Approaching Goldbach's conjecture using the asymmetric relationship between primes and composites within a limited even boundary

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Abstract (247 Words)

When an arbitrary integer n is chosen, the set of consecutive numbers from 0 to n is considered the 1st boundary and it expands using an arithmetic sequence with n elements but limited to n^2, or the n^{th} boundary. Except for 1, each number in the 1st boundary can be expressed in the wave form of '$y_n = sin\,(\frac{180}{n} \cdot x)$', it connects to composites located between the 2nd and n^{th} boundaries. Through wave multiplication, the wave of a composite, such as y_4, overlaps with the wave of a prime, y_2, can be eliminated, leaving only the waves of primes. Except for the prime factors of n, the waves of primes in the 1st boundary now have an asymmetrical relationship with the composites located between the 2nd and n^{th} boundaries. If 'y_1' is divided by the product of prime waves, the asymmetrical relationship between primes and composites is eliminated, leaving only new primes passively remaining between the 2nd and n^{th} boundaries. Under the same conditions, the boundary can be limited to $2n$ (or 2nd boundary) instead of n^2 (or n^{th} boundary). When $n \geq 4$, there is at least one prime in the 1st boundary, excluding the even prime 2. This ensures that a new prime, which maintains the same distance from n and forms a symmetric relationship, can be paired. The sum of this pair - a caused prime in the 1st boundary and an effected new prime in the 2nd boundary - is always $2n$, thereby satisfying Goldbach's conjecture.

Keywords. Asymmetry, Composites, Goldbach's conjecture, New primes, Primes

Mathematics Subject Classification Number*:* 11P32

1. Introduction

Goldbach's conjecture has been studied with two major streams: strong conjecture and weak conjecture. The strong conjecture states that every even number greater than 2 is the sum of two primes while the weak conjecture is composed of odd numbers greater than 5 and the sum of three primes [2, p.702-706]. Goldbach's conjecture is an extension of the prime rules, so it has been studied for over 300 years to understand the primes, but there still remain unsolved problems in the field of number theory.

Goldbach's conjecture is governed by primes, so understanding the behaviour of primes is key addressing Goldbach's conjecture. Therefore, this paper aims to prove Goldbach's conjecture through the following three series: Series I. Characteristics of primes within a limited boundary, Series II. Approaching Goldbach's conjecture within an even boundary, and Series III. Verifying Goldbach's conjecture using mathematical expressions.

2. Materials and methods

The sine waves were visualized in the Desmos, online graphing calculator (www.desmos.com). The visualized graphs were exported and additional graph modification was performed in Illustrator (CS6, Adobe, CA, USA).

2.1 Procedure for approaching Goldbach's conjecture

Prior to approaching Goldbach's conjecture, the characteristics of primes were defined within a limited boundary in series I. Within this boundary, the asymmetric relationship between primes and composites was characterized, which led to an understanding of the symmetric relationship between primes and new primes in series II. In series III, mathematical expressions were used to validate the symmetrical relationship between primes and new primes.

In this paper, a boundary is defined as follows: When an arbitrary positive integer n is chosen, the set of consecutive numbers from 0 to n is considered the 1st boundary. Similar to the Sieve of Eratosthenes [3], the boundary can be expanded using an arithmetic sequence with n elements, and the expansion range is limited to n^2 or the n^{th} boundary (Figure 1).

2.2 Series I: Characteristics of primes within a limited boundary

When an arbitrary positive integer n is chosen, all natural numbers less than or equal to n, except for 1, can be expressed in the form of '$y_n = sin\,(\frac{180}{n} \cdot x)$' (Figure 1A).

$$y_2 = sin\,(\frac{180}{2} \cdot x),\ y_3 = sin\,(\frac{180}{3} \cdot x),\ y_4 = sin\,(\frac{180}{4} \cdot x),\ y_5 = sin\,(\frac{180}{5} \cdot x),\ \ldots,\ y_n = sin\,(\frac{180}{n} \cdot x)$$

The waves of primes less than or equal to n in the 1st boundary connect to the composites, and the remaining numbers are all new primes between 2nd and n^{th} boundaries. Except for the prime factors of n, all primes in the 1st boundary have an asymmetrical relationship with the composites located between the 2nd and n^{th} boundaries.

Based on the point where y_n is 0, the sine wave of a composite, such as y_4, overlaps with the wave of a prime, y_2. In this way, the waves of composites can be eliminated through wave multiplication, leaving only the waves of primes, which can be expressed as

$$y_2 \cdot y_3 \cdot y_5 \cdot \ldots \cdot y_n$$

or

$$\prod_{P \leq n} sin\,(\frac{180}{P} \cdot x)$$

, where *P* represents all primes less than or equal to *n*. The asymmetrical relationship between primes in the 1^{st} boundary and composites between the 2^{nd} and n^{th} boundaries is now represented by a single rhythmic wave (Figure 1B).

