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THE REASON FOR THE STRIKING SIMILARITY BETWEEN THE FORMULAS OF COULOMB'S LAW AND NEWTON'S LAW OF GRAVITATION. THE UNIVERSAL FORMULA OF FORCE.

Abstract: *It is shown that the similarity of the formulas of Newton's law of gravitation and Coulomb's law is not a coincidence. The reason for the similarity is that these laws are derived from a single law of force. The forces of inertia, gravitation, electric force, and magnetic force are represented by a single generalized law. A universal formula of force is derived for the generalized law of force interaction. Newton's law of gravity, Newton's second law, Coulomb's law of electrostatics, Ampere's law, Lorentz's law of force, and the centripetal force all follow from the universal formula of force as particular results. The interaction constant in the universal formula of force is the fundamental constant of force F0 = 29.0535101 N. This is the electromagnetic interaction force between two electrons. Despite the electromagnetic status of this constant, it enters both the laws of electromagnetism and the formulas of Newton's laws of mechanics. From the universal formula of force, the equation for calculating the Newtonian constant of gravitation G is derived. The formulas for calculating the Newtonian constant of gravitation G include Planck's constant, Sommerfeld's constant, and the fundamental constants of the electron. This is an unexpected result from the universal formula of force that affects the independent status of the constant G. The dependence of the Newtonian constant of gravitation G on the fundamental physical constants opens the way to obtain a more accurate value of the constant G by calculation. In solving the problem of increasing the accuracy of the Newtonian constant of gravitation G, an important role is assigned to large Dirac numbers. The universal formula of force allows one to elegantly and simply obtain the equation of any force interaction law in mechanics and in electromagnetism using the fundamental constant of force. The Universal formula of force will facilitate the study and understanding of the laws of mechanics and the laws of electromagnetism in the educational process*

Keywords: *Universal formula of force, Fundamental constant of force, Coulomb's law, Newton's law, Ampere's law, Lorentz force, fundamental physical constants, Newtonian constant of gravitation G, relationship of fundamental constants, large Dirac numbers.*

1. Introduction

Analogies between electromagnetism and gravitation have long interested scientists [1, 2]. Why are the formulas of Coulomb's law and Newton's law of universal gravitation so similar in appearance? Such different interactions turned out to be so similar in the mathematical representation of the force formulae. In one - electric charges, in the other - masses, but the formulas in their structure are identical (Fig. 1). What lies behind this striking similarity? The fundamental status of both laws does not allow us to consider the similarity of the formulas accidental. We proceed from the assumption that both Coulomb's law and Newton's law are fragments of some universal law of force.

If this similarity is not accidental, then there must be a single law of force, which only appears for electricity as Coulomb's law and for gravitation as Newton's law of gravitation.

$$F_N = G\frac{m_1 \bullet m_2}{r^2}$$

$$F_K = k\frac{q_1 \bullet q_2}{r^2}$$

Fig.1. Newton's law of gravitation and Coulomb's law. F_N, F_K - Newtonian and Coulomb interaction forces, G - Newtonian constant of gravitation, $k = 1/4\pi\varepsilon_0$, ε_0 *- electric constant, m_1, m_2 , q_1, q_2 - masses and charges of interacting bodies, r - distance.*

2. Generalized law of force interaction for laws of mechanics and electromagnetism

Physicists have always had a great interest in dimensionless quantities [3 - 5]. The research of some dimensionless quantities (large Dirac numbers) has become a separate direction in physics [4]. The idea of dimensionless unification of fundamental interactions is currently actively developed by Stergios Pellis [6, 7].

To reveal the reason for the similarity of the formulas of Coulomb's and Newton's laws, let us perform equivalent transformations in Coulomb's and Newton's laws. This transformation of parameters to dimensionless quantities. For this purpose, let us represent Newton's and Coulomb's laws by generalized equations. The generalized equations by their structure will contain the force constant and dimensionless quantities (Fig. 2). The dimensionless quantities will include masses, charges, and distance.

Interaction force F = (Force constant F_0) • (Dimensionless values $k_1...k_3$)

Fig. 2. Construction of the generalized equation for Newton's and Coulomb's laws using the force constant and dimensionless quantities.

The force constant F_0 for the generalized equations is not difficult to obtain from Coulomb's and Newton's laws by equivalent transformations of these laws. The reduction of masses m_1, m_2, charges q_1, q_2 and distance r to dimensionless quantities $k_1...k_3$ is not difficult using fundamental physical constants. From partial generalized equations it is not difficult to obtain a generalized law of force interaction in the form shown in Fig. 2.

