Approaching Goldbach's conjecture using the asymmetric relationship between primes and composites within a limited even boundary

Junho Eom*

Department of Zoology, University of British Columbia, Vancouver, BC, V6T 1Z4, Canada

E-mails: zuno3302@gmail.com

Abstract (202 Words)

The core of this paper is to reveal the structural necessity that causes primes and new primes to form a symmetry, occurring from the cause-and-effect relationship between primes and composites. Regarding the boundary, if an arbitrary integer *n* is chosen, the set of consecutive numbers from 0 to *n* is defined as the 1st boundary and it extends using an arithmetic sequence with *n* elements but limits to n^2. After selecting *n*, therefore, *n* boundaries are generated from the 1st to *n*th, and each boundary contains *n* elements. Each prime wave in the 1st boundary connects to the composites that use the prime as a factor, and the remaining numbers between the 2nd and *n*th boundaries on the *x-axis* are all new primes in *Series I*. Under this condition, the primes in the 1st boundary and the new primes in the 2nd boundary form symmetry around the midpoint of even 2*n* caused by the asymmetry between primes and composites, Goldbach's conjecture is satisfied in *Series II* and *III*. Therefore, *Series IV* explains the necessity for the primes and new primes to form a structural symmetry using 2 and 3, and discusses how this symmetry repeats at intervals of 30, generated by 5.

Keywords. Asymmetry, Composites, Goldbach's conjecture, New primes, Primes, Symmetry

Mathematics Subject Classification Number*:* 11P32

1. Introduction

Goldbach's conjecture has been studied with two major streams: strong conjecture and weak conjecture. The strong conjecture states that every even number greater than 2 is the sum of two primes while the weak conjecture is composed of odd numbers greater than 5 and the sum of three primes [3]. Goldbach's conjecture is an extension of the prime rules, so it has been studied for over 300 years to understand the primes, but there still remain unsolved problems in the field of number theory.

Goldbach's conjecture is governed by primes, so understanding the behavior of primes is a key to approach the conjecture. Thus, this paper initially identifies the characteristics of the primes and approaches Goldbach's conjecture through the following four series: *Series I*. Characteristics of primes within a limited boundary, *Series II*. Approaching Goldbach's conjecture within an even boundary, *Series III*. Approaching Goldbach's conjecture using mathematical expressions, and *Series IV*. The role of the primes 2 and 3 in Goldbach's conjecture. After that, the paper discusses the structural necessity that leads primes and new primes to form a symmetric structure based on the characteristics of primes described above in the four series.

2. Methods and Results

The sine waves are visualized in the Desmos, online graphing calculator (www.desmos.com). The visualized graphs are exported and additional graph modification is performed in Illustrator (CS6, Adobe, CA, USA).

2.1 Procedure for approaching Goldbach's conjecture

Prior to approaching Goldbach's conjecture, the characteristics of primes are defined within a limited boundary in *series I*. Within this boundary, the asymmetric relationship between primes and composites is characterized, which leads to an understanding of the symmetric relationship between primes and new primes in *series II*. In *series III*, mathematical expressions are used to validate the necessity of a symmetrical relationship between primes and new primes which satisfies Goldbach's conjecture. In *Series IV*, the role of primes 2 and 3 explains how all prime candidates are structurally formed within a symmetry structure.

In this paper, a boundary is defined as follows: When an arbitrary positive integer n is chosen, the set of consecutive numbers from 0 to n is defined as the 1^{st} boundary. Similar to the Sieve of Eratosthenes [4], the boundary extends using an arithmetic sequence with n elements, but limits to n^2 or the n^{th} boundary (Figure 1). After selecting n, therefore, n boundaries are generated from the 1^{st} to n^{th}, and each boundary contains n elements.

