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NEWTON'S LAW OF GRAVITY + LAW OF COSMOLOGICAL FORCE = LAW OF UNIVERSAL GRAVITATION.

Abstract. *The reason for the limitations of Newton's classical theory of gravitation is that classical gravitation remained an unfinished theory. Newton's formula F_N = GMm/r^2 gives the force of gravitational interaction between two bodies. Accordingly, Newton's law formula gives only part of the force of universal gravitation and does not apply to the universe. In classical gravitation the additional cosmological force of gravitational interaction of bodies with the mass of the Universe remained undiscovered. The additional cosmological force is represented by a new law of gravitation, different from Newton's law. The law of cosmological force is presented using the cosmological constant Λ: F_{Cos} = m•c^2•√Λ. The cosmological force has a linear dependence on the mass of the body and does not obey the law of inverse squares. On small scales, the additional cosmological force is much smaller than the Newtonian force. On the scale of the Universe, the cosmological force exceeds the Newtonian force and has a theoretical limit equal to the Planck force F_P = c^4/G = 1.21027•10^44 N. This large force was not represented in the law of universal gravitation. A new mathematical formula for the law of universal gravitation is given. The law of universal gravitation is represented by two equations, Newton's law F_N = GMm/r^2 and the law of cosmological force F_{Cos} = m•c^2•√Λ. The law of universal gravitation admits a quantum description of the gravitational interaction. It is shown that extended classical gravity has a high heuristic potential. The law of universal gravitation in extended form explains the mystery of galaxy rotation curves and the Pioneer Anomaly without involving the dark matter hypothesis.*

Keywords*: The law of universal gravitation; The law of cosmological force; Quantum theory of gravitation; Large numbers; Cosmological equations; Parameters of the observable universe; Galaxy rotation curves; Pioneer anomaly.*

1. Introduction

The dominant force in the universe is gravity. Newton's law of gravity was a real breakthrough in science. Newton's law of gravitation (1) impresses with its simplicity and mathematical perfection:

$$F_N = G\frac{Mm}{r^2} \qquad (1)$$

Gravitational interaction has become the fourth fundamental interaction. Newton's law of gravitation makes it possible to explain and predict with great accuracy the motions of celestial bodies. The attractive thing about Newton's law of gravitation is the simple dependence of force on the parameters of interacting bodies.

At the same time, simple and perfect in mathematical representation, Newton's law of gravitation has limitations and limits of applicability. Newton's law of gravitation describes the interaction of two point masses. But it does not account for the gravitational interaction of bodies with the universe. It does not answer: "with what force does any mass interact with the mass of the Universe distributed in space?", "on what parameters of the Universe does the cosmological force depend?".

The possibility of application of Newton's law of gravitation ends where masses cannot be considered as point masses. There were repeated attempts to modify Newton's law and make it applicable in cosmology. In 1745 Alexis Clairaut [1] proposed a modification of Newton's law in which the law of inverse squares was changed. In 1894 Hall A.[2] proposed the replacement of the square of distance by a slightly greater degree. Hugo von Seeliger and Carl Gottfried Neumann proposed a modification of the law with a faster than Newton's law of gravitation decreasing with distance [3]. Attempts to modify Newton's law of gravitation [4] and attempts to question the law of inverse squares [3]. do not stop.

In the MOND theory [4] it was shown that for small accelerations of the order of $2 \cdot 10^{-10}$ m/s² Newton's law of gravitation may not work. The limits of applicability of Newton's law of gravitation were especially evident in the study of spiral galaxies. The curves of spiral galaxies (Fig. 1) showed a significant discrepancy between the observed experimental curves (B) and the curve obtained theoretically (A).

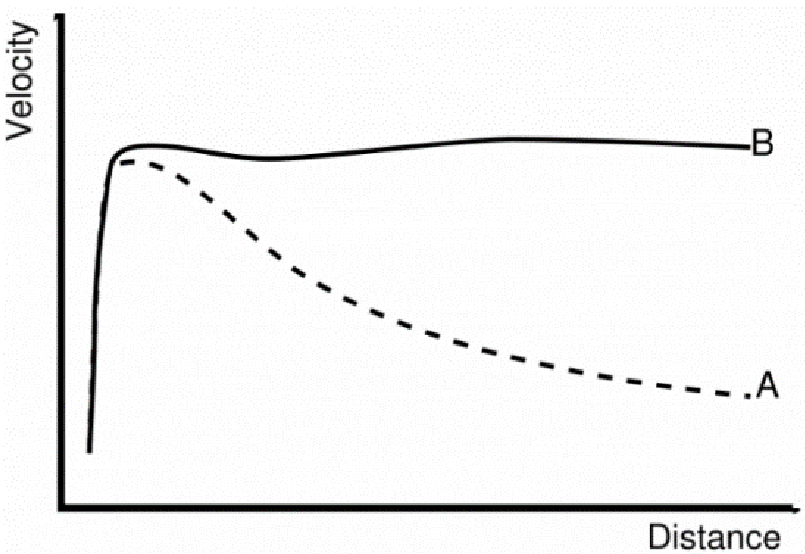

Fig. 1. Rotation curve of a typical spiral galaxy: predicted (A) and observed (B) [4, 5].