3. Generalized formula for Coulomb's law

Let's represent Coulomb's law as a product of some constant of electromagnetic interaction force F_0 by dimensionless quantities k_{q1}, k_{q2}, k_r. Let's represent electric charges and distance as dimensionless quantities. Let's represent dimensionless quantities k_{q1}, k_{q2} as the ratio of interacting electric charges to the fundamental physical constant - the electron charge ($k_{q1}=q_1/e$, $k_{q2} = q_2/e$). Let's represent the dimensionless quantity related to distance as the ratio of the square of the distance to the fundamental constant - the square of the classical radius of the electron ($k_r=r^2/r_e^2$). The stepwise transition from Coulomb's law to its equivalent generalized formula is as follows:

$$F_K = k\frac{q_1 \bullet q_2}{r^2} = F_0 \bullet \left\{ \frac{\frac{q_1}{e} \bullet \frac{q_2}{e}}{\frac{r^2}{r_e^2}} \right\} = k\frac{e^2}{r_e^2} \bullet \left\{ \frac{k_{q1} \bullet k_{q2}}{k_r} \right\} = \frac{\hbar \bullet c \bullet \alpha}{r_e^2} \bullet \left\{ \frac{k_{q1} \bullet k_{q2}}{k_r} \right\} = \quad (1)$$

$$= 29{,}0535101 \bullet \left\{ \frac{k_{q1} \bullet k_{q2}}{k_r} \right\}$$

where: F_0 is the force constant (F_0 = 29.0535101 N); e is the electron charge (e = 1.602 176 634 x 10^{-19} C); is the Planck constant ($\hbar$ = 1.054 571 817... x 10^{-34} J s); r_e - classical electron radius (r_e = 2.817 940 3262(13) x 10^{-15} m); c - speed of light in vacuum (c = 299 792 458 m s-1); α - fine-structure constant (α = 7.297 352 5693(11) x 10^{-3}).

In a generalized form Coulomb's law is represented by the constant F_0 = 29.0535101 N and the dimensionless quantities $k_{q1} = q_1/e$, $k_{q2} = q_2/e$, $k_r = r^2/r_e^2$. The electromagnetic interaction force constant F_0 in Coulomb's generalized law is represented by the formula:

$$F_0 = \frac{\hbar \bullet c \bullet \alpha}{r_e^2} = 29{,}0535101 \; N \qquad (2)$$

The generalized formula of Coulomb's law is the full equivalent of the original formula of Coulomb's law.

4. Generalized formula for Newton's law of gravitation

Let's represent Newton's law of gravitation as the product of a gravitational interaction force constant F_{0N} by dimensionless quantities k_{m1}, k_{m2}, k_r. Let's represent mass and distance as dimensionless quantities. Let's represent dimensionless quantities k_{m1}, k_{m2}, related to masses as the ratio of masses to the fundamental constant - mass of the electron ($k_{m1} = m_1/m_e$, $k_{m2} = m_2/m_e$). The dimensionless quantity kr related to distance will be represented as the ratio of the square of the distance to the fundamental constant - the square of the classical radius of the electron ($k_r = r^2/r_e^2$). The step-by-step transition from Newton's law to its generalized formula is as follows:

$$F_N = G\frac{m_1 \bullet m_2}{r^2} = F_{0N} \bullet \left\{ \frac{\frac{m_1}{m_e} \bullet \frac{m_2}{m_e}}{\frac{r^2}{r_e^2}} \right\} = F_{0N} \bullet \left\{ \frac{k_{m1} \bullet k_{m2}}{k_r} \right\} = \frac{G \bullet m_e^2}{r_e^2} \bullet \left\{ \frac{k_{m1} \bullet k_{m2}}{k_r} \right\} = \qquad (3)$$

$$= 6{,}97461 \bullet 10^{-42} \bullet \left\{ \frac{k_{m1} \bullet k_{m2}}{k_r} \right\}$$

where: F_{0N} is the gravitational force constant (F_{0N} = 6.97461x10^{-42} N); m_e is the electron mass (m_e = 9.109 383 7015(28) x 10^{-31} kg); G is the Newtonian constant of gravitation (G = 6.674 30(15) x 10^{-11} m3 kg-1 s-2); r_e is the classical electron radius (r_e = 2.817 940 3262(13) x 10^{-15} m).

In generalized form, Newton's law of gravitation is represented by the constant F_{0N} = 6,97461x10^{-42} N and the dimensionless quantities $k_{m1} = m_1/m_e$, $k_{m2} = m_2/m_e$, $k_r = r^2/r_e^2$.
The gravitational interaction force constant F_{0N} in Newton's generalized law is represented by the formula:

$$F_{0N} = G\frac{m_e^2}{r_e^2} = 6{,}97461 \bullet 10^{-42}\ N \qquad (4)$$

The generalized formula of Newton's law is the full equivalent of the original formula of Newton's law. It is known that the electromagnetic interaction force and the gravitational interaction force between two electrons are connected with each other via dimensionless number (D_0 = 4.16561...x 10^{42}), which belongs to the family of large Dirac numbers [2].