2.2 Series I: Characteristics of primes within a limited boundary

When an arbitrary positive integer n is chosen ($n \neq 1$), the 1st boundary is defined between 0 and n, and all integers less than or equal to n, except for 1, can be expressed in the form of '$y_n = sin\,(\frac{180}{n} \cdot x)$' (Figure 1A).

$$y_2 = sin\,(\frac{180}{2} \cdot x), y_3 = sin\,(\frac{180}{3} \cdot x), y_4 = sin\,(\frac{180}{4} \cdot x), y_5 = sin\,(\frac{180}{5} \cdot x), \ldots, y_n = sin\,(\frac{180}{n} \cdot x)$$

Each integer wave in the 1st boundary connects to the composites, and the remaining numbers on the *x-axis* which are not affected by the continuous waves are all new primes from the 2nd to nth boundaries (or from n to n^2).

Through the multiplication of waves, the composite wave, such as y_4, overlaps with the prime wave, y_2 on the *x-axis*. Thus, the intersected points on the *x-axis* are the same for both the product of integer waves and the product of prime waves only. Therefore, the multiplication of waves less than n can be expressed with the wave of primes as

$$y_2 \cdot y_3 \cdot y_5 \cdot y_7 \cdot y_{11} \ldots \cdot y_n$$

or

$$\prod_{P \leq n} sin\,(\frac{180}{P} \cdot x)$$

, where P is primes less than or equal to n (Figure 1B).

If y_1 is divided by the multiplication of prime waves, the primes and their associated composites on the *x-axis* cannot be defined. Only the integers that are not affected by the continuous multiplication of prime waves from the 1st boundary passively remain from the 2nd to the nth boundaries on the *x-axis*, and all of these are new primes. This can be expressed as

$$y = \frac{y_1}{y_2 \cdot y_3 \cdot y_5 \cdot \ldots \cdot y_n} = \frac{sin(180 \cdot x)}{\prod_{P \leq n} sin\,(\frac{180}{P} \cdot x)}$$

, where the boundary is limited to n^2, or the nth boundary (Figure 1C). Therefore, the composites between the 2nd and nth boundaries are caused by the primes in the 1st boundary, leading to an indirect cause-and-effect relationship between the primes in the 1st boundary and the new primes between the 2nd and nth boundaries.

2.3 Series II: Approaching Goldbach's conjecture within an even boundary

Under the same conditions as *Series I*,

$$y = \frac{sin(180 \cdot x)}{\prod_{P \le n} sin\,(\frac{180}{P} \cdot x)}$$

, the boundary can be limited to $2n$ (or 2nd boundary) within n^2 (or nth boundary) (Shaded area in Figure 1). With respect to n, if any of the prime wave y_p in the 1st boundary has an asymmetric relationship with the composites connected from the continuous wave of y_p in the 2nd boundary, the symmetric point $2n - P$ in the 2nd boundary is an odd number that is not factorized by P. If $2n - P$ is not affected (or not factorized) by any other primes less than $\sqrt{2n - P}$ in the 1st boundary, then $2n - P$ is confirmed as the new prime. Thus, the independence of $2n - P$ from P is the primary factor in determining the symmetry between P and $2n - P$. As the prime P and the new prime $2n - P$ maintain the same distance from n, the sum of P and $2n - P$ is $2n$.

2.4 Series III: Approaching Goldbach's conjecture using mathematical expressions

Goldbach's conjecture is satisfied when every even integer, except for 2, can be expressed as the sum of two primes. So, let $p + q = 2n$, where p and q are primes ($2 < p < q$). The equation can be organized around n.

$$n = \frac{p+q}{2}$$

The organized n can be written as follows between 0 and $2n$.

$$\mathbf{0}, 1, 2, \ldots, (n-3), (n-2), (n-1), \boldsymbol{n}, (n+1), (n+2), (n+3), \ldots, (2n-2), (2n-1), \boldsymbol{2n}$$

Two different boundaries are considered from n: the 1st boundary between 0 and n and the 2nd boundary between n and $2n$; each boundary contains n elements. In the 1st boundary, the largest number is $(n - 1)$ while the smallest one is $(n + 1)$ in the 2nd boundary. Suppose $(n - 1)$ and $(n + 1)$ are primes, and assume that they are each summed with themselves as follows.

$$(n-1) + (n-1) = 2n - 2 < 2n$$

or

$$(n+1) + (n+1) = 2n + 2 > 2n$$

If the primes are each summed with themselves in either the 1st or 2nd boundary, the sum of the two primes is not equal to $2n$. Therefore, two primes should be selected from each of the first and second boundary, respectively.