The reason of such a significant discrepancy between theory and observations has no convincing explanation. Is there a solution to this problem within the framework of classical gravitation? Below it is shown that there is a solution and this solution is not connected with the hypothesis of dark matter. The cause of the problem was hidden in the incompleteness of the classical theory of gravitation. Newton's formula (1) gives the force of gravitational interaction between two bodies. Accordingly, Newton's law formula gives only a part of the force of universal gravitation and does not apply to the universe. It should be noted that Newton's law of gravitation (1) is not the only formula for the force of gravitational interaction between two bodies. The law of gravitational interaction between two bodies can be represented by a formula that does not include the gravitational constant G and gravitational mass M:

$$\mathbf{Fg = mR^3/T^2r^2} \qquad (2)$$

where: Fg is the force represented by the Kepler relation, m is the mass of the body, R and T are orbit parameters, r is the distance.

Formula (2), as well as formula (1), gives the same dependence of the form (A), which shows a significant divergence from the curve (B). Newton's law of gravitation is clearly not sufficient to explain the dependence of the form (B). Formulas (1) and (2) “fail to see” a significant fraction of the gravitational force on the scale of the universe. This large difference indicates that for the missing gravitational force there is an unknown law of gravity different from Newton's law. The discovery of this unknown law of gravitation as an addition to Newton's law will make the classical theory of gravitation complete.

No refinements and revisions of Newton's law of gravitation have made it applicable in cosmology. The simple and mathematically perfect formula of Newton's law was not applicable in cosmology. The law of inverse squares and point masses are the main limiting factors in extending Newton's law to the Universe.Newton's law has limits of applicability. Beyond these limits it is necessary to search for another law of gravitation free from idealized point masses and the law of inverse squares.

The law of inverse squares, was formulated in 1645 by Ismail Bullialdus [3]. The law of inverse squares proved to be very productive for solving the two-body problem. This was shown by Newton's law of gravitation. The same law of inverse squares became an insurmountable obstacle in extending Newton's law of gravitation to the Universe.Obviously, with respect to the Universe there is another yet undiscovered law of gravitation, different from Newton's law of gravitation. To find a new law of gravitational interaction, we use new cosmological equations obtained by means of the law of scaling of large numbers.

2. The law of scaling of large numbers.

The coincidences of large numbers allowed us to derive the law of scaling of large numbers (Fig. 2). The law of scaling of large numbers includes two dimensionless constants: fine structure constant "alpha" and Weyl number. The law of scaling of large numbers has the form:

$$D_i = (\sqrt{\alpha D_0})^i$$

$$i = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7, \pm 8, \pm 9.$$

Fig. 2. The scaling law of large numbers. D_0 is a large Weyl number (D_0 = 4.16561...x 10^{42}), α - fine structure constant.

The scaling law (Fig. 2) provides a new method for calculating the values of large numbers from dimensionless constants. The scaling law generates large numbers up to scale 10^{180} with high accuracy. The large numbers obtained from the scaling law are close to the accuracy of the Newtonian constant of gravitation G. The values of the large numbers and the formulas for their calculation are given in Fig. 3.

$$(\sqrt{\alpha D_0})^0 = 1$$

$$D_{20} = (\sqrt{\alpha D_0})^1 = 1.74349... \bullet 10^{20}$$

$$D_{40} = (\sqrt{\alpha D_0})^2 = 3.03979... \bullet 10^{40}$$

$$D_{60} = (\sqrt{\alpha D_0})^3 = 5.29987... \bullet 10^{60}$$

$$D_{80} = (\sqrt{\alpha D_0})^4 = 9.24033... \bullet 10^{80}$$

$$D_{100} = (\sqrt{\alpha D_0})^5 = 16.1105... \bullet 10^{100}$$

$$D_{120} = (\sqrt{\alpha D_0})^6 = 28.088... \bullet 10^{120}$$

$$D_{140} = (\sqrt{\alpha D_0})^7 = 48.972... \bullet 10^{140}$$

$$D_{160} = (\sqrt{\alpha D_0})^8 = 85.383... \bullet 10^{160}$$

$$D_{180} = (\sqrt{\alpha D_0})^9 = 148.86... \bullet 10^{180}$$

Fig. 3. Large numbers and formulas for their calculation.