Taking into account the ratio $F_0/F_{0N} = D_0$ = 4.16561...x 10^{42} the generalized formula of Newton's law of gravitation will take the form:

$$F_N = F_{0N} \bullet \left\{ \frac{k_{m1} \bullet k_{m2}}{k_r} \right\} = G\frac{m_e^2 \bullet D_0}{r_e^2} \bullet \left\{ \frac{k_{m1} \bullet k_{m2}}{D_0 \bullet k_r} \right\} = 29{,}0535101 \bullet \left\{ \frac{k_{m1} \bullet k_{m2}}{D_0 \bullet k_r} \right\} \qquad (5)$$

In the generalized form Newton's law of gravitation is represented by the constant F_0 = 29.0535101 N and by dimensionless factors $1/D_0$ = 1/4.16561...x 10^{42} , $k_{m1} = m_1/m_e$, $k_{m2} = m_2/m_e$, $1/k_r = r_e^2/r^2$. Let us carry out similar equivalent transformations of other laws and make sure that all of them, as well as Coulomb's and Newton's laws, reduce to a single generalized law of force interaction.

5. Generalized formula for Newton's second law

Let us represent Newton's second law as the product of a force constant F_{0ma} by dimensionless quantities k_m, k_a. Let's represent dimensionless quantity k_m as the ratio of mass to the fundamental constant - the mass of the electron. The dimensionless quantity k_a will be represented as the ratio of acceleration a to the acceleration constant a_0. The stepwise transition from Newton's second law to its generalized formula is as follows:

$$F = ma = F_{0ma} \bullet \left\{ \frac{m}{m_e} \bullet \frac{a}{a_0} \right\} = \frac{m_e \bullet c}{t_0} \bullet \{k_m \bullet k_a\} = 29{,}0535101 \bullet \{k_m \bullet k_a\} \qquad (6)$$

where: a - acceleration; a_0 - acceleration constant ($a_0 = c^2/r_e$), represented by fundamental physical constants; c - speed of light in vacuum; t_0 - time constant $t_0=r_e/c$.

In generalized form, Newton's second law is represented by the constant F_0 = 29.0535101 N and the dimensionless dimension factors $k_m = m/m_e$, $k_a = a/a_0$. The force constant for the law of inertia F0ma in the generalized second law of Newton is represented by the formula

$$F_0 = \frac{m_e \bullet c}{t_0} = 29{,}0535101 \ N \qquad (7)$$

The generalized formula of Newton's second law is the complete equivalent of the original formula of Newton's law.

6. The generalized formula of Ampere's law

Let's represent Ampere's law for interacting conductors with current as the product of some force constant F_{0A} by dimensionless quantities k_{I1}, k_{I2} k_l, k_r. Let's represent dimensionless quantities k_{I1}, k_{I2} as the ratio of current to the electric current constant I_0. Let's represent dimensionless quantity k_l as the ratio of length to the classical radius of the electron. The dimensionless quantity kr will be represented as the ratio of the distance to the classical radius of the electron. The stepwise transition from Ampere's law to its generalized formula is as follows:

$$F_A = \frac{\mu_0}{4\pi} \bullet \frac{I_1 \bullet I_2 \bullet l}{r} = F_{0A} \bullet \left\{ \frac{\frac{I_1}{I_0} \bullet \frac{I_2}{I_0} \bullet \frac{l}{r_e}}{\frac{r}{r_e}} \right\} = F_0 \bullet \left\{ \frac{k_{I1} \bullet k_{I2} \bullet k_l}{k_r} \right\} = \frac{\mu_0 \bullet I_0^2}{4\pi} \bullet \left\{ \frac{k_{I1} \bullet k_{I2} \bullet k_l}{k_r} \right\} = \qquad (8)$$

$$= 29{,}0535101 \left\{ \frac{k_{I1} \bullet k_{I2} \bullet k_i}{k_r} \right\}$$

where: I_0 - electric current constant ($I_0 = ec/r_e$), represented by fundamental physical constants; μ_0 - vacuum magnetic permeability (μ_0 = 1.256 637 062 12(19) x 10^{-6} N A^{-2})

In generalized form Ampere's law is represented by the constant F_0 = 29,0535101 N and by dimensionless factors $k_{I1}=I_1/I_0$, $k_{I2} = I_2/I_0$, $k_l = l/r_e$, $1/k_r=r_e/r$. The generalized formula for Ampere's law is the complete equivalent of the original formula for Ampere's law.