Suppose that $n - \alpha$ and $n + \beta$ are primes p and q, respectively, and they are selected from the 1st and 2nd boundaries, where α and β are any positive integers less than n. Then, primes p and q are expressed as

$$p = n - \alpha \text{ and } q = n + \beta$$

, where $0 < n - \alpha < n$ and $n < n + \beta < 2n$.

Using the definition of Goldbach's conjecture, two primes are summed.

$$p + q = (n - \alpha) + (n + \beta) = 2n - \alpha + \beta$$

At first, the sum of primes, $p + q$, is defined by $2n$. Thus, the above equation is expressed as follows.

$$2n = 2n - \alpha + \beta$$

$$\alpha = \beta$$

As a result, two primes, p and q, from their respective boundaries can be written as follows.

$$p = n - \alpha \text{ and } q = n + \alpha$$

or

$$p = n - \beta \text{ and } q = n + \beta$$

Therefore, Goldbach's conjecture is satisfied when the two primes, p and q, are placed in the 1st and 2nd boundaries, respectively, maintaining the same distance from n. This is consistent with the conclusion derived from *Series II*.

2.5 Series IV: The role of the primes 2 and 3 in Goldbach's conjecture

The role of the primes 2 and 3 is to generate all prime candidates at $6x \pm 1$ and arrange them in positions that have a value difference of 2, which is a twin prime position. Therefore, all primes are selected within the symmetrically paired twin prime candidates. For example, using the wave of primes 2 and 3 (or y_2 and y_3), any composite, which uses 2 and/or 3 as a factor, are produced and the passively remaining numbers, which are not affected by the continuous waves of 2 and 3, are regularly formed at $6x$

± 1 ($x \geq 1$), on the *x-axis*. Since the primes 2 and 3 are already used, the role of producing additional composites within $6x \pm 1$ is now transferred from the wave of primes 2 and 3 to the wave of primes 5 and 7, which is the first set of twin primes within $6x \pm 1$ ($x = 1$).

Due to the primes 2 and 3, the even boundary that satisfies Goldbach's conjecture extends from 16 down to 8. For example, as the symmetrical twin prime candidates repeat every multiple of 6, the midpoint of any even number is limited to the following six cases

$$6x,\ 6x + 1,\ 6x + 2,\ 6x + 3,\ 6x + 4,\ 6x + 5$$

, where $x \geq 0$, then the even number is

$$2 \cdot 6x,\ 2 \cdot (6x + 1),\ 2 \cdot (6x + 2),\ 2 \cdot (6x + 3),\ 2 \cdot (6x + 4),\ 2 \cdot (6x + 5)$$

, respectively. As the midpoint shifts from $6x$ to $6x + 5$, the paired candidates that are symmetrical around the midpoint and whose sum is the even number are also relatively shifted. If the midpoint is $6x$, then the symmetric pair of candidates around $6x$ and whose sum is $12x$ are expressed as

$$(6x + (6y + 1) \text{ and } 6x - (6y + 1)) \text{ or } (6x + (6y - 1) \text{ and } 6x - (6y - 1)).$$

If the midpoint is $6x + 1$, then the symmetric pair of candidates and whose sum is $2 \cdot (6x + 1)$ are expressed as

$$(6x + 1 + 6y \text{ and } 6x + 1 - 6y) \text{ or } (3 \text{ and } 6(2x) - 1).$$

If the midpoint is $6x + 2$, then the symmetric pair of candidates and whose sum is $2 \cdot (6x + 2)$ are expressed as

$$(6x + 2 + (6y + 3) \text{ and } 6x + 2 - (6y + 3)) \text{ or } (3 \text{ and } 6(2x) + 1).$$

If the midpoint is $6x + 3$, then the symmetric pair of candidates and whose sum is $2 \cdot (6x + 3)$ are expressed as

$$(6x + 3 + (6y + 4) \text{ and } 6x + 3 - (6y + 4)) \text{ or } (6x + 3 + (6y - 4) \text{ and } 6x + 3 - (6y - 4)).$$