3. The set of coincidences of large numbers involving the cosmological constant Λ.

The table in Fig. 4 summarizes the relations of dimensional quantities containing the constant Λ. These relations are equal to large numbers. Many coincidences of large numbers involving the cosmological constant Λ make it possible to derive new cosmological equations for various combinations of cosmological parameters and fundamental physical constants.

Ratios of dimensional constants	Scale
$\frac{Gm_e^2}{r_e\alpha^2\hbar\sqrt{\Lambda}c}=\frac{G\hbar}{r_e^3\sqrt{\Lambda}c^3}=\frac{Gm_e}{r_e^2\alpha\sqrt{\Lambda}c^2}=\frac{Gm_e^3}{\alpha^3\hbar^2\sqrt{\Lambda}}=\frac{c^2}{M_UR_UG\Lambda}=\frac{c^2}{M_UG\sqrt{\Lambda}}=\frac{c^2R_U}{M_UG}=$ $=\frac{l_{Pl}^4}{\Lambda r_e^6}=\frac{R_U}{M_UGT_U^2\Lambda}=\frac{c^3T_U^3\Lambda}{R_U}=\frac{cr_e^3A_0}{G\hbar}=\frac{c^2}{M_UR_U\Lambda G}=\frac{M_UR_U\sqrt{\Lambda}A_0G}{c^4}=1$	10^0
$D_{20}=\frac{r_e}{l_{Pl}}=\frac{t_0}{t_{Pl}}=\frac{\alpha\ m_{Pl}}{m_e}=\frac{l_{Pl}}{r_e^2\sqrt{\Lambda}}=\frac{l_{Pl}R_U}{r_e^2}=\frac{c^2l_{Pl}}{r_e^2A_0}=\sqrt{\alpha D_0}$	10^{20}
$D_{40}=\frac{T_U}{t_0}=\frac{R_U}{r_e}=\frac{m_ec}{\alpha\hbar\sqrt{\Lambda}}=\frac{r_e\alpha c^2}{Gm_e}=\frac{1}{t_0c\sqrt{\Lambda}}=\frac{r_e^2}{l_{Pl}^2}=\frac{t_0^2}{t_{Pl}^2}=\frac{\alpha^2m_{Pl}^2}{m_e^2}=\frac{l_{Pl}^2}{r_e^4\Lambda}=\frac{c^2}{r_eA_0}=\frac{1}{r_e\sqrt{\Lambda}}=(\sqrt{\alpha D_0})^2$	10^{40}
$D_{60}=\frac{T_U}{t_{Pl}}=\frac{R_U}{l_{Pl}}=\frac{M_U}{m_{Pl}}=\frac{1}{l_{Pl}\sqrt{\Lambda}}=\frac{r_e^3}{l_{Pl}^3}=\frac{t_0^3}{t_{Pl}^3}=\frac{c^2}{Gm_{Pl}\sqrt{\Lambda}}=\frac{c^2}{l_{Pl}A_0}=(\sqrt{\alpha D_0})^3$	10^{60}
$D_{80}=\frac{R_U^2}{r_e^2}=\frac{\sqrt{\Lambda}M_U^2\alpha G}{c^2m_e}=\frac{cT_U}{r_e^2\sqrt{\Lambda}}=\frac{r_e}{\sqrt{\Lambda}l_{Pl}^2}=\frac{1}{r_e^2\Lambda}=(\sqrt{\alpha D_0})^4$	10^{80}
$D_{100}=\frac{m_ec}{l_{Pl}\alpha\hbar\Lambda}=\frac{r_e\alpha M_U}{l_{Pl}m_e}=\frac{\sqrt{\Lambda}M_U^2\alpha Gr_e}{c^2m_el_{Pl}}=\frac{R_U^2}{r_el_{Pl}}=\frac{1}{r_el_{Pl}\Lambda}=(\sqrt{\alpha D_0})^5$	10^{100}
$D_{120}=\frac{T_U^2}{t_{Pl}^2}=\frac{R_U^2}{l_{Pl}^2}=\frac{M_U^2}{m_{Pl}^2}=\frac{R_U}{r_e^3\Lambda}=\frac{M_Uc}{\hbar\sqrt{\Lambda}}=\frac{GM_U^2}{\hbar c}=\frac{c^3}{G\hbar\Lambda}=\frac{1}{l_{Pl}^2\Lambda}=(\sqrt{\alpha D_0})^6$	10^{120}
$D_{140}=\frac{r_e^2m_ec}{l_{Pl}^3\alpha\hbar\Lambda}=\frac{r_e^3\alpha M_U}{l_{Pl}^3m_e}=\frac{R_U^3}{l_{Pl}r_e^2}=\frac{1}{c^3t_{Pl}t_0^2\sqrt{\Lambda}\Lambda}=\frac{1}{l_{Pl}r_e^2\Lambda\sqrt{\Lambda}}=\frac{c^2}{l_{Pl}r_e^2A_0\Lambda}=(\sqrt{\alpha D_0})^7$	10^{140}
$D_{160}=\frac{M_Uc^2\alpha^2}{Gm_e^2\sqrt{\Lambda}}=\frac{M_U^2G\alpha}{c^2r_e^2m_e\sqrt{\Lambda}}=\frac{1}{r_e^4\Lambda^2}=\frac{r_e^2}{l_{Pl}^4\Lambda}=(\sqrt{\alpha D_0})^8$	10^{160}
$D_{180}=\frac{r_e^4m_ec}{l_{Pl}^5\alpha\hbar\Lambda}=\frac{r_e^5\alpha M_U}{l_{Pl}^5m_e}=\frac{R_U}{l_{Pl}^3\Lambda}=\frac{1}{l_{Pl}^3\Lambda\sqrt{\Lambda}}=\frac{c^2}{l_{Pl}^3A_0\Lambda}=\frac{GM_UT_U^2l_{Pl}}{\Lambda r_e^6}=\frac{1}{r_e^3\Lambda^2l_{Pl}}=(\sqrt{\alpha D_0})^9$	10^{180}