7. Generalized formula for centripetal force

The generalized formula for centripetal force is presented as a product of force constant F_{0mRv} by dimensionless quantities k_m, k_R, k_v. Let's represent dimensionless quantity k_m as the ratio of body mass to the fundamental constant m_e - to the mass of an electron. The dimensionless quantity k_R will be represented as the ratio of the radius to the classical radius of the electron. The dimensionless quantity k_v will be represented as the ratio of frequency to the frequency constant ($v_0 = c/r_e$). The step-by-step transition to the generalized formula for the centripetal force has the form:

$$F_R = mR\nu^2 = F_{0mVR} \bullet \left\{ \frac{m}{m_e} \bullet \frac{R}{r_e} \bullet \frac{\nu^2}{\nu_0^2} \right\} = m_e \bullet r_e \bullet \nu_0^2 \bullet \{k_m \bullet k_R \bullet k_\nu\} = 29{,}0535101 \bullet \{k_m \bullet k_R \bullet k_\nu\} \qquad (9)$$

where: v_0 is a frequency constant ($v_0 = c/r_e$), represented through fundamental physical constants.

Generalized formula of centripetal force is represented by constant F_0 = 29.0535101 N and dimensionless factors $k_m=m/m_e$, $k_R = R/r_e$, $k_v = v^2/v_0^2$. Generalized formula of centripetal force is full equivalent of original formula of centripetal force.

8. Generalized formula for Lorentz magnetic force

Let's represent the generalized formula for Lorentz magnetic force as a product of some force constant F_{0L} by dimensionless quantities k_q, k_V, k_B. Let's represent the dimensionless quantity kq as the ratio of the charge to the fundamental constant, the charge of the electron. We will represent the dimensionless quantity k_V as the ratio of the velocity to the speed of light. The dimensionless quantity k_B is the ratio of magnetic induction to the magnetic induction constant B_0. The step-by-step transition to the generalized formula for the Lorentz magnetic force is as follows:

$$F_{Lm} = q \bullet V \bullet B = F_{0L} \bullet \left\{ \frac{q}{e} \bullet \frac{V}{c} \bullet \frac{B}{B_0} \right\} = e \bullet c \bullet B_0 \bullet \{k_q \bullet k_V \bullet k_B\} = 29{,}0535101 \bullet \{k_q \bullet k_V \bullet k_B\} \qquad (10)$$

where: B_0 is the magnetic induction constant ($B_0 = \mu_0 \bullet e \bullet c / 4\pi r_e^2$), represented by fundamental physical constants; e is the electron charge; c is the speed of light in vacuum.

The generalized formula for the Lorentz magnetic force is represented by the constant F_0 = 29.0535101 N and the dimensionless factors $k_q=q/e$, $k_V = V/c$, $k_B = B/B_0$. The generalized formula for the Lorentz magnetic force is the full equivalent of the original formula for the Lorentz magnetic force.

9. Generalized formula for the Lorentz electric force

Let us represent the generalized formula for Lorentz force as the product of some force constant F_{0LE} by dimensionless quantities k_q, k_E. The dimensionless quantity k_q will be represented as the ratio of the charge to the fundamental constant, the charge of the electron. The dimensionless quantity k_E will be represented as the ratio of intensity to the electric field strength constant. The step-by-step transition to the generalized formula for the Lorentz electric force is as follows:

$$F_{LE} = q \bullet E = F_{0LE} \bullet \left\{ \frac{q}{e} \bullet \frac{E}{E_0} \right\} = e \bullet E_0 \bullet \{k_q \bullet k_E\} = 29{,}0535101 \bullet \{k_q \bullet k_E\} \qquad (11)$$

where: E_0 is the electric field strength constant ($E_0 = e / 4\pi\varepsilon_0 r_e^2$), represented by fundamental physical constants; e is the electron charge.

The generalized formula for Lorentz magnetic force is represented by the constant F_0 = 29.0535101 N and dimensionless factors $k_q=q/e$, $k_E = E/E_0$. The generalized formula for the Lorentz electric force is the full equivalent of the original formula for the Lorentz electric force.

Thus, the known laws of force interaction in both mechanics and electromagnetism are represented by single-type generalized equations. The same-type equations allow these force interaction laws to be represented by a single formula.

10. Fundamental constant of force

Generalized equations (1) - (11) for all above interaction types include the interaction constant F_0 (Fig. 3):

F_0 = 29,0535101(50) N

Fig. 3. Fundamental constant of force.

