value B=1.6682 To achieve this goal the power for π_2 must be -5/6. This way, power of E in formula $D_{DM} = A \cdot E^B$ will be 5/3 and the formula of Density with two pi monomial $D = \frac{J}{\sqrt[7]{G^9 \cdot h^2}} \cdot E^{\frac{10}{7}} \cdot f(\pi_2)$ becomes $D = \frac{M}{\sqrt[6]{G^7 \cdot c^5 \cdot h \cdot}} E^{\frac{5}{3}}$ being M a dimensionless number.

9. RECALCULATING FORMULAS IN M31 HALO WITH B = 5/3

Findings got through Buckingham theorem are crucial. It is clear that a physic formula has to be dimensionally coherent .Therefore it is needed to rewrite all the formulas considering B=5/3. Furthermore, with B= 5/3, a lot of parameters of the theory become simple fraction numbers. In other words, the theory gains simplicity.

In the chapter 5 was shown that the relation between a & b parameters and A&B parameters are:

$$A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G} \ \& \ B = \frac{2b-2}{2b-1}.$$

Now considering B= 5/3 It is right to get $b = \frac{B-2}{2B-2} = -\frac{1}{4}$ and $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$

Therefore, the central formula of theory becomes $D_{DM} = A \cdot E^{\frac{5}{3}} = \frac{a^{\frac{-4}{3}}}{8 \cdot \pi \cdot G} \cdot E^{\frac{5}{3}}$.

In chapter 11 will be studied the rotation curve of Milky Way according the data published by Sofue in 2020, and it will be shown that in the halo region the rotation curve decreases with the same exponent b = -1/4. This fact is crucial for DM by gravitation theory because both giant galaxies are the only ones with accuracy measures in the halo region.

9.1 RECALCULATING THE PARAMETER a IN M31 HALO

Table below comes from chapter 3 and represents regression curve of velocity depending on radius.

Table 2- Power Regression for M31 rot. curve	
$V=a \cdot r^b$	
a	$4.32928 \cdot 10^{10}$
b	-0.24822645
Correlation coeff.	0.96

Due to the Buckingham theorem it is needed that b= -1/4. Therefore it is needed to recalculate the parameter **a** in order to find a new couple of values a &b that fit perfectly to the experimental measures in M31 halo.

RECALCULATING a WITH THE MINUMUN SQUARE METHOD

When it is searched the parameter **a**, a method widely used is called the minimum squared method. So it is searched a new parameter **a** for the formula $v= a \cdot r^{-0.25}$ on condition that $\sum_e (v-v_e)^2$ has a minimum value.

Where v represents the value fitted for velocity formula and v_e represents each measure of velocity. It is right to calculate the formula for a.

$$a = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}}$$

Table 8- New parameters a &b and A & B for M31 galaxy	
B	5/3
$b = \frac{B-2}{2B-2}$	b = -1/4
a new	$4.727513 \cdot 10^{10}$ $m^{5/4}/s$
A new	$3.488152 \cdot 10^{-6}$

Where r_e represents each radius measure and v_e represents its velocity associated.

So using the data set in table page 5, columns in grey it is right to get the value for parameter **a**. From now on it is possible to consider the dimension precise for parameter a, as this parameter is the coefficient for the formula of velocity $v = a \cdot r^{-1/4}$ so its dimension will be $m^{5/4}/s$ and the parameter a^2 its dimension will be $m^{5/2}/s^2$.

$$a = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}} = 4.727513 \cdot 10^{10} \frac{m^{5/4}}{s}$$

This parameter a^2 will be used continuously in the following chapters.

9.2 FORMULAS OF DIRECT D.M. FOR DENSITY FOR MASS AND FIELD E

With these new parameters recalculated it is going to get the direct formulas got at the beginning of paper.

Function of Density DM depending on radius.

$$D_{DM}(r) = L \cdot r^{2b-2} = L \cdot r^{\frac{-5}{2}} \text{ being } L = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot = \frac{a^2}{8 \cdot \pi \cdot G} = 1.3326 \cdot 10^{30} \frac{Kg}{m^{1/2}}$$

Function of E depending on radius $E = a^2 \cdot r^{2b-1} = a^2 \cdot r^{\frac{-3}{2}}$ being $a^2 = 2.235 \cdot 10^{21} \frac{m^{5/2}}{s^2}$

Mass enclosed by a sphere of radius r, known as dynamical mass.

$M_{DYN}(< r) = \frac{v^2 \cdot R}{G}$ When velocity is replaced by its fitted function it is got $M_{DIRECT}(< r) = \frac{a^2 \cdot r^{2b+1}}{G} = \frac{a^2 \cdot \sqrt{r}}{G}$

9.3 BERNOULLI SOLUTION FOR E AND DENSITY IN M31 HALO

In chapter 7 was got the solution for field in the halo region, now thanks dimensional analysis it is possible to simply the Bernoulli formulas. Namely:

$E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$ being $\alpha = 2B - 2 = \frac{4}{3}$ and $\beta = \frac{1}{1-B} = \frac{-3}{2}$ By other side, the initial condition C

$C = \frac{E_0^{1/\beta} - D \cdot R_0}{R_0^{\alpha}}$ becomes $C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$ and $D = \left(\frac{4 \cdot \pi \cdot G \cdot A(1-B)}{3-2B}\right) = 8 \cdot \pi \cdot G \cdot A$ and as $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$ then

$D = a^{\frac{-4}{3}} = 5.85 \cdot 10^{-15}$

Therefore $E(r) = \left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-3}{2}}$ being C the initial condition of differential equation solution for E and $D = a^{\frac{-4}{3}}$ is a parameter closely related to the global rotation curve at halo region, being parameter **a** = $4.7275 \cdot 10^{10}$ $m^{5/4}$ /s

BERNOULLI SOLUTION FOR DENSITY IN HALO REGION

As the formula for DM density is $D_{DM} = A \cdot E^B$, being B= 5/3 it is right to get the new formula for DM density.

$$D_{BERNI}(r) = A \cdot E^{\frac{5}{3}} = A\left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-5}{2}} = \frac{D}{8\pi G}\left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-5}{2}}$$

9.4 DARK MATTER AT A SPHERICAL CORONA BY BERNOULLI SOLUTION IN HALO REGION

Formula below express the dark matter contained inside a spherical corona defined by R_1 and R_2 belonging at halo.

$$M_{BERNI} = \int_{R1}^{R2} 4\pi \cdot r^2 \cdot D_{BERNI} dr = \int_{R1}^{R2} 4\pi \cdot r^2 AE^B dr = 4\pi A \int_{R1}^{R2} r^2 \left[C \cdot r^{4/3} + D \cdot r\right]^{\frac{-5}{2}} \cdot dr$$ The indefinite integral

$$I = 4\pi A \cdot \int \frac{r^2}{\left(C \cdot r^{4/3} + D \cdot r\right)^{2.5}} = \frac{8\pi A\sqrt{r}}{D \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}} = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$$

As $\frac{8\pi A}{D} = \frac{1}{G}$. Calling $M_{BER}(r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ and by the Barrow`s rule, it is got

$M_{BERNI}{}_{R1}^{R2}$ = $M_{BERNI}(R_2)$-$M_{BERNI}(R_1)$ that provided the DM contained inside the spherical corona defined by R_2 and R_1.

9.5 NEWTON`S THEOREM WITH BERNOULLI MASS FORMULA

The name for this theorem has been chosen because the relation between field E and total mass M (<r) is the same that in Newton's theory.

From Bernoulli field $E_{BERNI}(r) = \left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-3}{2}} = \frac{1}{r^{3/2} \cdot \left(C \cdot r^{1/3} + D\right)^{3/2}}$

Bernoulli mass formula $M_{BERNI}(r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ so $G \cdot M_{BER}(r) = \frac{\sqrt{r}}{\left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ and

$$\frac{G \cdot M_{BER}(r)}{r^2} = \frac{\sqrt{r}}{r^2\left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}} = \frac{1}{r^{3/2} \cdot \left(C \cdot r^{1/3} + D\right)^{3/2}} = E(r)$$

Therefore $E_{BERNI}(r) = \frac{G \cdot M_{BER}(r)}{r^2}$ this is the intensity of field in Newton´s theory. This identity shows how the DMbQG theory**,** adding an extra of mass depending on radius, being the halo region unlimited, is able to explain the DM measures in galaxies and cluster in the Newtonian framework.