Fig. 4. Relations of dimensional quantities containing the constant Λ. M_U is the mass of the observable Universe, α is the fine structure constant, ħ is Planck's constant, G is the Newtonian gravitational constant, Λ is the cosmological constant, R_U is the radius of the observable Universe, T_U is the time of the Universe, H is the Hubble constant, A_0 is the cosmological acceleration, re is the classical radius of the electron; c - speed of light in vacuum; $t_0 = r_e/c$, m_e - electron mass, D_0 - large Weyl number, t_{pl} - Planck time, l_{pl} - Planck length, m_{pl} - Planck mass.

4. Systems of algebraic equations of the Universe

Several systems of equations can be formed from the cosmological equations (Fig. 4). In Fig . 5 shows four systems of cosmological equations.

$$
\begin{cases}
G\hbar / r_e^3 A_0 = c^1 \\
1/T_U^2 \Lambda = c^2 \\
M_U A_0 G = c^4 \\
M_U R_U A_0 G / T_U = c^5 \\
M_U R_U A_0^2 G = c^6
\end{cases}
$$

a)

$$
\begin{cases}
M_U \Lambda G = A_0 \\
c^5 r_e^3 / M_U G^2 = \hbar \\
M_U G T_u^2 = R_u^3 \\
G M_U = c^2 R_U \\
M_U \Lambda G T_u^2 = R_U
\end{cases}
$$

b)

$$
\begin{cases}
\dfrac{c^5 r_e^3}{M_U G^2} = \hbar \\
M_U \Lambda c r_e^3 = \hbar \\
\dfrac{R_U \Lambda c^3 r_e^3}{G} = \hbar \\
\dfrac{c r_e^3 A_0}{G} = \hbar \\
\dfrac{M_U r_e^3}{R_U T_U} = \hbar
\end{cases}
$$

c)

$$
\begin{cases}
\dfrac{M_U G^2 m_e}{c^4 r_e^2} = \alpha, \\
\dfrac{m_e}{M_U \Lambda r_e^2} = \alpha, \\
\dfrac{G m_e}{R_U \Lambda c^2 r_e^2} = \alpha, \\
\dfrac{G m_e}{r_e^2 A_0} = \alpha, \\
\dfrac{G m_e T_U}{r_e^2 c} = \alpha.
\end{cases}
$$

d)

Fig. 5. Systems of cosmological equations for calculating the parameters of the observed Universe. Where : α - fine-structure constant, ħ - Planck constant, M_U - mass of the observable Universe, G - Newtonian constant of gravitation, Λ - cosmological constant, R_U - radius of the observable Universe, A_0 - cosmological acceleration, r_e - classical electron radius; c - speed of light in vacuum; m_e - electron mass, D_0 - large number.

5. Parameters of the Universe

In these systems of cosmological equations (Fig. 5) only the fundamental physical constants **ħ, r_e, G, c, α, m_e** are known quantities. The unknown cosmological parameters **M_U, R_U, Λ, A_0, T_U** are easily found by solving the system of equations. All the given systems of cosmological equations (Fig. 5) give the same values of the Universe parameters (Fig. 6).

$$
\begin{aligned}
M_U &= 1.15348\ldots \bullet 10^{53}\,kg \\
R_U &= 0.856594\ldots \bullet 10^{26}\,m \\
T_U &= 2.85729\ldots \bullet 10^{17}\,s \\
\Lambda &= 1.36285\ldots \bullet 10^{-52}\,m^{-2} \\
A_0 &= 10.4922\ldots \bullet 10^{-10}\,m/s^2
\end{aligned}
$$

Fig. 6. Identical values of the Universe parameters obtained from different systems of algebraic equations of the Universe.