The interaction constant F0 in the generalized equations is defined by the following equivalent formulas:

$$F_0 = \frac{\hbar \bullet c \bullet \alpha}{r_e^2} = G\frac{m_e^2 \bullet D_0}{r_e^2} = \frac{m_e \bullet c}{t_0} = \frac{\mu_0 \bullet I_0^2}{4\pi} = m_e \bullet r_e \bullet v_0^2 = e \bullet c \bullet B_0 = e \bullet E_0 = 29{,}0535101\ N \qquad (12)$$

We will call the constant F_0 = 29.0535101 N: "Fundamental constant of force". This constant is the electron constant. It directly follows from Coulomb's law. It claims fundamental status. The fundamental status of the constant F0 is indicated by its presence in the formulas of the laws of mechanics and in the formulas of the laws of electromagnetism. Its value is the force of electromagnetic interaction between two electrons. The electron is a fundamental particle of the microcosm. Moreover, its fundamental status is confirmed by the fact that the value of constant F_0 is determined by the most important fundamental physical constants: the Planck constant, the fine-structure constant , the speed of light, the classical radius of the electron. Taking into account the interrelation of fundamental physical constants [8 - 11] it is possible to propose the following equivalent formulas for calculating the constant F0 (Fig. 4):

$$F_0 = \left\{ \begin{matrix} \dfrac{\hbar \bullet c \bullet \alpha}{r_e^2} & G\dfrac{m_e^2 \bullet D_0}{r_e^2} & \dfrac{m_e \bullet c}{t_0} & \dfrac{\hbar \bullet c}{l_{pl}^2 \bullet D_0} \\ \dfrac{e^2}{4\pi\varepsilon_0 \bullet r_e^2} & \dfrac{E_h}{r_e \bullet \alpha^2} & G\dfrac{m_{pl}^2 \bullet \alpha}{r_e^2} & \\ \dfrac{m_e \bullet c^2}{r_e} & \dfrac{\hbar \bullet \alpha}{r_e \bullet t_0} & \dfrac{2\mu_B \bullet m_e \bullet c \bullet \alpha}{e \bullet r_e^2} & \end{matrix} \right\} = \mathbf{29{,}0535101(50)\ N}$$

Fig. 4. Equivalent formulas to calculate the fundamental constant of force F_0 using the fundamental physical constants. e - electron charge; $\hbar$ - Planck constant; G - Newtonian constant of gravitation , r_e - classical electron radius; c - speed of light in vacuum; α - fine-structure constant;

t_0 - time constant for electron ($t_0 = r_e/c$); number π, m_e - electron mass, D_o - large Dirac number, μ_B - Bohr magneton, E_h - Hartree energy, l_{pl} - Planck length, m_{pl} - Planck mass.

11. Universal formula of force

Using the fundamental constant of force F_0 the generalized formulas of Newton's laws of mechanics and electromagnetism laws can be represented as one formula (Fig. 5):

$$F = 29{,}0535101(50) \bullet \{k_1 \bullet k_2 \bullet k_3 \bullet k_4\}$$

Fig. 5. Universal formula of force

This is the Universal formula of force for all types of force interaction considered above. Its structure is represented by the product of the Fundamental constant of force F_0 = 29,0535101 N on dimensionless values (Fig. 6). Fundamental constant of force (F_0 = 29,0535101 N) acts as a force interaction constant in the universal formula of force. Dimensionless values k_1 - k_4 are presented as the ratio of physical characteristics of the interacting bodies to their physical constants.

$$F = F_0 \bullet \left\{ \begin{array}{l} \text{The dimensionless quantities of mass} \\ \text{(m), charge (q), current (I), distance} \\ \text{(r), magnetic induction (B), field} \\ \text{strength (E), speed (V), acceleration} \\ \text{(a), frequency } (\nu). \end{array} \right\} = 29{,}0535101.. \bullet \{k_1 \bullet k_2 \bullet k_3 \bullet k_4\}$$

Fig. 6. Structure of the Universal formula of force

If the characteristics of interacting bodies are electric charges q_1, q_2, the dimensionless quantities in the universal formula of force are presented as follows: $k_1 = q_1/e$, $k_2 = q_2/e$, $k_3 = r_e^2/r^2$, $k_4 = 1$. The physical constant for electric charges is the charge of the electron. The physical constant for distance r is the classical radius of the electron. In this case Coulomb's law follows from Universal formula of force (Fig. 7).

$$F = 29{,}0535101 \bullet \{k_1 \bullet k_2 \bullet k_3 \bullet k_4\} = k\frac{q_1 \bullet q_2}{r^2}$$

where: $k_1 = q_1/e$, $k_2 = q_2/e$, $k_3 = r_e^2/r^2$, $k_4 = 1$

$$F = k\frac{q_1 \bullet q_2}{r^2}$$

Fig. 7. From the Universal formula of force Coulomb's law follows.