If the midpoint is $6x + 4$, then the symmetric pair of candidates and whose sum is $2 \cdot (6x + 4)$ are expressed as

$$(6x + 4 + (6y - 3) \text{ and } 6x + 4 - (6y - 3)) \text{ or } (3 \text{ and } 6(2x + 1) - 1)$$

If the midpoint is $6x + 5$, then the symmetric pair of candidates and whose sum is $2 \cdot (6x + 5)$ are expressed as

$$(6x + 5 + 6y \text{ and } 6x + 5 - 6y) \text{ or } (3 \text{ and } 6(2x + 1) + 1)$$

, where $x \geq y$. Excluding 3, the consecutive even number initiates at 16, with prime pairs occurring between $6x + 2 + (6y + 3)$ and $6x + 2 - (6y + 3)$, where $x = 1$ and $y = 0$. Thus, the prime pair is 11 and 5, centered around the midpoint $6x + 2$, which is 8. With the inclusion of 3, however, the consecutive even number initiates with prime pairs between 3 and $6(2x + 1) - 1$, such as 3 and 5 when $x = 0$ and $y = 0$, centered around the midpoint $6x + 4$, which is 4. Also, the prime 3 satisfies 10 and 14 Goldbach's even number when the midpoint is 5 and 7. Due to the prime 3, therefore, the consecutive even numbers satisfying Goldbach's conjecture extends from 16 down to 8.

Overall, the primes 2 and 3 form the basic symmetrical structure of potential primes that satisfies Goldbach's conjecture. As the prime 3 is caused by the prime 2, both the primes 2 and 3 play a role in extending the Goldbach's even number from 16 down to 8. If the repeated use of primes is allowed, the number is further extended from 8 down to 4 using the primes 2 and 3.

3. Discussions

The core of this paper is to prove Goldbach's conjecture by understanding the symmetrical structure between the primes and passively remaining new primes caused by the asymmetrical relationship between the primes and composites; as a result, the cause-and-effect relationship between the primes and the new primes with respect to the 1st and 2nd boundaries can satisfy Goldbach's conjecture in the wave analysis described in *Series I* and *II*. Using the wave of primes 2 and 3, all prime candidates are produced at $6x \pm 1$ ($x \geq 1$), confirmed 5 and 7 as the first new primes, and the role of primes 2 and 3 is completed or no composites exist that use 2 and/or 3 as factors in $6x \pm 1$. This means that all primes, greater than or equal to 5, occur within the symmetry structure of twin prime position, thus, Goldbach's conjecture can be proven by structurally verifying the symmetrical structure between the primes and new primes within the symmetry structure of prime candidates.

3.1. The structural characteristics of prime candidates at 6x ± 1

Two types of prime candidates exist, Type A ($6x - 1$) and Type B ($6y + 1$). If $x = y$, then Type A and B form the twin prime structure, while $x \neq y$ and the center of symmetry is still formed in the multiple of 6, but does not form the twin prime structure. Regardless, the symmetry center of two different prime candidates is formed around the average between Type A and B, and it is expressed as follows.

$$\frac{Type\ A + Type\ B}{2}$$

Each Type A and B produce the composites through the product of either Type A′ (or $6x' - 1$) (Type A ≤ Type A′) or Type B′ (or $6y' + 1$) (Type B ≤ Type B′); in results, the composites are paired between Type A·Type A′ and Type A·Type B′ or between Type B·Type A′ and Type B·Type B′. The symmetry center of paired composites is expressed as

$$\frac{\text{Type A} \cdot \text{Type A'} + \text{Type A} \cdot \text{Type B'}}{2}\ or\ \frac{\text{Type B} \cdot \text{Type A'} + \text{Type B} \cdot \text{Type B'}}{2}$$