The values of the Universe parameters and formulas for their calculation are given in Fig. 7.

$$M_U = m_e \alpha D_0^2 = 1.15348...\bullet 10^{53}\,kg$$
$$R_U = r_e \alpha D_0 = 0.856594...\bullet 10^{26}\,m$$
$$T_U = \frac{r_e \alpha D_0}{c} = 2.85729\,...\bullet 10^{17}\,s$$
$$\Lambda = \frac{1}{r_e^2 \alpha^2 D_0^2} = 1.36285\,...\bullet 10^{-52}\,m^{-2}$$
$$G = \frac{r_e^3}{t_0^2 m_e D_0} = 6.67430\,...\bullet 10^{-11}\,kg^{-1} m^3 s^{-2}$$
$$A_0 = \frac{c^2}{r_e \alpha D_0} = 10.4922\,...\bullet 10^{-10}\,m/s^2$$

Fig. 7. Values of the Universe parameters and formulas for their calculation.

6. Cosmological acceleration constant.

One of the solutions of the systems of algebraic equations of the universe is the value of the cosmological acceleration constant as one of the parameters of the universe:

$$\mathbf{A_0 = 10.4922... \bullet 10^{-10}\ ms^{-2}} \qquad (3)$$

This parameter of the universe is unknown in Newton's law of gravitation. For this reason, Newton's theory of gravitation has limitations and does not apply to cosmology.

Using Newton's second law, we obtain the value of the cosmological force F_{cos}, which acts on all bodies in the universe and causes the acceleration A_0.

$$F_{Cos} = mA_0 = m \bullet 10.4922...\bullet 10^{-10}\,N \qquad (4)$$

The cosmological acceleration constant A_0 can be represented by various equivalent formulas, such as:

$$\mathbf{A_0 = GM_U\Lambda = H^2/\sqrt{\Lambda} = c^2\sqrt{\Lambda} = R_U^3\Lambda/T_U^2 = c^2/r_e\alpha D_0.} \qquad (5)$$

Where: G - Newtonian constant of gravitation, M_U - mass of the observable Universe, Λ - cosmological constant, H - Hubble constant, R_U is the radius of the universe, T_U is the time of the universe, α - fine-structure constant, D_0 - large Weyl number r_e - classical electron radius; c - speed of light in vacuum.

7. Law of cosmological force

The cosmological acceleration constant A_0 allows us to derive equations for the cosmological force. Fig. 8 shows three equivalent formulas for the cosmological force:

$$F_{Cos} = mGM_U\Lambda \qquad (6)$$

$$F_{Cos} = \frac{mH^2}{\sqrt{\Lambda}} \qquad (7)$$

$$F_{Cos} = mc^2\sqrt{\Lambda} \qquad (8)$$

$$F_{Cos} = \frac{mR_U^3\Lambda}{T_U^2} \qquad (9)$$

$$F_{Cos} = \frac{mc^2}{r_e\alpha D_0} \qquad (10)$$

FIG. 8. Equivalent formulas for the law of cosmological force. Where: F_{cos} is the cosmological force, G is the Newtonian constant of gravitation, M_U is the mass of the observable Universe, Λ is the cosmological constant, H is the Hubble constant, R_U is the radius of the universe, T_U is the time of the universe, α is the fine-structure constant, D_0 is the large Weyl number, r_e is the classical electron radius; c is the speed of light in vacuum, m is the mass of a body.

Formula (9) contains three parameters of the universe. Formulas (6) and (7) include two parameters of the Universe each, formula (8) includes one parameter of the Universe. Of the three equivalent formulas, equation (8) is the simplest and most mathematically perfect.

$$F_{Cos} = mc^2\sqrt{\Lambda}$$

Fig. 9. Formula of the law of cosmological force.

The law of cosmological force gives a linear dependence of the gravitational interaction force on the body mass m. The coupling constant in the new law of gravitation is the cosmological constant Λ.

The law of cosmological gravitational force of the Universe is represented by a simple formula, which is not inferior in simplicity and perfection to the formula of Newton's law of gravitation. The cosmological constant **Λ** fulfills the role of the coupling constant in the law of cosmological force. This determines the name of the law. The law shows with what force a body interacts with the mass of the Universe distributed in space. The law of cosmological force shows that any body of mass **m** is affected by the cosmological force of the Universe proportional to the mass of the body **m**. The status of the cosmological constant **Λ** in the law of cosmological force is not less significant than the status of the constant **G** in Newton's law.