If the characteristics of interacting bodies are masses, then the dimensionless quantities in the Universal formula of force are presented as follows: $k_1 = m_1/m_e$, $k_2 = m_2/m_e$, $k_3 = r_e^2/r^2$, $k_4 = 1/D_0$. In this case Newton's law of gravitation follows from the Universal formula of force (Fig. 8).

$$F = 29,0535101 \bullet \{k_1 \bullet k_2 \bullet k_3 \bullet k_4\} = G\frac{m_1 \bullet m_2}{r^2}$$

where: $k_1 = m_1/m_e$, $k_2 = m_2/m_e$, $k_3 = r_e^2/r^2$, $k_4 = 1/D_0$

$$F = G\frac{m_1 \bullet m_2}{r^2}$$

Fig. 8. Newton's law of gravitation follows from the Universal formula of force.

If the characteristics of interacting bodies are electric currents then the dimensionless quantities in Universal formula of force are presented as follows: $k_1 = I_1/I_0$, $k_2 = I_2/I_0$, $k_3 = l/r_e$, $k_4 = r_e/r$. In this case from the Universal formula of force follows the Ampere's law (Fig. 9).

$$F = 29,0535101 \bullet \{k_1 \bullet k_2 \bullet k_3 \bullet k_4\} = ma$$

where: $k_1 = m/m_e$, $k_2 = a/a_0$, $k_3 = 1$, $k_4 = 1$

$$F = ma$$

Fig. 9. From the Universal formula of force follows Newton's second law.

If the characteristics are mass and acceleration, the dimensionless quantities in the Universal formula of force are represented as follows: $k_1 = m/m_e$, $k_2 = a/a_0$, $k_3 = 1$, $k_4 = 1$. In this case from the Universal formula of force follows Newton's second law (Fig. 10).

$$F = 29,0535101 \bullet \{k_1 \bullet k_2 \bullet k_3 \bullet k_4\} = \frac{\mu_0}{4\pi} \bullet \frac{I_1 \bullet I_2 \bullet l}{r}$$

where: $k_1 = I_1/I_0$, $k_2 = I_2/I_0$, $k_3 = r_e/r$, $k_4 = l/r_e$

$$F = \frac{\mu_0}{4\pi}\frac{I_1 \bullet I_2 \bullet l}{r}$$

Fig. 10. Ampere's law follows from the universal formula of force.

If the characteristics are mass, radius of rotation and frequency of rotation, the dimensionless quantities in the universal formula of force are represented as follows: $k_1 = m/m_e$, $k_2 = R/r_e$, $k_3 = v/v_0$, $k_4 = 1$. In this case, the centripetal force formula follows from the universal formula of force (Fig. 11).

$$F = 29,0535101 \bullet \{k_1 \bullet k_2 \bullet k_3 \bullet k_4\} = mRv^2$$

where: $k_1 = m/m_e$, $k_2 = R/r_e$, $k_3 = v^2/v_0$, $k_4 = 1$

$$F = mRv^2$$

Fig. 11. The formula for the centripetal force follows from the Universal formula of force.

If the characteristics are electric charge, velocity, and magnetic induction, the dimensionless quantities in the Universal formula of force are represented as follows: $k_1 = q/e$, $k_2 = V/c$, $k_3 = B/B_0$, $k_4 = 1$. In this case from the Universal formula of force follows the law of magnetic Lorentz force (Fig. 12).

$$F = 29,0535101 \bullet \{k_1 \bullet k_2 \bullet k_3 \bullet k_4\} = q \bullet V \bullet B$$

where: $k_1 = q_1/e$, $k_2 = V/c$, $k_3 = B/B_0$, $k_4 = 1$

$$F = qVB$$

Fig. 12: The law of magnetic Lorentz force follows from the Universal formula of force.

If the characteristics are electric charge and electric field strength, the dimensionless quantities in the Universal formula of force are represented as follows: $k_1 = q/e$, $k_2 = E/E_0$, $E_0 = e / 4\pi\varepsilon_0 r_e^2$, $k_3 = 1$, $k_4 = 1$. In this case from the Universal formula of force follows the law of electric Lorentz force (Fig. 13).

$$F = 29,0535101 \bullet \{k_1 \bullet k_2 \bullet k_3 \bullet k_4\} = q \bullet E$$

where: $k_1 = q_1/e$, $k_2 = E/E_0$, $k_3 = 1$, $k_4 = 1$

$$F = q \bullet E$$

Fig. 13. From the Universal formula of force follows the law of the Lorentz electric force.