COROLLARY

According the Newtonian framework, the function of mass included in the formula of field E means the total mass included inside the sphere with radius r. Therefore $M_{BER}(r)$ must be renamed as $M_{BER}(< r) = M_{TOTAL}(< r)$ where r ranges in the halo region i.e. $M_{BER}(r)$ gives the total mass enclosed by the sphere with radius r.

In conclusion, the total mass enclosed by the sphere with radius r is given by the formula:

$$M_{TOTAL}(<r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$$ where r belong to halo region. i.e. r > 40 kpc for M31 galaxy.

Notice how in the formula the variable M(r) has been changed by M(<r)

9.6 CALCULUS OF PARAMETER C

$$C_O = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$$

This parameter is calculated by the data set of rotation curve in halo region, see table 1.

Theoretically any point belonging to the data set may be used as initial condition, and the value got must be the same.

E_0 is the gravitational field at R_O radius. Considering that data measures are in dynamical equilibrium, it is possible to estimate the field by $E_0 = v_0^2/r_0$ using data in table 1.

Finally the parameter D=5.85E-15 was calculated in epigraph 9.3 using the value B =5/3 got by dimensional analysis.

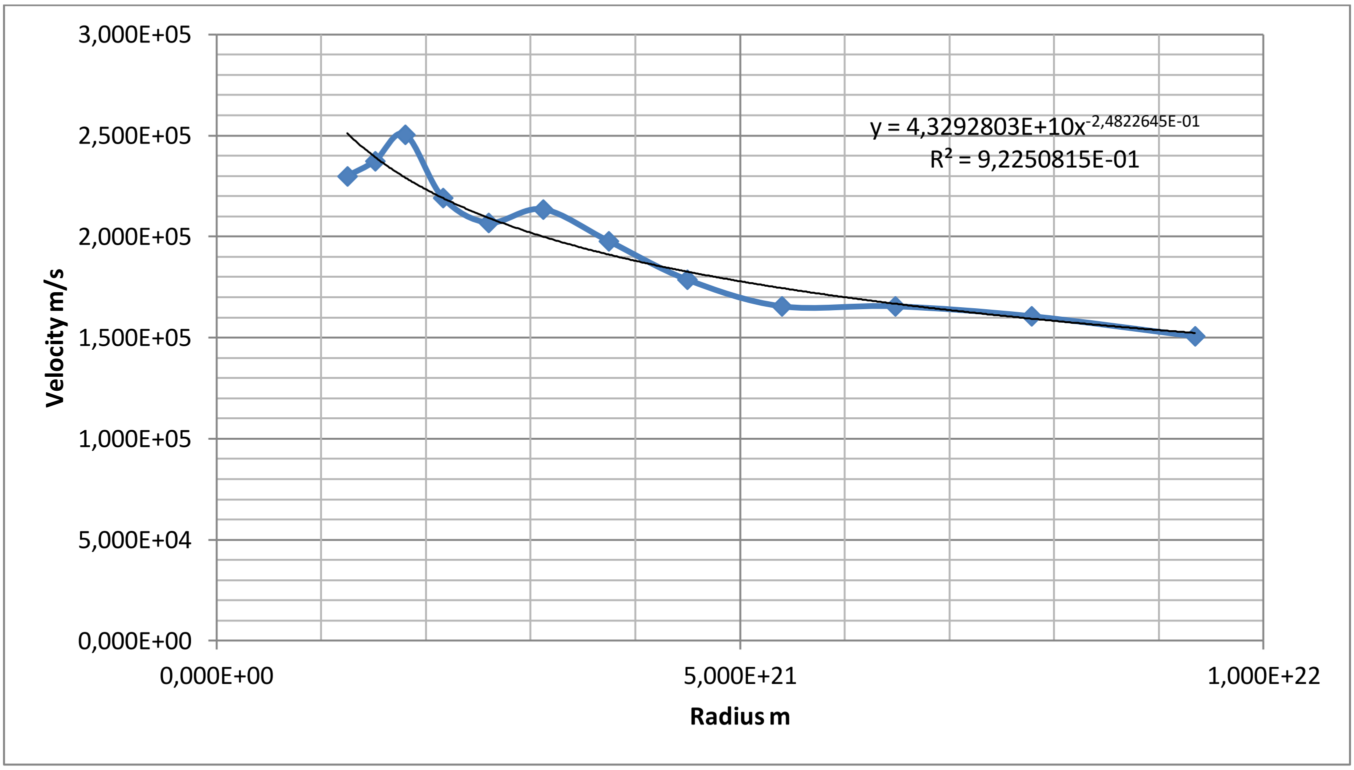
It is clear that the data set are not in perfect dynamical equilibrium, but data selected in the halo region are quite close to the dynamical equilibrium.

In table 9 it is calculated Co for every point of data set.

Table 9	Radius	Radius	Velocity	Field Eo	Eo^(-2/3)	Eo^(-2/3)-D*R_0	Parameters C_0
Points	kpc	m	m/s	m/s^2			
1	40,5	1,250E+21	2,299E+05	4,23E-11	8239256,95	9,27E+05	6,8830E-23
2	49,1	1,515E+21	2,374E+05	3,72E-11	8973615,749	1,11E+05	6,3753E-24
3	58,4	1,802E+21	2,505E+05	3,48E-11	9377625,482	-1,16E+06	-5,3082E-23
4	70,1	2,163E+21	2,192E+05	2,22E-11	12654654,13	1,20E+03	4,2804E-26
5	84,2	2,598E+21	2,069E+05	1,65E-11	15443489,73	2,45E+05	6,8683E-24
6	101,1	3,120E+21	2,135E+05	1,46E-11	16732951,68	-1,52E+06	-3,3317E-23
7	121,4	3,746E+21	1,978E+05	1,04E-11	20928758,45	-9,85E+05	-1,6934E-23
8	145,7	4,496E+21	1,788E+05	7,11E-12	27043372,43	7,42E+05	9,9988E-24
9	175	5,400E+21	1,656E+05	5,08E-12	33846627,1	2,26E+06	2,3822E-23
10	210,1	6,483E+21	1,656E+05	4,23E-12	38232870,38	3,08E+05	2,5446E-24
11	252,3	7,785E+21	1,607E+05	3,32E-12	44959131,06	-5,83E+05	-3,7771E-24
12	302,9	9,347E+21	1,508E+05	2,43E-12	55281485,72	6,02E+05	3,0572E-24

It is remarkable that all the values C_0 are very close all of them and they are close to 0 as well.
Values in yellow are negatives because these points are above the fitted curve. See graph below. In addition, the more close to curve the point is, the smaller, in absolute value, the parameter C_0 is. The cyan value is the smaller. Afterwards will be shown that this fact explain that data measures are close to dynamical equilibrium. For example points 1 and 3 are quite far away from dynamical equilibrium despite to be placed in halo region.

In the graph below the graph point are numbered from left to the right, so the first one to the left is the number one.



The close oscillation around the fitted curve suggests strongly that this curve might be the ideal curve of perfect ideal dynamical equilibrium for the celestial bodies belonging to M31 galaxy.

In the following epigraph it will be demonstrated that if it is considered as initial condition a point belonging to the ideal fitted curve at halo region, this point will have parameter C = 0. In addition in the epigraph 9.8 it will be shown that Bernoulli mass formula becomes direct mass if parameter C = 0.

9. 7 PARAMETER C EQUAL ZERO THEOREMS

These theorems mean an important step forward in the theoretical development of DMbQG theory. When parameter C is zero then Bernoulli formula becomes direct mass, with only the parameter $\mathbf{a^2}$**.** The reader can check that in the paper [2] Abarca, M.2019 the DMbQG theory is essentially developed but the formula for total mass that is the Bernoulli instead of Direct mass. In practice, the relative differences between both formulas are totally negligible. However to demonstrate that Direct mass is the Bernoulli formula when parameter C is zero means a better understanding of DM phenomenon. In addition allow to continue the development of the theory more easily, for example when the theory is extended to cluster of galaxies. See chapter 13.

Definition. Hereafter, it will be named Buckingham halo curve to the points **(r, v)** **r** belonging to the halo region and the velocity $v = a \cdot r^{-1/4}$ being **a** the parameter associated to galactic halo. It is an ideal curve because its points are in perfect dynamical equilibrium.
As the exponent -1/4 was got by the Buckingham theorem, it has been select such name for that curve.