, and the interval is

$$\frac{\text{Type } A_{n+1} \cdot \text{Type} A'_{n+1} + \text{Type } A_{n+1} \cdot \text{Type } B'_{n+1}}{2} - \frac{\text{Type } A_n \cdot \text{Type } A'_n + \text{Type } A_n \cdot \text{Type } B'_n}{2}$$

or

$$\frac{\text{Type } B_{n+1} \cdot \text{Type} A'_{n+1} + \text{Type } B_{n+1} \cdot \text{Type } B'_{n+1}}{2} - \frac{\text{Type } B_n \cdot \text{Type } A'_n + \text{Type } B_n \cdot \text{Type } B'_n}{2}$$

For example, the first confirmed prime 5 is Type A and it routinely produces the composites by the product of Type A′ and Type B′, so the paired composites $5(6x' - 1)$ and $5(6y' + 1)$, such as (25, 35), (55, 65), (85, 95), and more, are produced by the interval of 30. Similarly, the prime 7, which is Type B, also produces the composites, such as (35, 49), (77, 91), (119, 133), and more, by the interval of 42. If Type A and/or Type B is non-prime, the generated composites are disregarded as they overlap with the wave of Type A prime and/or Type B prime on the *x-axis*. In this way, the composite pairs produced by either prime 5 or prime 7 are symmetrically positioned at $30x \pm 5$ or $42x \pm 7$ and the effect is that the potential new primes are also symmetrically positioned on the opposite side of composite pairs, $30x \pm 7$ or $42x \pm 5$. In other words, the structural characteristics of $6x \pm 1$ enable the Type A and B primes to produce the symmetrically paired potential Type B′ and A′ new primes. Meanwhile, the remaining other prime candidates are placed in the twin prime positions.

3.2. The repeated coexistence of symmetry and asymmetry by the prime 5

Within symmetrically structured twin prime candidates, the initial prime 5 routinely produces symmetry composites between $30x$ and $30(x + 1)$, where $x \geq 1$; as a result, maximum 8 prime candidates, consisting of 2 pairs of twin prime candidates and two of each prime candidates at the position of Type A and Type B, are produced. Both Type A and B prime candidates can form symmetry with Type B′ and A′, forming $30x \pm 1$ and/or $30(x + 1) \pm 1$, and it leads the increasing potential pair of twin prime candidates from 2 to 4. The primes less than $\sqrt{30x}$ produce the paired composites from the respective symmetry center of each prime, and remaining candidates which are not affected by the symmetrically produced composites are determined as new primes less than 30*x*. If the 1st boundary is defined between 0 and 30*x*, the 2nd boundary is ranged between 30*x* to 30(*x* + 1); additional new primes are selected by the primes between $\sqrt{30x}$ and $\sqrt{30(x+1)}$; as a result, additional new composites are involved between 30*x* and 30(*x* + 1). For example, if the 1st boundary is defined between 0 and 30, the 2nd boundary is ranged between 30 to 60; additional new primes between $\sqrt{30}$ (or 5.47) and $\sqrt{60}$ (or 7.75) are selected, which is the prime 7, and this produces the new composites between 30 and 60. Mathematically, the center of prime 7 passes in an equation of $\left\lfloor\frac{60}{42}\right\rfloor - \left\lfloor\frac{30}{42}\right\rfloor = 1$, and it means that the symmetry center of 7, which is 42, passes between 30 and 60 once, thereby paired candidates from 42, which is (35, 49), are confirmed as additional composites only for the 2nd boundary. From the perspective of a wave, decimal points can be rounded down. The new composite pair (35, 49) affects the previously paired prime candidates (35, 37) and (47, 49). As 35 is overlapped with the prime 5, only 49 is confirmed as an additional composite between 30 and 60. Since (35, 49) is paired around 42, the remaining prime candidates (37, 47) which are located on the opposite side of the composite pair are confirmed as symmetrically paired new primes. As a result, one pair of twin primes (41, 43), three Type A (47, 53, 59), and two Type B (31, 37) primes are confirmed as new primes between 30 and 60.