Newton's laws of mechanics and laws of electromagnetism (Fig. 14) follow from the Universal formula of force as particular results.

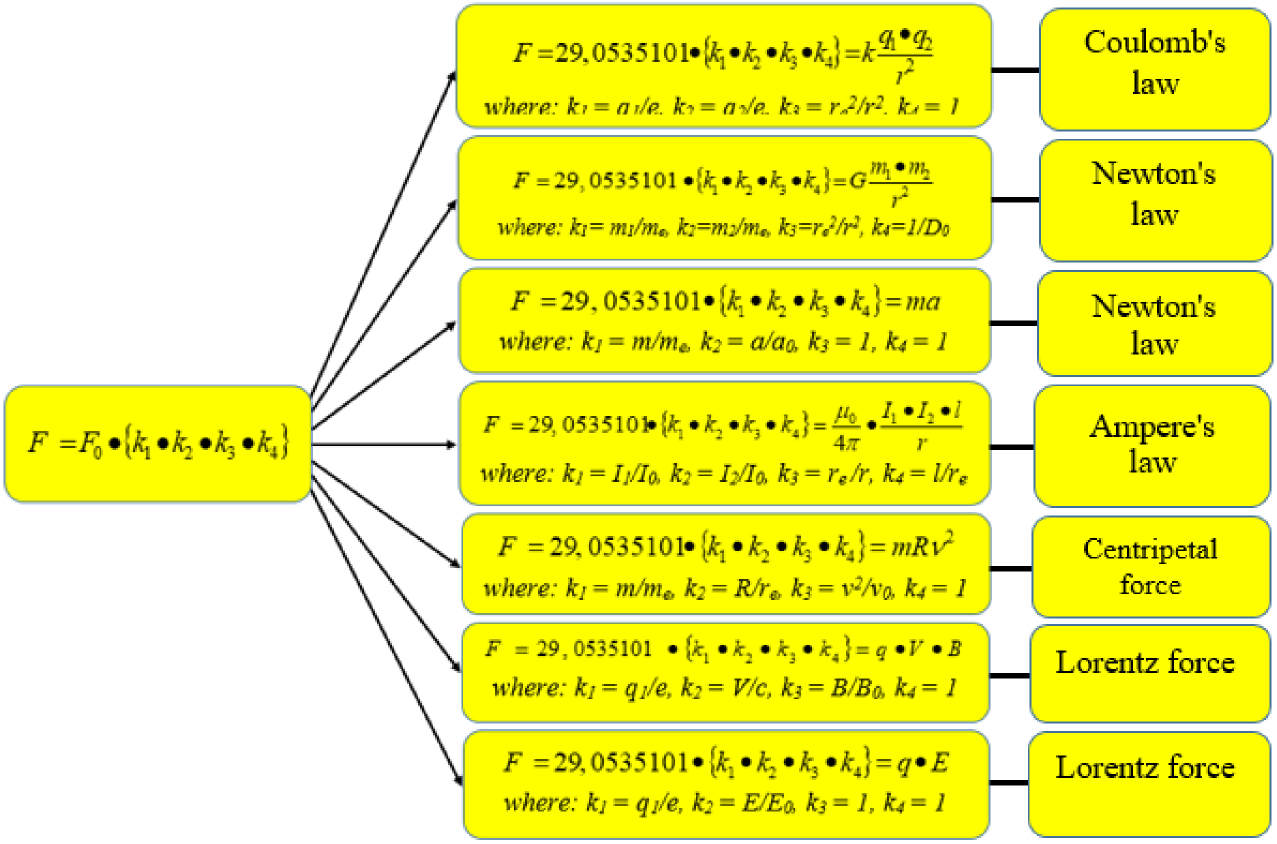

Fig. 14. From the Universal formula of force, as individual results, follow the fundamental laws of mechanics and electromagnetism.

The value of electric, magnetic, mechanical or gravitational force is a function of dimensionless quantities. The dimensionless quantities are the ratios of characteristics of interacting bodies to the constants of these characteristics. These are the ratios of mass m, charge q, current I, distance r, magnetic induction B, etc. to the corresponding fundamental constants. To the constants: to the mass of the electron (m_e), to the charge of the electron (e), to the current constant (I_0), to the classical radius of the electron (r_e), to the magnetic induction constant (B_0), etc.