Direct Theorem: If it is supposed that a point belonging to Buckingham halo curve is in dynamical equilibrium and if it is selected such point as initial point to calculate C, then such parameter is zero.
Proof : Suppose a point (Ro, Vo) belonging to Buckingham halo curve, then $V_0 = a \cdot R_0^{-1/4}$ As dynamical equilibrium leads to $E_0 = \frac{GM(<r)}{r^2} = \frac{V_0^2}{r}$ then $E_0 = a^2 \cdot R_0^{-3/2}$ and $E_0^{\frac{-2}{3}} = a^{-4/3} \cdot R_0 = D \cdot R_0$ because $D = a^{\frac{-4}{3}}$ when B=5/3 as it was shown at epigraph 9.3. Therefore C= 0 because its numerator is zero. See the formula: $C = \frac{E_0^{\frac{-2}{3}} - D \cdot R_0}{R_0^{\frac{4}{3}}}$

It is important to highlight that such points are in perfect dynamical equilibrium, whereas the data measures are **close** to dynamical equilibrium. In addition, the real gravitational field never has a perfect spherical symmetry. See in table below the relative difference between data and Buckingham points.

Reverse Theorem

If it is selected a point (Ro,Vo) which is supposed to be in dynamical equilibrium and its parameter C=0 then such point belong to Buckingham halo curve. Suppose that $V_0 = a \cdot R_0^b$ being exponent b unknown.

Proof: If C = 0 then $E_0^{-2/3} = D \cdot R_0$ and as there is dynamical equilibrium $E_0 = \frac{V_0^2}{r}$ then $E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$

and $E_0^{-2/3} = a^{-4/3} \cdot R_0^{\frac{2-4b}{3}}$ so $a^{-4/3} \cdot R_0^{\frac{2-4b}{3}} = D \cdot R_0$ or $D \cdot R_0^{\frac{2-4b}{3}} = D \cdot R_0$ which leads to $R_0 = R_0^{\frac{2-4b}{3}}$ so b= - 1/4

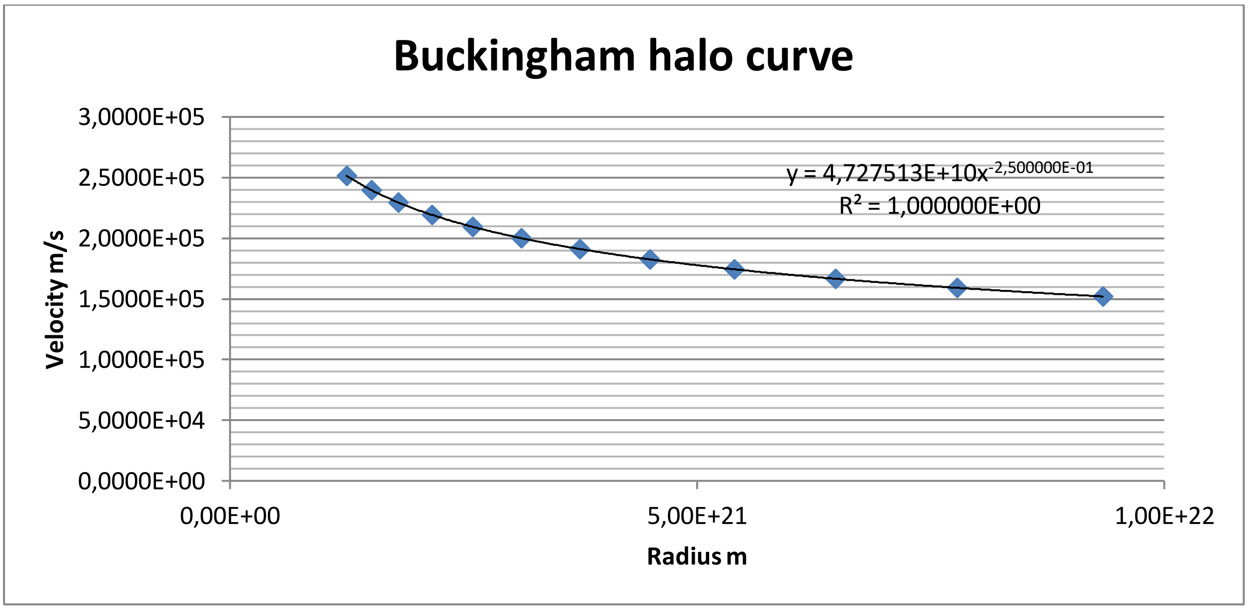


Table 10				
Radius kpc	Radius m	Buckingham Velocity m/s	Data velocity m/s	Relative Diff. %
40,5	1,25E+21	2,5144E+05	2,2990E+05	8,57E+00
49,1	1,52E+21	2,3962E+05	2,3740E+05	9,27E-01
58,4	1,80E+21	2,2945E+05	2,5050E+05	-9,17E+00
70,1	2,16E+21	2,1921E+05	2,1920E+05	5,58E-03
84,2	2,60E+21	2,0939E+05	2,0690E+05	1,19E+00
101,1	3,12E+21	2,0004E+05	2,1350E+05	-6,73E+00
121,4	3,75E+21	1,9109E+05	1,9780E+05	-3,51E+00
145,7	4,50E+21	1,8257E+05	1,7880E+05	2,06E+00
175	5,40E+21	1,7440E+05	1,6560E+05	5,04E+00
210,1	6,48E+21	1,6660E+05	1,6560E+05	6,03E-01
252,3	7,79E+21	1,5915E+05	1,6070E+05	-9,72E-01
302,9	9,35E+21	1,5204E+05	1,5080E+05	8,18E-01

The third column got with the formula $V_0 = a \cdot R_0^{-1/4}$ is called Buckingham velocity because such formula is calculated by the Buckingham theorem. See the graph above.

Parameter $a_{M31} = 4.727513E+10\ m^{5/4}$ /s

The fourth column is the measures of velocity.

Data at radius 302.9 kpc has the lowest relative difference. Although data radius 252.3 kpc has almost the same relative difference but negative.

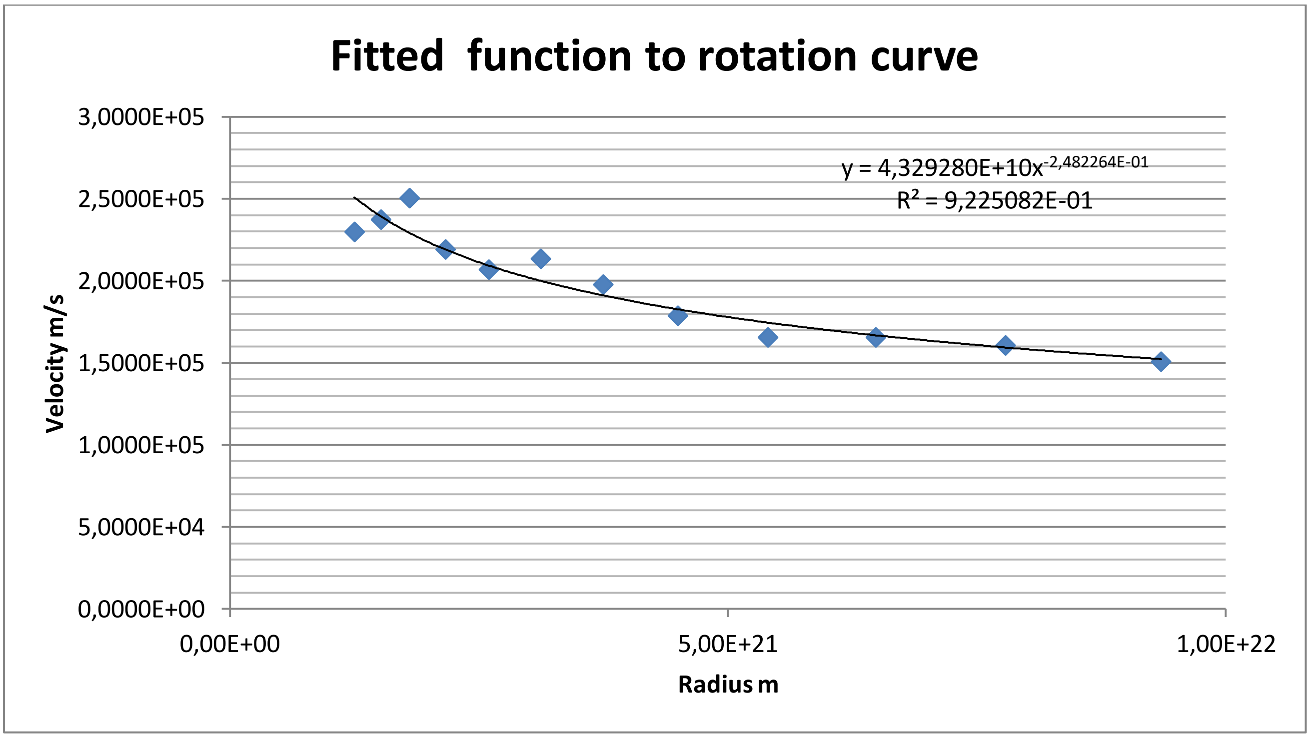
Final comments

Data measures do not belong to Buckingham halo curve by two reasons:

The first one it is simple: measures have experimental errors. The second one is more subtle, the celestial bodies are not in perfect dynamical equilibrium. It is right to think that celestial bodies which belong to M31 gravitational system from its formation times, more than ten billions years ago, will be closer to dynamical equilibrium regarding other ones that were captivated by the gravitational field of M31 afterwards.