Each prime 5 and 7 produces symmetrically paired composites but due to the different symmetry center, 30 and 42, the partial irregularities (or asymmetry) occurs until it reaches the least common multiple (LCM), which is 210; the composites which have a different symmetry center, such as (5·Type A′, 5·Type B′) and (7·Type A′, 7·Type B′), form symmetry in both directions from 210. If LCM of 210 is defined as the midpoint between the 1st (from 0 to 210) and 2nd (from 210 to 420) boundaries, additional new composites are selected in the 2nd boundary by the primes between $\sqrt{210}$ (or 14.49) and $\sqrt{420}$ (or 20.49), which are 17 and 19. Using the symmetry center of 17 and 19, which is 102 and 114, it is calculated that both symmetry centers pass through the 2nd boundary 4 times more than the 1st boundary, based on the equation of $\left(\left\lfloor\frac{420}{102}\right\rfloor - \left\lfloor\frac{210}{102}\right\rfloor\right) + \left(\left\lfloor\frac{420}{114}\right\rfloor - \left\lfloor\frac{210}{114}\right\rfloor\right) = 4$; it is equivalent to 8 composites. This means that the 1st boundary has 8 more potential primes than the 2nd boundary and this shows an accuracy of 72.7% compared to the actual difference value of 11. As 210*x* linearly increases, where $x \geq 1$, by producing more primes, additional composites are also increased, and they affect the decreasing number of new primes in the 2nd boundary. However, the increased number of composites is limited by the primes between $\sqrt{210x}$ and $\sqrt{420x}$. As the size of the boundary increases, therefore, the number difference between the primes in the 1st boundary and the new primes in the 2nd boundary becomes negligible and it is expressed as follows.

$$\lim_{n\to\infty} \frac{\frac{n}{ln(n)}}{\left(\frac{2n}{ln(2n)} - \frac{n}{\ln(n)}\right)} = \lim_{n\to\infty} \frac{ln(n) + ln2}{ln(n) - ln2} \to 1$$

, where n is $210x$, and $\pi(\frac{n}{ln(n)})$ and $\pi(\frac{2n}{ln(2n)}) - \pi(\frac{n}{\ln(n)})$ are estimated the number of primes in the 1st boundary and new primes in the 2nd boundary, respectively.

Overall, as the composites are symmetrically produced within the symmetric structure of prime candidates at $6x \pm 1$, the primes in the 1st boundary and the new primes in the 2nd boundary also form symmetry around the center of the LCM. However, due to the additional composites in the 2nd boundary, a limited number of primes and new primes form an asymmetry, resulting in the coexistence of symmetry and asymmetry. Considering the constant ratio between primes and new primes as the boundary size increases, both the number of primes and new primes increases while maintaining the coexistence of symmetry and asymmetry from the LCM. The smallest LCM that satisfies the above conditions is 30, produced by the prime 5, representing an interval of 30 and including twin primes as well as additional Type A and B primes. Therefore, Type B′ and A′ new primes including pairs of twin primes, which form partial symmetry to the primes in the 1st boundary, are consistently produced and maintained within the interval of 30.

3.3. A numerical approach to the symmetrical structure that satisfies Goldbach's conjecture

In *Series IV*, the midpoint of even Goldbach's number is limited by 6 cases: $6x$, $6x + 1$, $6x + 2$, $6x + 3$, $6x + 4$, and $6x + 5$. For satisfying Goldbach's conjecture, each midpoint should consist of at least one pair of primes, such as (Type A + Type B or B′) around the midpoint of $6x$, (Type B + Type B′) around $6x + 1$, (Type A + Type A′) around $6x + 2$, (Type A′ + Type B′ or B) around $6x + 3$, (Type B′ + Type B) around $6x + 4$, and (Type A′ + Type A) around $6x + 5$; a total of 12 primes, consisting of 6 Type A-like and 6 Type B-like primes, are required. Since duplicate use of primes is allowed, the required number of primes can be reduced from 12 down to 6 with the assistance of twin primes. If the midpoint of $6x$ increases to $12x$ by defining the 1st boundary between 0 and $12x$, the 2nd boundary is ranged between $12x$ and $24x$. Among the prime pairs between the 1st and 2nd boundaries, 50% of the primes should be used from the 1st boundary. Thus, only the remaining 50% of primes, which are new primes and maintain symmetry with the primes in the 1st boundary, are required in the 2nd boundary. Therefore, as the midpoint increases from $12x$ to $12x + 5$ by shifting the 1st boundary, the 2nd boundary is also shifted from $2(12x)$ to $2(12x + 5)$; the minimal required number of new primes in the 2nd boundary can also be reduced from 6 to less than 3, with the assistance of the repeated use of increasing primes and new primes. Consequently, as the size of boundary increases, both Type A and B primes in the 1st boundary as well as Type B′ and A′ new primes in the 2nd boundary increases, so the probability of prime and new prime pairs satisfying Goldbach's conjecture also increases. However, considering the repeated symmetry and asymmetry of new composites, the number of new primes will oscillate. Thus, the number of prime and new prime pairs will also oscillate but show an increasing pattern as the boundary size increases.