Combining Newton's law of gravitation and the law of cosmological force give a new Law of universal gravitation. The resulting force of gravitational interaction is defined as a vector sum of two forces: the Newtonian force and the cosmological force.

8. Planck's constant and Planck units in classical gravitation.

The system of cosmological equations (Fig. 5-c.) shows that the parameters of the Universe are related to the Planck constant. A consequence of this is that all parameters of the universe can be represented using Planck units. From the coincidence of large numbers (Fig. 4) such equations are obtained:

$$M_U = m_{Pl}(\sqrt{\alpha D_0})^3 \qquad (11)$$

$$R_U = l_{Pl}(\sqrt{\alpha D_0})^3 \qquad (12)$$

$$T_U = t_{Pl}(\sqrt{\alpha D_0})^3 \qquad (13)$$

$$\Lambda = \frac{l_{Pl}^4}{r_e^6} \qquad (14)$$

$$A_0 = \frac{l_{Pl}}{t_{pl}^2(\sqrt{\alpha D_0})^3} = \frac{l_{Pl}^2 c^2}{r_e^3} \qquad (15)$$

Formulas (11) - (15) give the same values of the Universe parameters as the systems of cosmological equations (Fig.10).

$$M_U = 1.15348... \bullet 10^{53}\, kg$$
$$R_U = 0.856594... \bullet 10^{26}\, m$$
$$T_U = 2.85729\, ... \bullet 10^{17}\, s$$
$$\Lambda = 1.36285\, ... \bullet 10^{-52}\, m^{-2}$$
$$A_0 = 10.4922\, ... \bullet 10^{-10}\, m/s^2$$

Fig. 10. Values of the Universe parameters obtained from Planck units.

The connection of the parameters of the Universe with Planck's constant and with Planck units allows us to represent the Law of universal gravitation in quantum form.

9. The law of cosmological force law explains the Pioneer anomaly.

The cosmological force for small values of masses is much smaller than the Newtonian force. Therefore, for small values of masses it is not pronounced and can be masked by effects of a non-gravitational nature. The unknown force was first experimentally detected in the Pioneer effect [6, 7, 8]. The Pioneers effect still has no convincing explanation. An attempt has been made to explain the effect by thermal recoil [9].

The new force that follows from the cosmological force law surprisingly turned out to be close to the Pioneer anomaly, which casts doubt on the thermal nature of the Pioneer effect. The significance of the cosmological acceleration that follows from the cosmological force law:

$$A_0 = c^2\sqrt{\Lambda} = 10.4922... \bullet 10^{-10} m/s^2 \qquad (16)$$

The significance of the cosmological force:

$$F_{Cos} = m \bullet (10.4922 \bullet 10^{-10}) N \qquad (17)$$

Value of unknown force found in the pioneer effect:

$$F_{Pioneer} = m \bullet (8.74 \pm 1.33) \bullet 10^{-10} N \qquad (18)$$

In addition to the Pioneer-10 and Pioneer-11 experiment, there is anomalous acceleration data from Galileo and Ulysses [10 - 13].

The value of the unknown force for Galileo:

$$F_{Galileo} = m \bullet (8 \pm 3) \bullet 10^{-10} N \qquad (19)$$

Value of unknown force for Ulysses:

$$F_{Ulysses} = m \bullet (12 \pm 3) \bullet 10^{-10} N \qquad (20)$$

The value of the cosmological force F = m(10.4922... × 10^{-10})N is very close to the experimental values F = m((8.74 ± 1.33) x 10^{-10})N, F = m((8±3) x 10^{-10})N, F = m((12±3) x 10^{-10})N. The coincidence of the force values casts doubt on the explanation of the pioneer anomaly by the temperature effect. The law of cosmological force points to the gravitational nature of the Pioneer Anomaly. The gravitational nature of the Pioneer anomaly was also pointed out by Hasmukh K. Tank in a study of critical-acceleration of MOND [14].