Watching the graph, it is clear that point 1 and point 3 at 40.5 kpc and 58.4 kpc are the points more distant regarding Buckingham halo curve. This important difference regarding dynamical equilibrium curve may be explained by the asymmetries of gravitational field during the history of dynamic evolution.

Anyway it is undeniable that in general data are very close to dynamical equilibrium.



As it is shown in the graph the exponent of fitted function differs 18 thousands regarding -0.25

9.8 BERNOULLI FORMULAS BECOME DIRECT FORMULAS WHEN PARAMETER C = 0

Thanks demonstration made above, it is clear why data measures close to Buckingham halo curve give values for C very close to zero. The more close point measure to Buckingham halo curve is, the more close to zero parameter C is. Now parameter C will be zero, because at C = 0 are got the formulas with initial point belonging to Buckingham halo curve. i. e. a initial point which is in perfect dynamical equilibrium.

FOR FIELD E

When in formula $E(r) = \left(Cr^{\frac{4}{3}} + Dr \right)^{\frac{-3}{2}}$ C= 0 then it is got $E = a^2 \cdot r^{\frac{-3}{2}}$ because $D = a^{\frac{-4}{3}}$, being $a^2 = 2.235 \cdot 10^{21}$

which is precisely direct formula for E. Which is the field calculated by perfect dynamical equilibrium.

FOR D.M. DENSITY

As $D_{DM} = A \cdot E^{5/3}$ Using field got by Bernoulli solution it is right to get

$D_{DM}(r) = A\left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-5}{2}}$ Being $A = \dfrac{D}{8\pi G}$ and $D = a^{\frac{-4}{3}}$ if **C** = 0 then formula becomes

$D_{DM}(r) = A \cdot D^{\frac{-5}{2}} \cdot r^{\frac{-5}{2}} = L \cdot r^{\frac{-5}{2}}$ being $L = \dfrac{a^2}{8 \cdot \pi \cdot G}$ = $1.3326 \cdot 10^{30}$ which is direct DM density formula.

FOR DIRECT MASS FORMULA

If C=0 then $M_{BERNI}(<r) = \dfrac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$ becomes $M_{DIRECT}(<r) = \dfrac{a^2 \cdot \sqrt{r}}{G}$ being $\dfrac{a^2}{G} = 3.349 \cdot 10^{31}$

Final comment

As it is clear Direct formulas are a particular case of Bernoulli formulas when parameter C =0.

At the beginning of this paper, the direct formulas were only right up to 300 kpc, and they were statistical formulas to calculate the extra additional amount of matter needed to explain the rotation curves. However, thanks to findings got in this chapter, Direct formulas are the Bernoulli formulas when it is considered as initial point a theoretical point belonging to the Buckingham curve.
Therefore the dominion of Direct formulas becomes unbounded as the gravitational field.
In addition the direct formulas depend on parameter **a** solely, instead two parameters C and D associated to Bernoulli formulas, which is a magnificent simplification of the theory.

10. MASSES IN M31

In this chapter, it will be calculated and compared three different types of masses related to M31.

10.1 DYNAMICAL MASS VERSUS DIRECT MASS

As it is known, dynamical mass represents the total mass enclosed by a sphere with a radius r in order to produce a balanced rotation with a specific velocity at such radius, so it is right to consider dynamical mass as the total mass, baryonic and DM mass, enclosed at radius R. Ranging radius in the interval of radius measured.

The formula of dynamical mass is $M_{DYN}(<r) = \dfrac{V^2 \cdot r}{G}$ and $M_{DIRECT}(<r) = \dfrac{a^2 \cdot \sqrt{r}}{G}$ being $a^2/G = 3.35 \cdot 10^{31}$

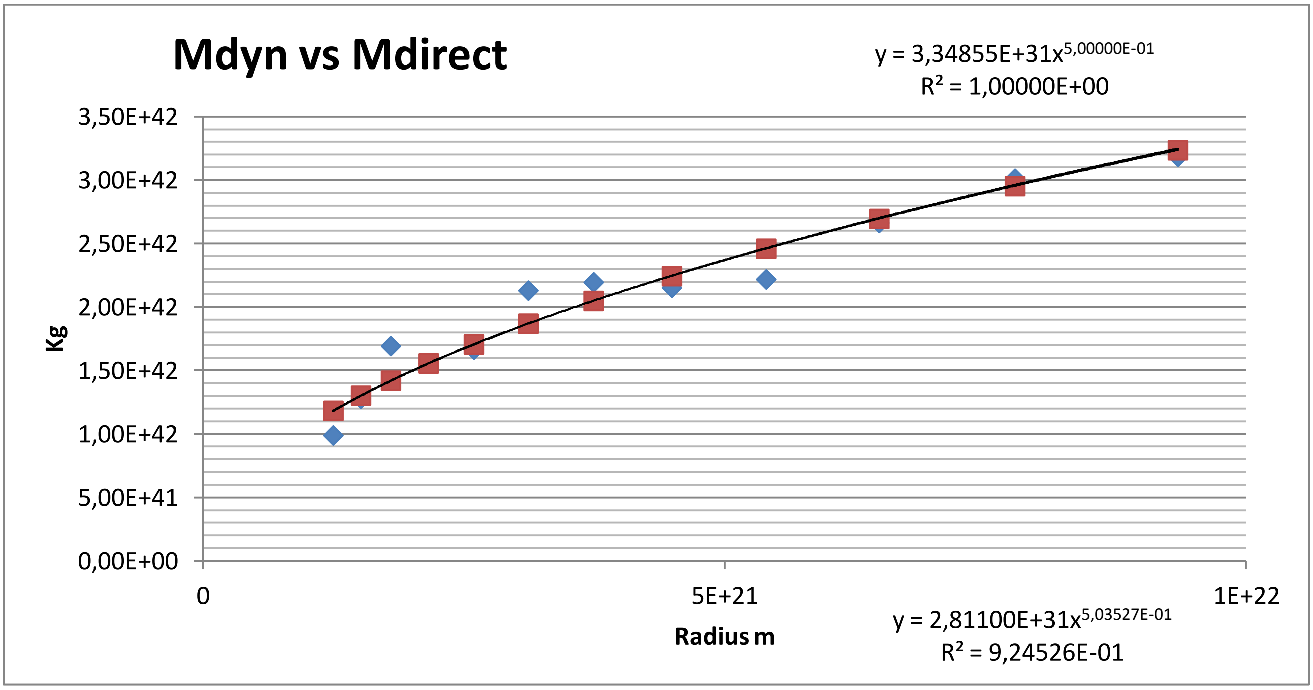
Table11 Radius	Radius	Velocity	Dyn Mass	Direct mass	Relative diff.
kpc	m	m/s	M_Θ	M_Θ	%
40,5	1,250E+21	2,299E+05	4,974E+11	5,95E+11	1,639E+01
49,1	1,515E+21	2,374E+05	6,429E+11	6,55E+11	1,849E+00
58,4	1,802E+21	2,505E+05	8,514E+11	7,14E+11	-1,919E+01
70,1	2,163E+21	2,192E+05	7,825E+11	7,83E+11	1,419E-02
84,2	2,598E+21	2,069E+05	8,373E+11	8,58E+11	2,373E+00
101,1	3,120E+21	2,135E+05	1,071E+12	9,40E+11	-1,392E+01
121,4	3,746E+21	1,978E+05	1,103E+12	1,03E+12	-7,143E+00
145,7	4,496E+21	1,788E+05	1,082E+12	1,13E+12	4,087E+00
175	5,400E+21	1,656E+05	1,115E+12	1,24E+12	9,833E+00
210,1	6,483E+21	1,656E+05	1,339E+12	1,35E+12	1,204E+00
252,3	7,785E+21	1,607E+05	1,514E+12	1,48E+12	-1,951E+00
302,9	9,347E+21	1,508E+05	1,600E+12	1,63E+12	1,629E+00

In the fifth column is tabulated the direct masses in order to be compared with dynamical masses.