4. Conclusions

All prime candidates are structurally positioned at $6x \pm 1$, thus all primes are fundamentally selected based on a symmetrical structure and the repeated smallest symmetry unit is established in the interval of 30 produced by the prime 5. As the number of primes increases, the number of composites also increases, and it breaks the symmetry between the primes and new primes into asymmetry. However, considering the number ratio between the primes in the 1st boundary and the new primes in the 2nd boundary, the symmetry and asymmetry coexist between the primes and new primes, consisting of twin primes and additional Type A and B primes. The limited six cases of the midpoint of Goldbach's even numbers also limit the number of possible cases of Goldbach's conjecture to six with a combination of Type A and B primes. As Type A and B primes are partially but consistently produced in the interval of 30, the limited six cases of Goldbach's conjecture can be satisfied.

If an arbitrary integer n is selected and defined as the 1st boundary with n elements, each wave of primes P less than n produce the composites and evenly distribute the composites from the 2nd to nth boundaries. From the perspective of waves, if the value of prime P increases, the wavelength (λ_P) also increases, causing a decrease in frequency, $sin(\frac{360}{\lambda_P} \cdot x)$. As the value of a number increases, the number of factors may also increase. In other words, as frequency decreases, the probability of factor overlap increases, producing irregularity that affects the number of new primes oscillating between the 2nd and nth boundaries. Since only primes within the 1st boundary produce and distribute the composites up to the nth boundary, the produced number of composites is limited and cannot exceed the prime candidates per boundary. Therefore, the oscillating number of new primes per boundary cannot converge to 0; instead, it approaches a constant value as it moves from the 2nd to the nth boundary; this adjusts the ratio of the new primes in the 2nd to those in the nth boundary to a constant value, specifically 2 [2]. As the size of a boundary increases, therefore, it is concluded that the number of prime and new prime pairs, consisting of twin primes and additional Type A and B primes, between the 1st and 2nd boundaries that satisfy Goldbach's conjecture will also show an oscillate-like increasing pattern.

4. Declaration of interests

The author does not work for, advise, own shares in, or receive funds from any organization that could benefit from this article, and has declared no affiliation other than their research organisations.

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Figure 1. If the arbitrary positive integer *n* is set to 4, the 1st boundary is defined between 0 and 4. A) In the 1st boundary, the wave of integers, except for 1, can be expressed in the form of '$y_n = sin\,(\frac{180}{n} \cdot x)$' and extends to 4^2 or 4th boundary. B) Considering the overlapped waves between y_4 and y_2 on the *x-axis*, the multiplication of integer waves is identical with the multiplication of prime waves in the 1st boundary. C) If y_1 is divided by multiplication of prime waves, $y_2 \cdot y_3$, the primes in the 1st boundary and the composites which are affected by the continuous wave of primes within the 4th boundary cannot be defined, remaining numbers between the 2nd and 4th boundaries are all new primes on the *x-axis*. Under the same conditions, the boundary can be limited from the 4th to 2nd; the midpoint of the 2nd boundary is 4. With respect to 4, the wave of prime 2 is symmetrical to the composites in the 2nd boundary, so the number symmetrical to 2 in the 2nd boundary is necessarily composite. This is why prime 3, whose wave is asymmetrical to 4, is symmetrically paired with prime 5 by satisfying Goldbach's conjecture.