10. The theoretical limit of the cosmological force is equal to the Planck force c^4/G.

The study of the equivalent equations of the new law of the cosmological force shows that the value of the cosmological force in the limit is equal to the Planck force:

$$\lim_{m \to M_U} F_{Cos} = \lim_{m \to M_U} mc^2\sqrt{\Lambda} = 1.21027 \bullet 10^{44} N = \frac{c^4}{G} \qquad (21)$$

$$\lim_{m \to M_U} \frac{mH^2}{\sqrt{\Lambda}} = 1.21027 \bullet 10^{44} N = \frac{c^4}{G} \qquad (22)$$

$$\lim_{m \to M_U} mGM_U\Lambda = 1.21027 \bullet 10^{44} N = \frac{c^4}{G} \qquad (23)$$

$$\lim_{m \to M_U} \frac{mR_U^3\Lambda}{T_U^2} = 1.21027 \bullet 10^{44} N = \frac{c^4}{G} \qquad (24)$$

$$\lim_{m \to M_U} \frac{mc^2}{r_e\alpha D_0} = 1.21027 \bullet 10^{44} N = \frac{c^4}{G} \qquad (25)$$

The theoretical limit of the cosmological force at m → M_U reaches the enormous value c^4/G = 1.21027 x 10^{44} N.

11. The law of cosmological force explains Galaxy rotation curve

Newton's law does not reveal the presence of cosmological force of gravitational interaction of bodies with the mass of the Universe. The coupling constant in Newton's law of gravitation is the constant G. The cosmological force of gravitational interaction is an additional force to the Newtonian force of gravitation of two bodies. The additional cosmological force is represented by a different law of gravitation from Newton's law. The law of cosmological force (Fig. 9) is represented using the cosmological constant Λ. The cosmological force has a linear dependence on the mass of the body (Fig. 11) and does not obey the law of inverse squares. Fig. 11 conventionally shows the contribution of the cosmological force to the Galaxy rotation curve.

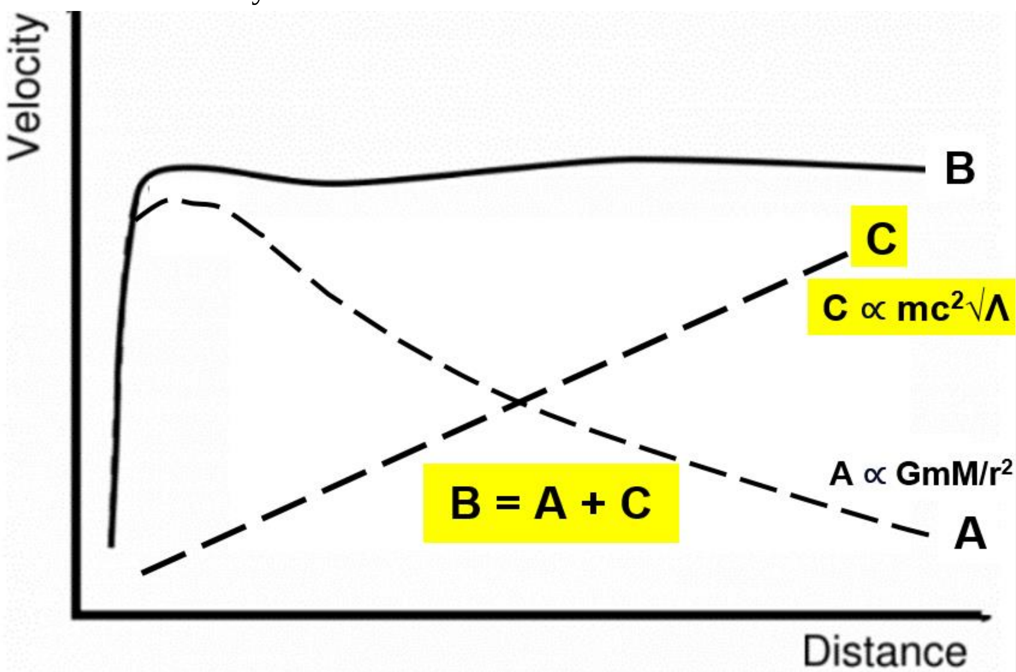

Fig. 11. Galaxy rotation curve (B) as a result of two forces: the contribution of the Newtonian force (A) and the contribution of the cosmological force (C).

On small scales, the additional cosmological force is much smaller than the Newtonian force. On the scale of the Universe, the cosmological force is enormous. For large distances it exceeds the Newtonian force and has a theoretical limit equal to the Planck force F_P = c^4/G = 1.21027•10^44 N. The total force of universal gravitation, which acts on a body, consists of two forces. This is the Newtonian force of gravitational interaction of two bodies and the additional cosmological force of gravitational interaction of a body with the mass of the Universe. Accordingly, the Galaxy rotation curve (B) (Fig. 11) is represented by the sum B = A + C. Combining the two laws of forces F_N and F_{Cos} gives the law of universal gravitation. At small distances, the main share of the force of universal gravitation is represented by the Newtonian force F_N. At large distances, the cosmological force F_{Cos} represents the major fraction of the universal gravitational force. As a result of the two forces, the velocity in the graph (Fig. 11) is represented by curve (B).