Below in the graph are plotted both functions, blue points are dynamical masses and brown point are direct masses.

The first and third points have the maximum difference regarding fitted curve whereas relative differences decreased as radius increased. Namely, relative differences are below 10% for radius bigger than 120 kpc and are below 2 % for radius bigger than 210 kpc.

So direct mass is a very good approximation for dynamical mass enclosed at radius R. Ranging radius in the interval of radius measured.



10.2 BERNOULLI MASS VERSUS DIRECT MASS

In this epigraph will be shown that relative difference between both kinds of formulas is negligible.

Below are both function formulas.

$M_{DIRECT}(<r) = \dfrac{a^2 \cdot \sqrt{r}}{G}$ being $a^2/G = 3.35 \cdot 10^{31}$, as was pointed in previous paragraph, will be used to approximate total mass at radius R.

In the corollary of Newton´s theorem, epigraph 9.5, was demonstrated that the Bernoulli mass is the total mass enclosed by a sphere with radius r. Now it will be shown that relative differences between Bernoulli and direct mass are quite small, even for extended haloes.

In the paper [13] Abarca,M.2023, epigraph 9.6 it is developed a method to calculate the optimal parameter C

M31 PARAMETERS C & D	This selected value C_{M31} belong to the initial point radius equal to 252. kpc. See epigraph 9.6
$C_{M31} = -3.777 \cdot 10^{-24}$	
$D_{M31} = a^{\frac{-4}{3}} = 5.85 \cdot 10^{-15}$	

$$M_{BERNI}(<r) = \frac{\sqrt{r}}{G \cdot \left(C \cdot \sqrt[3]{r} + D\right)^{\frac{3}{2}}}$$

Below are tabulated both function and its relative difference. It is remarkable that even at 2 Mpc its difference is only 3.8 %, despite the fact that its dominion has been extended 7 times.

	Table 12	Direct mass C=0	Bernoulli mass $C \neq 0$	Relative diff.
kpc	m	Msun	Msun	%
40,5	1,250E+21	5,949E+11	6,011E+11	1,04E+00
60	1,851E+21	7,240E+11	7,327E+11	1,19E+00
80	2,469E+21	8,361E+11	8,471E+11	1,31E+00
100	3,086E+21	9,347E+11	9,481E+11	1,41E+00
200	6,171E+21	1,322E+12	1,346E+12	1,77E+00
385	1,188E+22	1,834E+12	1,875E+12	2,20E+00
500	1,543E+22	2,090E+12	2,142E+12	2,40E+00
770	2,376E+22	2,594E+12	2,668E+12	2,77E+00
1000	3,086E+22	2,956E+12	3,048E+12	3,02E+00
1500	4,629E+22	3,620E+12	3,750E+12	3,46E+00
2000	6,171E+22	4,180E+12	4,346E+12	3,80E+00

Bernoulli mass have been got with the DM by gravitation theory so its dominion is the unlimited haloes.

It is obvious that direct mass is easier to calculate than Bernoulli mass because it has only one parameter. Hereafter it will be used Direct mass instead Bernoulli mass.

11. DARK MATTER BY GRAVITATION THEORY IN MILKY WAY

In the new rotation curve published by [6] Sofue.2020, the radius of data range from 0.1 kpc up to 95.5 kpc whereas in the previous rotation curve [5] Sofue.2015 the radius range up to 300 kpc. Afterwards will be discussed the importance to reduce the dominion up to 95 kpc. However firstly it is needed to calculate the lowest radius for the halo region, where the baryonic density is negligible versus DM density.

11.1 AN ESTIMATION FOR THE HALO RADIUS

As it is known, Bernoulli solution is only right into the halo region, where the baryonic mass is negligible.
To calculate baryonic volume density has been used model provided by Sofue for baryonic disc.

This Table comes from Sofue [6], see table 3, page 12. Parameters for baryonic matter at disc in Milky Way.

Component	Parameter	Fitted Value	χ^2
Expo. disk	a_d	4.38 ± 0.35 kpc	
	Σ_0	$(1.28 \pm 0.09) \times 10^3 M_\odot \mathrm{pc}^{-2}$	

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents superficial density at disc region. To convert superficial baryonic density to volume density it is right to get the formula $D_{BARYONIC}^{VOLUME} = \dfrac{\Sigma(r)}{2r}$ so $D_{BARYONIC}^{VOLUME}(30.5\,kpc) = 1.34 \cdot 10^{-24}\, kg/m^3$.

The formula of Direct Dark matter density was got in page 17. $D_{DM}(r) = L \cdot r^{\frac{-5}{2}}$ being $L_{MILKY\text{-}WAY}$ = 9.1E+29, according with parameter **a** got in epigraph 11.3. For example D_{DM} (30.5 kpc) = 3.35E-23 kg/m^3 . So the ratio of both volume density at 30.5 kpc is 0.04 .In conclusion it is right to consider negligible the baryonic density for radius bigger than 30.5 kpc, therefore it is possible to estate that halo dominion begins at 30.5 kpc for Milky Way. According with DM by gravitation theory, the halo region is unbounded as it is the gravitational field.

11. 2 ROTATION CURVE OF MILKY WAY BY SOFUE 2020 DATA

Table 13	Veloc.	Radius	Veloc.
kpc	km/s	m	m/s
30,448	229,6	9,40E+20	229600
33,493	222,5	1,03E+21	222500
36,842	215	1,14E+21	215000
40,527	207,1	1,25E+21	207100
44,579	200,3	1,38E+21	200300
49,037	194,7	1,51E+21	194700
53,941	189,8	1,66E+21	189800
59,335	186,2	1,83E+21	186200
65,268	184,7	2,01E+21	184700
71,795	183,9	2,22E+21	183900
78,975	181,4	2,44E+21	181400
86,872	175,5	2,68E+21	175500
95,56	167,7	2,95E+21	167700

This table of rotation curve of Milky Way comes from [6] Sofue.2020, and there have been selected data with radius bigger than 30 kpc.

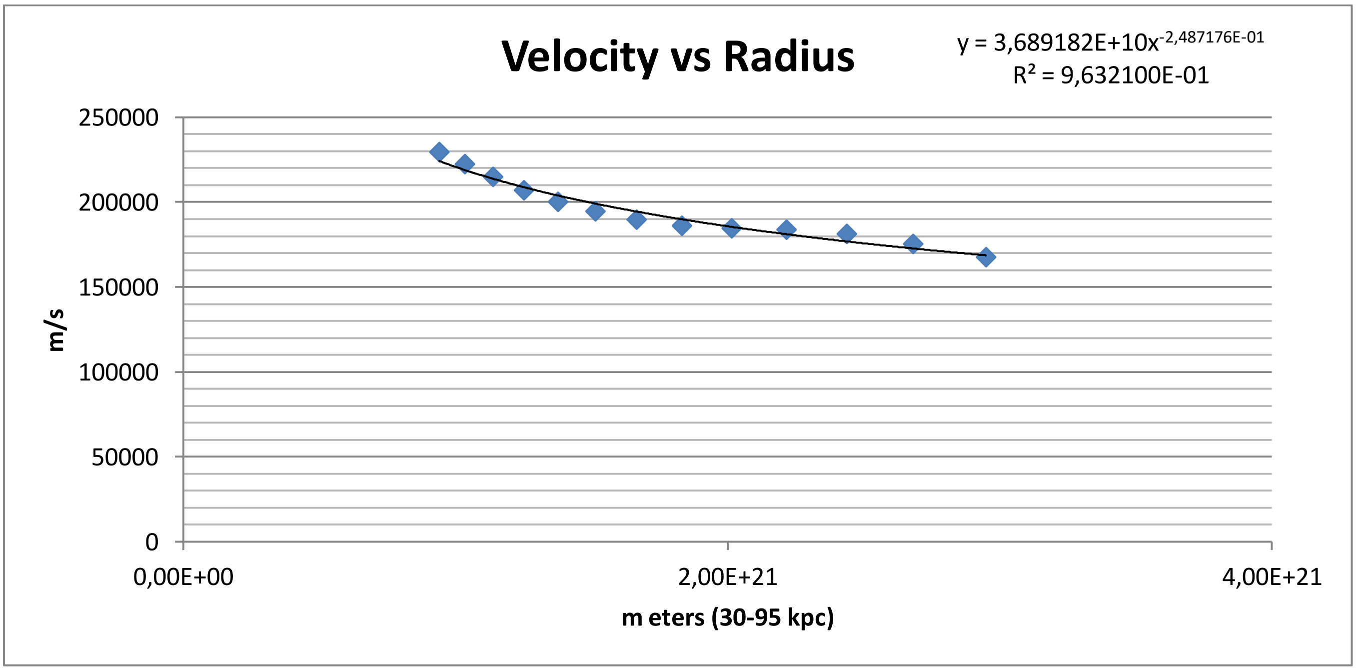
This new set of Sofue data is very important for the theory of DM by gravitation theory because gives a rotation curve at halo region with a power for radius very close to -1/4 which is the same for M31. This fact backs strongly the hypothesis of this theory.