If 'y_1' or '$sin\,(\frac{180}{1} \cdot x)$' is divided by the product of prime waves, '$y_2 \cdot y_3 \cdot y_5 \cdot \ldots \cdot y_n$', the asymmetric relationship between primes and composites is eliminated. Only new primes will remain between the 2^{nd} and n^{th} boundaries on the *x-axis* where *y* is 0, and this can be expressed as

$$y_n = \frac{y_1}{y_2 \cdot y_3 \cdot y_5 \cdot \ldots \cdot y_n}$$

or

$$y_n = \frac{sin(180 \cdot x)}{\prod_{P \leq n} sin\,(\frac{180}{P} \cdot x)}$$

, where *P* represents all primes less than or equal to *n* and the boundary is limited to n^2, or the n^{th} boundary (Figure 1C).

2.3 Series II: Approaching Goldbach's conjecture within an even boundary

Under the same conditions as *Series I*, Goldbach's conjecture can be defined as

$$y_n = \frac{sin(180 \cdot x)}{\prod_{P \leq n} sin\,(\frac{180}{P} \cdot x)}$$

, where *P* represents all primes less than or equal to *n* and the boundary is limited to 2*n* within n^2 (Shaded area in Figure 1). Except for the prime factors of *n* and prime 2, the waves of all primes in the 1^{st} boundary have an asymmetrical relationship with composites, remaining numbers which are not affected by the continuous waves of primes less than or equal to n are all new primes in the 2nd boundary. Within an even boundary of $2n$ or the 2nd boundary, if n is greater than or equal to 4 in the 1st boundary, it ensures that a new prime, maintaining the same distance from n and forming a symmetric relationship, may also exist. Therefore, the sum of a paired prime in the 1st boundary and a new prime in the 2nd boundary will always be $2n$.

2.4 Series III: Approaching Goldbach's conjecture using mathematical expressions

Goldbach's conjecture is satisfied when every even integer can be expressed as the sum of two primes. So, let $p + q = 2n$, where p and q are primes ($2 < p < q$). The equation can be organized around n.

$$n = \frac{p+q}{2}$$

Based on the boundary's characteristics [1], the organized equation can be written as follows between 0 and $2n$.

$$\mathbf{0}, 1, 2, \ldots, (n-3), (n-2), (n-1), \boldsymbol{n}, (n+1), (n+2), (n+3), \ldots, (2n-2), (2n-1), \mathbf{2}\boldsymbol{n}$$

Two different boundaries are considered from n: the 1st boundary between 0 and n and the 2nd boundary between n and $2n$. In the 1st boundary, the largest number is $(n-1)$ while the smallest one is $(n+1)$ in the 2nd boundary.

Suppose $(n-1)$ and $(n+1)$ are primes, and assume that they are each summed with themselves.

$$(n-1) + (n-1) = 2n - 2 < 2n$$

or

$$(n+1) + (n+1) = 2n + 2 > 2n$$

If the primes are each summed with themselves in each first or second boundary, the sum of the two primes is not equal to $2n$. Therefore, two primes should be selected from each of the first and second boundary respectively.

Suppose that $(n - \alpha)$ and $(n + \beta)$ are primes, p and q, selected from the first and second boundary respectively, where α and β are any positive integers (α and $\beta < n$).

$$p = (n - \alpha)$$

$$q = (n + \beta)$$

, where $0 < (n - \alpha) < n$ and $n < (n + \beta) < 2n$.

Using the definition of Goldbach's conjecture, two primes are summed.

$$p + q = (n - \alpha) + (n + \beta)$$

$$p + q = 2n - \alpha + \beta$$

At first, the sum of primes, $p + q$, is defined by $2n$. Thus,

$$2n = 2n - \alpha + \beta$$

$$\alpha = \beta.$$

As a result, two primes, p and q, from their respective boundaries can be written as follows.

$$p = n - \alpha \text{ and } q = n + \alpha$$

or

$$p = n - \beta \text{ and } q = n + \beta$$

Overall, it is concluded that Goldbach's conjecture is satisfied when the two primes, p and q, are placed in the 1st and 2nd boundaries respectively by maintaining the same distance from n. Considering *Series I* and *II*, there is at least one prime, p, in the 1st boundary when the arbitrary integer n is greater than or equal to 4. This ensures that a new prime, q, which maintains the same distance from n and forms a symmetric relationship, can be paired. The sum of this pair, $p + q$, therefore, is always $2n$ by satisfying Goldbach's conjecture.