The assumption about the influence of the rest of the visible Universe on the rotation curves of galaxies was first made by Philip D. Mannheim [15].

12. New equations for the law of universal gravitation.

The law of cosmological force allows to present the law of universal gravitation in a new form. The law of universal gravitation should include not only the Newtonian force of interaction between two bodies, but also an additional cosmological force.

Fig. 12 shows the equivalent formulas of the law of universal gravitation.

$$\begin{cases} F_N = G\dfrac{Mm}{r^2} \\ F_{Cos} = mGM_U\Lambda \end{cases} \qquad \begin{cases} F_N = G\dfrac{Mm}{r^2} \\ F_{Cos} = \dfrac{mH^2}{\sqrt{\Lambda}} \end{cases} \qquad \begin{cases} F_N = G\dfrac{Mm}{r^2} \\ F_{Cos} = mc^2\sqrt{\Lambda} \end{cases}$$

$$\begin{cases} F_g = \dfrac{mR^3}{T^2r^2} \\ F_{Cos} = \dfrac{mR_U^3\Lambda}{T_U^2} \end{cases} \qquad \begin{cases} F_N = G\dfrac{Mm}{r^2} \\ F_{Cos} = \dfrac{mc^2}{r_e\alpha D_0} \end{cases}$$

Fig. 12. Equivalent formulas of the law of universal gravitation.

The most simple and mathematically perfect is the following form of the Law of universal gravitation:

$$\begin{cases} F_N = G\dfrac{Mm}{r^2} \\ F_{Cos} = mc^2\sqrt{\Lambda} \end{cases}$$

Fig. 13. Law of universal gravitation.

The law of universal gravitation contains two components. The first formula in the law of universal gravitation is Newton's well-known law of gravitation, and the second formula is the new law of cosmological force. Together the two laws of gravitation give the law of universal gravitation. The coupling constants in the law of universal gravitation are two constants: the gravitational constant G and the cosmological constant Λ.

The contribution of the Newtonian force and the cosmological force to the total gravitational force depends on the distance and mass of the interacting bodies. For small masses and small distances, Newton's law of gravitation makes a significant contribution. The fraction of the cosmological force for small masses is much smaller than the Newtonian force. The reason is the small value of the cosmological constant Λ (Λ = 1.36285...x 10^{-52} m^{-2}). As the mass of the interacting bodies increases and the distance increases, the Newton force's fraction of the law of universal gravitation decreases and the cosmological force's fraction increases. The total gravitational force acting on a body is a vector sum of two forces: the Newton force and the cosmological gravitational force of the Universe.

13. Quantum law of universal gravitation.

All equivalent formulas of the Law of universal gravitation (Fig. 12) can be represented in quantum form (Fig. 14):

$$\begin{cases} F_N = \dfrac{\hbar c \alpha M m}{m_e^2 D_0 r^2} \\ F_{Cos} = \dfrac{\hbar c}{m_e r_e^2 D_0} m \end{cases}$$

Fig. 14. The law of universal gravitation in quantum form. Where: M and m - body masses, r - distance, α - fine-structure constant, $\hbar$ - Planck constant, r_e - classical electron radius; c - speed of light in vacuum; m_e - electron mass, D_0 is a large Weyl number (D_0 = 4.16561...x 10^{42}).

It is possible that classical gravitation is not so far from quantum gravitation as it is commonly believed. On the background of unsuccessful attempts to construct the theory of quantum gravitation it is necessary to take a closer look at the classical gravitation, allowing the quantum description of the gravitational interaction.

14. Conclusion

The reason for the limitations of Newton's classical theory of gravitation is that classical gravitation remained an unfinished theory. There is no doubt about the correctness of Newton's mathematical model for the gravitational interaction of two bodies. At the same time, Newton's formula gives only a part of the total force of gravitational interaction. The rest of the total force is the gravitational interaction of bodies with the mass of the universe. This component of the law of universal gravitation Newton's formula “does not see”. The force not accounted for by Newton's law is a significant fraction of the universal gravitational force for large distances and large masses. As the distance increases, the error in the determination of the force increases. The discrepancy between the theoretical value and the observed value is acutely demonstrated by the Galaxy rotation curve. The reason for the discrepancy is not in the imperfection of Newton's formula, but in the fact that the law of universal gravitation is expressed by a more complex mathematical model. Newton's formula is part of the law of universal gravitation. Therefore, to call Newton's law of gravitation the law of universal gravitation is an over exaggeration. The law of universal gravitation has a more complex form. It includes two laws of gravitational force: Newton's mathematically perfect formula for the gravitational interaction of two bodies and the formula of the law of cosmological force of the Universe. Accordingly, the force of universal gravitation is represented by a vector sum of two forces - newtonian and cosmological.

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