In the previous paper [5] Sofue, Y.2015, the author gave an extended dominion up to 300 kpc, However data with radius bigger than 100 kpc have too high velocity and fitted power function did not fit properly with exponent -1/4.

The logical explanation about the "bad" behaviour of these data is to consider that such celestial bodies are not in dynamical equilibrium. Perhaps they came from the outskirts of MW and were captivated by MW gravitational field afterwards so it is right to consider that these data are far away to dynamical equilibrium, whereas celestial bodies below 100 kpc of radius are properly in dynamical equilibrium with Milky Way.

Anyway, the important data are those closer, because it is right to think that celestial objects with lower radius belong to MW from times of MW formation so these objects may have a better dynamic equilibrium.

11.3 FITTED FUNCTION VELOCITY VERSUS RADIUS AT HALO REGION



According the statistical procedure $v=a\cdot r^b$ Being a = 3.68918E+10 and b = - 0.248717

It is remarkable that the exponent b is almost identical to the one associated to M31 galaxy and very close to - 1/4

$$a_{OPTIMAL} = \frac{\sum_e Ve \cdot r_e^{-0.25}}{\sum_e r_e^{-0.5}} = 3.90787373 \cdot 10^{10}$$

Using Buckingham theorem it has been stated b = - 1/4 so it is needed to recalculate the parameter **a** through the formula as it was made with M31 rotation curve, using the formula for **a** optimal, where V_e is the experimental velocity and r_e is its associated radius.

Table 14 New parameters a&b - A&B for MW	
B	5/3
$b = \frac{B-2}{2B-2}$	b = -1/4
a optimal	$3.90787373 \cdot 10^{10}$ $m^{5/4}/s$
$A = \frac{a^{\frac{-4}{3}}}{8\pi G}$	New parameter A $4.496262 \cdot 10^{-6}$

So the optimal parameter $\mathbf{a_{M\text{-}W}}$ = 3.9E+10 $m^{5/4}/s$

Which is lightly bigger compared with the which one associated to b = - 0.248717

The parameter **a** is similar for similar galaxies, for example $\mathbf{a_{M31}}$ = 4.7275E+10 $m^{5/4}/s$

However parameter b is the same, so it is right to consider that parameter b is constant for different types of galaxies.

Table beside shows the most important parameters for MW at the halo region.

11.4 MASSES SSOCIATED TO MILKY WAY UP TO 2 MPC

Table 15 Radius	Radius	Direct Mass
kpc	m	M_Θ
30,448	9,3953E+20	3,524E+11
33,493	1,033E+21	3,696E+11
36,842	1,137E+21	3,877E+11
40,527	1,251E+21	4,066E+11
44,579	1,376E+21	4,264E+11
49,037	1,513E+21	4,473E+11
53,941	1,664E+21	4,691E+11
59,335	1,831E+21	4,920E+11
65,268	2,014E+21	5,160E+11
71,795	2,215E+21	5,412E+11
78,975	2,437E+21	5,676E+11
86,872	2,681E+21	5,953E+11
95,56	2,949E+21	6,244E+11
770	2,38E+22	1,772E+12
1000	3,09E+22	2,020E+12
2000	6,17E+22	2,856E+12

As it has been demonstrated previously, the direct mass is the Bernoulli mass when parameter C is zero.

$$M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$$ being $a^2/G = 2.2885.10^{31}$

11.5 COMPARING DIRECT MASS WITH RESULTS FROM GAIA DR2 PUBLISHED IN JCAP 2020

In this section will be compared result got by direct mass formula with result published in the prestigious Journal of Cosmology and Astroparticle Physics by [15] E.V. Karukes et al. 2020 in the paper *A robust estimate of the Milky Way mass from rotation curve data.*
These results come from GAIA DR2 and others remarkable sources.

In table below is made the comparison only with the four radiuses bigger than 30 kpc, as DM by gravitation theory only works in the halo region.

In the last column is shown the relative difference between both kind of mass, being quite small indeed.

Table 16 Radius kpc	Radius m	Direct Mass M_Θ	Karukes et al. M_Θ	Relative difference %
45,79	1,4129E+21	4,305E+11	4,27E+11	8,23E-01
74	2,2834E+21	5,473E+11	5,68E+11	-3,78E+00
119,57	3,6896E+21	6,957E+11	7,26E+11	-4,35E+00
193,24	5,9628E+21	8,845E+11	8,95E+11	-1,19E+00

It is awesome how a simple theory which associates only one parameter **a** to the galactic halo, which has been calculated with a data set from 30 kpc up to 95 kpc, is able to give results so close with results got by GAIA DR2 which have been got with the highest current technology and processed through sophisticated software. Notice how the relative difference at 193 kpc is even lower that the ones at 74 kpc or 119 kpc.

The table below comes from **[**15**]** E.V. Karukes et al. In page 25

Table 17 Total mass in MW at some radius
E.V. Karukes et al. 2020 in page 25

Radius kpc	Total mass x 10^{11} M_Θ
45.79	$4.27^{+0.22\,(0.43)}_{-0.19\,(0.37)}$
74.0	$5.68^{+0.40\,(0.83)}_{-0.37\,(0.65)}$
119.57	$7.26^{+0.66\,(1.40)}_{-0.58\,(1.03)}$
193.24	$8.95^{+0.98\,(2.07)}_{-0.84\,(1.48)}$

Notice that there is a perfect concordance, between direct mass and measures if it is considered the interval of errors.

Notice that the parameter **a** was got using a data set ranging from 30 kpc up to 95 kpc. However Direct formula give accuracy calculus for mass not only at 193 kpc but up to 770 kcp and beyond as it will shown in the next chapter.

11.6 RESULTS GOT BY JEFF SHEN ET AL.ApJ.2022 VERSUS DIRECT MASS AT MW HALO

In this epigraph will be compared the results published in The astrophysical journal 2022. See [11] Jeff Shen, with the results calculated by the Direct mass formula in Milky Way.

Abstract

The mass of the Milky Way is a critical quantity that, despite decades of research, remains uncertain within a factor of two. Until recently, most studies have used dynamical tracers in the inner regions of the halo, relying on extrapolations to estimate the mass of the Milky Way. In this paper, we extend the hierarchical Bayesian model applied in Eadie & Juri to study the mass distribution of the Milky Way halo; the new model allows for the use of all available 6D phase-space measurements. We use kinematic data of halo stars out to 142 kpc, obtained from the H3 survey and Gaia EDR3, to infer the mass of the Galaxy. Inference is carried out with the No-U-Turn sampler, a fast and scalable extension of Hamiltonian Monte Carlo. We report a median mass enclosed within 100 kpc of $M(<100\ \mathrm{kpc}) = 0.69^{+0.05}_{-0.04} \times 10^{12}\,M_\odot$ (68% Bayesian credible interval), or a virial mass of $M_{200} = M(<216.2^{+7.5}_{-7.5}\ \mathrm{kpc}) = 1.08^{+0.12}_{-0.11} \times 10^{12}\,M_\odot$, in good agreement with other recent estimates. We analyze our results using posterior predictive checks and find limitations in the model's ability to describe the data. In particular, we find sensitivity with respect to substructure in the halo, which limits the precision of our mass estimates to ~15%.

Above is placed rightly the abstract of the paper where it is possible to see two masses results at different radiuses.

Now it will be calculated the total mass at the same radius with direct mass formula $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$ being $\frac{a^2}{G} = 2.2885 \cdot 10^{31}$, (units in I.S.). In the last column is shown the relative difference between both results.