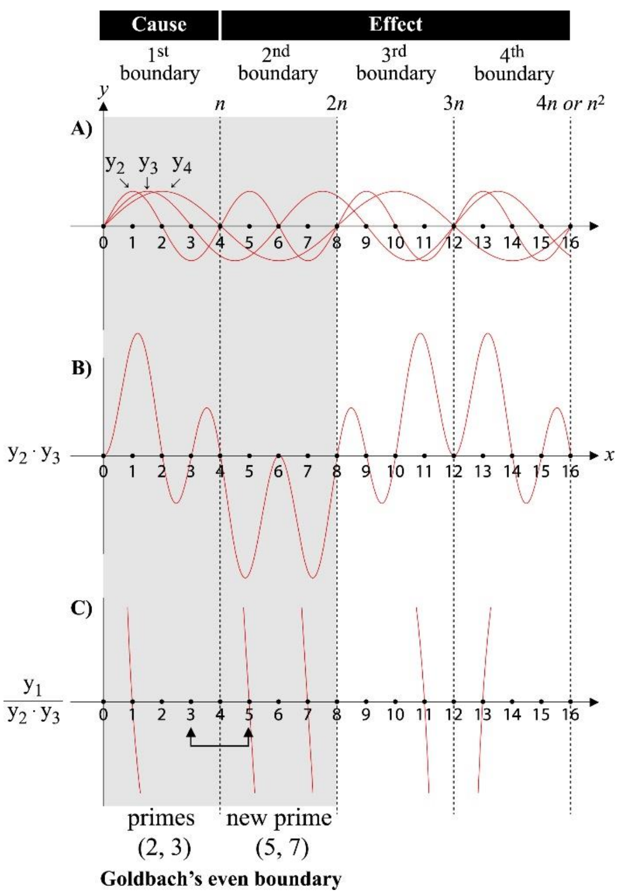
3. Results and conclusions

If the number 4 is chosen for an arbitrary positive integer n (1st boundary), the even boundary is limited to 8 (2nd boundary) and waves of 'y_2', 'y_3', and 'y_4' can be generated (Figure 1A). Since the wave of composite, y_4, overlaps with the wave of prime, y_2, the y_4 can be eliminated through wave multiplication, leaving only the waves of primes. The asymmetrical relationship, represented by a single rhythmic wave between primes in the 1st boundary and composites in the 2nd boundary (Figure 1B), is eliminated when 'y_1' is divided by the product of the prime waves, $y_2 \cdot y_3$, leaving only new primes (5 and 7) in the 2nd boundary. The new primes 3 and 5 are generated by the prime waves of 2 and 3. Since 3 and 5 are positioned at the same distance from 4, the sum of this prime pair satisfies Goldbach's conjecture (Figure 1C).

If the number 3 is chosen for an arbitrary positive integer n (1st boundary), the even boundary is limited to 6 (2nd boundary), and only the new prime 5 is generated in the 2nd boundary due to the wave of prime 2 from the 1st boundary. The primes 2 and 5 do not maintain the same distance from the midpoint of the even boundary, which is 3. Therefore, the smallest positive integer n that satisfies Goldbach's conjecture is 4.

Overall, it is concluded that if n is greater than or equal to 4, then there is at least one prime in the 1st boundary, excluding the even prime 2. This guarantees that new primes in the 2nd boundary, forming a symmetric relationship by maintaining the same distance from n, may exist. Therefore, the sum of each pair, consisting of a caused prime in the 1st boundary and an effected new prime in the 2nd boundary, is always $2n$. This suggests that the asymmetrical relationship between primes and composites forms a symmetrical cause-and-effect relationship between primes and new primes, thereby satisfying Goldbach's conjecture.

Figure 1. The arbitrary positive integer n is set to 4 for convenience in the explanation. A) All natural numbers less than or equal to 4, except for 1, can be expressed in the form of '$y_n = sin\ (\frac{180}{n} \cdot x)$'. The set of consecutive numbers from 0 to 4 is considered the 1st boundary, and it can be expanded to 16 (4^2) or 4th boundary. B) Based on the point where y is 0, the sine wave of a composite, such as y_4, overlaps with the wave of a prime, y_2, after the waves are multiplied. In this way, the wave of composites can be eliminated, leaving only the primes where y is not 0 between 2nd and 4th boundaries. C) If 'y_1' is divided by the product of prime waves, $y_2 \cdot y_3$, then the asymmetric relationship between primes and composites will be excluded. Only new primes will remain on the *x-axis* where y is 0. If the boundary is limited to 2nd under the same conditions, asymmetrical relationship between primes and composites still remains and it forms a symmetrical cause-and-effect relationship between a pair of prime and a new prime, thereby satisfying Goldbach's conjecture.



4. Declaration of interests

The author does not work for, advise, own shares in, or receive funds from any organization that could benefit from this article, and have declared no affiliation other than their research organisations.

5. References

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