Table 18 Radius kpc	Total mass x 10^{12} M_Θ Direct formula	Data total mass x 10^{12} M_Θ Jeff Shen et al.	Relative difference %
100	0.64	0.69 ± 0.04	7
216 ± 7.5			
216+7.5	0.96	1.08 ± 0.11	11

As it is shown the relative difference is small especially at 100 kpc. In addition both results match if it is considered the range of errors of measures.

It is important to notice that results by the direct mass formula at 216 kpc is calculated through Dark Matter by gravitation theory using a formula which was got with a data set whose dominion ranges between 30 kpc and 100 kpc.

12. THE MASS CALCULUS FOR THE LOCAL GROUP OF GALAXIES

According [16] Azadeh Fattahi, Julio F. Navarro.2020. The pair MW, M31 has a mass around $5 \cdot 10^{12} M_\Theta$. The authors claim that to suppose there is dynamical equilibrium in the Local Group of galaxies is a plausible hypothesis supported by the fact that only one third of dwarf galaxies belonging to the L.G. are satellite associated to M31 or MW, whereas the other two thirds are linked to global gravitational field of LG. See [17] Azadeh Fattahi, et al. 2020.

According [5] Sofue, (see epigraph 4.6 of his paper) the mutual velocity MW- M31 is 170 Km/s which correspond a dynamical mass equal to $5 \cdot 10^{12}$ M_Θ, however using the current models of DM, the total mass of M31 and Milky Way is approximately $3 \cdot 10^{12}$ M_Θ.

In conclusion, there is a scientific consensus about a lack of mass equal to $2 \cdot 10^{12} M_\Theta$ in the L.G. that the current DM models are not able to explain.
In this epigraph will be demonstrated by the DMbQG theory that the total mass MW-M31 system is $5 \cdot 10^{12}$ M_Θ, so this result is a remarkable success of the theory.
Up to now, in order to do calculus with data of rotation curve, the border of M31 is right to be placed at a half the distance to Milky Way because it is supposed that up to such distance its gravitational field dominates whereas for bigger distances is Milky Way field which dominates. However when it is considered the gravitational interaction between both giant galaxies it is needed to extend their haloes up to 770 kpc, because according DMbQG theory the phenomenon of Dark matter is linked to gravitational field, which is unlimited.

Therefore the M31 halo is extend up to 770 kpc and reciprocally the Milky Way halo is extend up to 770 kpc, when it is calculated the gravitational interaction between both galaxies, and dominion may be extend beyond as the gravitational field is unbounded.

In the following paragraph it will considered MW and its main satellite galaxy LMC as well as M31 and its main satellite M33 in order to do calculus at different radii.

12.1 STIMATING TOTAL MASS FOR THE L.G. AT DIFFERENT RADII

In order to estimate the total mass of Local Group will be considered only M31, M33, MW and LMC. The rest of galaxies have a mass negligible to estimate the total mass of Local Group. In fact M33 add only a 9% of total mass approx. and LMC add only a 2.8 % . It will be used Direct mass formulas to do calculus.

The values of parameters $\mathbf{a^2}$ for the LMC and M33 have been got in the paper [4] Abarca,M.2024 using its rotation curves.

By the formula of Direct mass it is right to get the table of masses at different distances using parameters a^2 associated to galaxies.

Table 19 Galaxies	Parameter a^2 $m^{5/2}/s^2$
M31	2.235E+21
MW	1.527E+21
M33	4E+20
LMC	1.1881E+20
Parameter a^2 Local Group	4.28E+21

Calculus written in table below are only an estimation, as the gravitational interaction between the four galaxies is quite complex. The masses calculated below are in M_Θ units.

Table 20 Radius kpc	Radius m	MW M_Θ	LMC M_Θ	M31 M_Θ	M33 M_Θ	Local Group Total Mass M_Θ
770	2,376E+22	1,772E+12	1,379E+11	2,594E+12	4,63E+11	4,967E+12
1000	3,086E+22	2,020E+12	1,571E+11	2,956E+12	5,27E+11	5,660E+12
1500	4,629E+22	2,474E+12	1,924E+11	3,620E+12	6,46E+11	6,932E+12
2000	6,171E+22	2,856E+12	2,222E+11	4,180E+12	7,46E+11	8,004E+12

In conclusion adding MW+LMC+M31+M33 at 770 kpc the mass calculated is $4.97\cdot 10^{12} M_\Theta$ that match fully with the stated in [16] Azadeh Fattahi, Julio F. Navarro. et al. 2020 and [5] Sofue,Y.2015.

As DMbQG theory stated an unbounded dominion of DM, it is possible to extend the radius. For example at 2Mpc the total mass is $8\cdot 10^{12} M_\Theta$.

These results are a magnificent success of Dark Matter by Quantum Gravitation theory.

13. VIRIAL THEOREM AS A METHOD TO CALCULATE THE DIRECT MASS IN CLUSTERS

In the paper [3] Abarca,M.2024, it is developed fully the DMbQG theory in cluster of galaxies, in this chapter it will be shown a method to calculate the parameter $\mathbf{a^2}$ associate to the cluster, solely.
As the Direct mass formula contains only the parameter $\mathbf{a^2}$ then is enough to know the data pair virial radius and virial mass associated to the cluster.
The property of dynamical equilibrium is crucial to be able to calculate the parameter a^2 with a formula so simple.
If it is considered that the virial radius is the border of halo cluster where galaxies are in dynamical equilibrium and at the same time is negligible the amount of Baryonic matter outside the sphere with such radius, then it is possible to apply the equation $M_{VIRIAL} = M_{DIRECT}(< R_{VIRIAL})$. Then from this equation will be possible to clear up a^2.

$M_{VIRIAL} = M_{DIRECT}(< R_{VIRIAL}) = \dfrac{a^2 \cdot \sqrt{R_{VIRIAL}}}{G}$ Getting the value for $a^2 = \dfrac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}}$ This formula is only a way to estimate parameter a^2 because outside the virial radius always there will be a fraction of the galaxies belonging to cluster. Anyway, this method may estimate a lower bound of parameter a^2 associated to the cluster.

13.1 PARAMETER a^2 ASSOCIATED TO LOCAL GROUP

For example, in the Local Group of galaxies, the dynamical data according [5] Sofue, Y.2015, are 770 kpc for distance between M31 and MW and 170 km/s for its relative velocity, so using $M_{DYNAMIC}(< r) = \frac{v^2 \cdot R}{G}$ it is got a dynamical mass = $M_{LOCAL\text{-}GROUP} = 5.17 \cdot 10^{12} M_\Theta$ = 1.03E+43 kg. Supposing that there is dynamical equilibrium between MW and M31, we have the equation: $Dynamicalmass = \frac{a^2 \cdot \sqrt{r}}{G} = 1.03 \cdot 10^{43}$ kg for a radius = 770 kpc, which leads to the value a^2 = 4.45E+21, which is very close to the parameter a^2 = 4.28E+21 got in the previous chapter adding parameters a^2 associated to M31 plus M33 and MW plus LMC and even these values would be closer if it was considered another galaxies such as the Small Cloud of Magellan and the others dwarfs satellite galaxies of MW and M31. Anyway its relative difference is lower than 4%.

It is remarkable the fact that the parameter a^2 of M31 and MW were calculated with data in the halo whose radius range from 40 kpc up to 300 kpc in M31 case and range from 35 kpc up to 100 kpc in MW case, whereas calculus for parameter a^2 for Local Group has been made with only one data radius 770 kpc and velocity 170 km/s.

So, despite the fact that both methods are independent they give a value to the parameter a^2 a value whose relative difference is only 4%. ***So this fact is another validation to the theoretical findings of DMbQG theory.***

13.2 PARAMERER a^2 ASSOCIATED TO COMA AND VIRGO CLUSTERS

Table 21	Virial Radius	Virial mass
Cluster of galaxies	Mpc	x 10^{14} M_Θ
Virgo [7]Kashibadze 2020	1.7	6.3±0.9
Coma [10] Seong-A. 2023	2.8	27

Beside there are recent data of Coma and Virgo cluster. With such data will be calculated its parameter $\mathbf{a^2}$ with the formula $a^2 = \frac{G \cdot M_{VIRIAL}}{\sqrt{R_{VIRIAL}}}$. Using the data [7] Kashibadze 2020, the Virgo cluster, which is 17 Mpc far away from MW, it is right to get the parameter $\mathbf{a^2}$ = 3.65E+23 $m^{5/2}/s^2$

As it has been comment, this value is a lower bound, because always there will be a fraction of baryonic mass outside from the virial radius. Also, it is clear that parameter $\mathbf{a^2}$ error depend on the measure errors for virial mass and radius.

Similarly using the data for the Coma cluster [10] Seong-A.2023, it is right to get the parameter $\mathbf{a^2}$ = 1.22E+24 $m^{5/2}/s^2$

14. DARK MATTER IS COUNTER BALANCED BY DARK ENERGY

The paper [3] Abarca,M.2024 may be considered as an extension of this chapter. In that paper the DMbQG theory co working with the DE finds unexpected theoretical findings which are able to explain some important open problems for the current cosmology.

The basic concepts about DE on the current cosmology can be studied in [9] Chernin,A.D. 2013

As currently there is a tension regarding the experimental value of Hubble constant, in this paper will be used H =70 Km/s/Mpc and $\Omega_{DE} = 0.7$ as the fraction of Universal density of DE.

14.1 ZERO GRAVITY RADIUS DEPENDING ON PARAMETER a^2 FORMULA

According [9] Chernin,A.D. in the current cosmologic model ΛCDM , dark energy has an effect equivalent to antigravity i.e. the mass associated to dark energy is negative and the dark energy have a constant density for all the Universe equal to $\varphi_{DE} = \varphi_C \cdot \Omega_{DE} = -6.444 \cdot 10^{-27} kg/m^3$ being $\Omega_{DE} = 0.7$ and $\rho_C = \frac{3H^2}{8\pi G}$ = 9.205E-27 kg/m^3 the critic density of the Universe.

As DE density is constant, the total DE mass is proportional to Radius with power 3, whereas DM mass grows with radius power 0.5 so it is right to get a radius where DM is counter balanced by DE.

According [9] Chernin, A.D. The mass associated to DE is $M_{DE}(<R) = -\frac{\rho_{DE} 8\pi R^3}{3}$,(5.1) , or equivalently $M_{DE}(<R) = -\varphi_{DE} \frac{8\pi R^3}{3} = \frac{-H^2 \cdot \Omega_{DE}}{G} \cdot R^3$ (5.2). Notice that the author multiplies by two the volume of a sphere by reasons explained in his work.

[9] Chernin defines gravitating mass $M_G(<R) = M_{DE}(<R) + M_{TOTAL}(<R)$ (5.3), where M_{TOTAL} is baryonic plus dark matter mass, and defines R_{ZG} , Radius at zero Gravity as the radius where $M_{DE}(<R_{ZG}) + M_{TOTAL}(<R_{ZG}) = 0$. i.e. where the gravitating mass is zero.

According DMbQG theory this definition leads to equation $M_{TOTAL}(<R_{ZG}) = \varphi_{DE} \frac{8\pi R_{ZG}^3}{3}$,(5.4).

Using (4.1) formula $M_{TOTAL}(<R_{ZG}) = \frac{a^2 \cdot \sqrt{R_{ZG}}}{G}$ the expression (5.4) leads to $\frac{a^2 \cdot \sqrt{R_{ZG}}}{G} = \rho_{DE} \cdot \frac{8\pi \cdot R_{ZG}^3}{3}$, (5.5), where it is possible to clear up $R_{ZG} = \left[\frac{3a^2}{8\pi G \rho_{DE}}\right]^{2/5}$ (5.6) and as $\varphi_{DE} = \frac{3 \cdot H^2}{8\pi G}\Omega_{DE}$ (5.7) then by substitution $R_{ZG} = \left[\frac{a^2}{H^2 \cdot \Omega_{DE}}\right]^{2/5}$ (5.8) This formula will be called R_{ZG} (parameter a^2).

As the radius R_{ZG} is the distance to cluster centre where is zero the gravitating mass, it is right to consider R_{ZG} as the halo radius and its sphere defined as the halo cluster.

14.2 ZERO GRAVITY RADIUS FOR SOME IMPORTANT CLUSTERS OF GALAXIES

According the parameter a^2 value got in chapter 12 $a_{L-G}^2 = 4.28 \cdot 10^{21}$ it is right to get R_{ZG} =2.19 Mpc. So at that radius the gravitating mass is zero, in other words, for radius under 2,19 Mpc dark matter dominates and for bigger radius dark energy dominates and it is not possible for the Local Group to have any dwarf galaxy linked gravitationally beyond this radius.

According the parameter a^2 value got in chapter 13 for the Coma cluster it is right to get R_{ZG} = 21 Mpc .In other words 21 Mpc is the radius of region where the DM of Coma Cluster dominates versus dark energy.

Similarly for the Virgo cluster the parameter $\mathbf{a^2}$ = 3.65E+23 leads to R_{ZG} = 12,97 Mpc.

Table 22	parameter $\mathbf{a^2}$ Units $m^{5/2}/s^2$	Zero Gravity Radius
Local Group	4.28E+21	2.19 Mpc
Virgo Cluster	3.65E+23	12.97 Mpc
Coma Cluster	1.22E+24	21 Mpc

Beside it is summarized the previous calculus.

With these three important clusters of

galaxies, it has been illustrated how the total mass, estimated by $M_{DIRECT}(<r) = \frac{a^2 \cdot \sqrt{r}}{G}$, is counter balanced by the dark energy at mega parsecs scale, and precisely this Radius at zero gravity determines the region size where the cluster has gravitational influence.

15. CONCLUDING REMARKS

As it has been outlined at the introduction, this work is the consequence of the new set of data for rotation curve of Milky Way published by Sofue in 2020. With these new data, it is possible to state that the rotation curve of MW at halo is governed by the ideal curve named Buckingham halo curve, which has the same exponent for M31 and Milky Way galaxies in the framework of DMbQG theory.

This fact back strongly the main hypothesis of Dark gravitation theory i.e. Dark matter is generated according an unknown quantum gravitational mechanism, which depend on the gravitational field, so it is a Universal law.

Through the first ten chapters is developed the theory using M31 rotation curve. This chapters are identical to the previous paper [2] Abarca,M.2019, excepting the process of getting the parameter C. See epigraph 9.7. In this work, it has been found that the ideal curve called *Buckingham halo curve* linked to dynamical equilibrium of systems leads to the value of parameter C equal zero and then the Bernoulli mass becomes direct mass formula.

In the chapter 10 are compared the Bernoulli mass versus Direct mass and it is shown that its relative difference is negligible so it is preferable to consider the direct mass because it contains only the parameter $\mathbf{a^2}$.
In the chapter 11 has been calculated parameter $\mathbf{a^2}$ associated to Milky Way halo using the Sofue data set of rotation curve into the halo region and it is calculated Direct mass at different radii up to 2 Mpc. These results got by the Direct mass are compared with data published by two prestigious astrophysics teams in a dominion radius which ranges from 45 kpc up to 220 kpc. The relative differences are below 4 % regarding a team and below 11% regarding the other team, so these tests are a successfully experimental validation of the Direct mass formula.

In the chapter 12 is calculated the direct mass associated to the Local Group, considering the parameter $\mathbf{a^2}$ associated to MW and its satellite galaxy, the LMC as well as M31 and its satellite M33. The direct mass calculated for the Local Group is $5 \cdot 10^{12}\ M_\Theta$ at 770 kpc. There is no any other theory of DM able to justify theoretically such amount of mass. For example with the model NFW the total mass for the Local Group is $3 \cdot 10^{12} M_\Theta$, so this calculus is a big success of DMbQG theory.

In the chapter 13 it is shown a method to estimate the Direct mass formula for a cluster of galaxies, only with its Virial Mass and radius. Using this method it is estimated the parameter a^2 of L.G. which match with the one calculated in previous chapter. Also are calculated the parameters a^2 associated to Virgo and Coma clusters.
In the chapter 14, it is introduced the Radius Zero gravity, defined by [9] Chernin,A.D. et all.2013 and it is calculated its formula in the framework of DMbQG theory. Also it is found that this radius is 2.2 Mpc for the LG, 13 Mpc for Virgo and 21 Mpc for Coma cluster.
This chapter is crucial to demonstrate how the Dark energy is able to counterbalance the DM at cluster scale, because the Direct mass grows up with the square root of radius whereas the DE grows up with the cubic power. In the paper [3] Abarca,M.2024 it is fully developed the DMbQG theory for cluster of galaxies and there were found remarkable theoretical results validated with measures in cluster of galaxies published by well known researchers.

This theory introduces a powerful method to study DM in the halo region of galaxies and cluster of galaxies and conversely measures in galaxies and clusters offer the possibility to validate the theory.

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