From the Universal formula of force, it is easy to derive a formula for a particular law of force. It depends on which physical characteristics of the interacting bodies act as parameters in the formula. For example, Universal formula of force turns into Newton's law of gravitation if the parameters in the formula are dimensionless masses. Universal formula of force turns into Coulomb's law if the parameters in the formula are dimensionless electric charges. Universal formula of force turns into Ampere's law if the parameters in the formula are dimensionless electric currents. From the Universal formula of force follows Newton's second law if the parameters in the formula are dimensionless mass and acceleration.

12. Relation of the Newtonian constant of gravitation G with fundamental physical constants.

From the Universal formula of force directly follows the equation for the Newtonian constant of gravitation G. The equation for the constant G is a combination of fundamental physical constants.

Let's show this. Given a large Dirac number D_0 and the relation for F_0 (formula (2)), let us represent Newton's law of gravitation (formulas (3), (5)) as:

$$F_N = F_{0N} \bullet \left\{ \frac{k_{m1} \bullet k_{m2}}{k_r} \right\} = \frac{\hbar \bullet c \bullet \alpha}{r_e^2 \bullet D_0} \bullet \left\{ \frac{k_{m1} \bullet k_{m2}}{k_r} \right\} \qquad (13)$$

After substituting the values $k_{m1} = m_1/m_e$, $k_{m2} = m_2/m_e$, $k_r = r^2/r_e^2$ into equation (13) we obtain:

$$F_N = \frac{\hbar \bullet c \bullet \alpha}{r_e^2 \bullet D_0} \bullet \left\{ \frac{k_{m1} \bullet k_{m2}}{k_r} \right\} = \frac{\hbar \bullet c \bullet \alpha}{D_0 \bullet m_e^2} \bullet \frac{m_1 \bullet m_2}{r^2} = G \frac{m_1 \bullet m_2}{r^2} \qquad (14)$$

In formula (14) the Newtonian constant of gravitation G is represented by a combination of fundamental physical constants:

$$G = \frac{\hbar \bullet c \bullet \alpha}{D_0 \bullet m_e^2} \qquad (15)$$

From equation (15) it follows that the Newtonian constant of gravitation G is a compound constant. The formula of constant G includes Planck's constant, the fine structure constant α, the speed of light and the constants that relate to the electron (m_e, D_0). This is not the only formula for the Newtonian constant of gravitation G. This constant can be represented by other fundamental physical constants. Since there is a deep interconnection between fundamental physical constants [8 - 11], other equivalent formulas for constant G can be proposed [12 - 16]. In [8] there are 9 formulas, in [16] there are 25 formulas. Here, as an example, we present 10 equivalent formulas for the calculation of constant G (Fig. 15).

$$G = \left\{ \begin{matrix} \dfrac{\hbar \bullet c \bullet \alpha}{D_0 \bullet m_e^2} & \dfrac{r_e^3}{t_0^2 \bullet m_e \bullet D_0} & \dfrac{r_e^5}{t_0^3 \bullet \alpha \bullet \hbar \bullet D_0} \\ \dfrac{c^3 \bullet l_{pl}^2}{\hbar} \quad \dfrac{\hbar \bullet c}{m_{pl}^2} & \dfrac{c^4 \bullet r_e \bullet t_0}{\alpha \bullet \hbar \bullet D_0} & \dfrac{c^4}{F_0 \bullet D_0} \\ \dfrac{E_h \bullet r_e}{\alpha^2 \bullet m_e^2 \bullet D_0} & \dfrac{4\mu_B^2 \bullet \alpha^2 \bullet 10^{-7}}{r_e^2 \bullet m_e^2 \bullet D_0} & \dfrac{c^5 \bullet t_{pl}^2}{\hbar} \end{matrix} \right\} = \mathbf{6.6743x10^{-11} m^3 kg^{-1} s^{-2}}$$

Fig. 15. Equivalent formulas for the calculation of the Newtonian constant of gravitation G using the fundamental physical constants. $\hbar$ *- Planck constant;* r_e *- classical electron radius; c - speed of light in vacuum;* α *- fine-structure constant;* t_0 *- time constant for the electron (*$t_0 = re/c$*); number π,* m_e *- electron mass,* D_o *- large Dirac number,* μ_B *- Bohr magneton,* E_h *- Hartree energy,* t_{pl} *- Planck time,* l_{pl} *- Planck length,* m_{pl} *- Planck mass.*

One of the equivalent formulas for calculating the Newtonian constant of gravitation G is represented exclusively by the electron constants:

$$G = \frac{r_e^3}{t_0^2 \bullet m_e \bullet D_0} \qquad (16)$$

Note that formula (16) includes the ratio r_e^3/t_0^2. This ratio is very similar to the ratio of the cube of the large semi-axes to the squares of the period in Kepler's third law.

All equivalent formulas for calculating the constant G contain electromagnetic constants [12-16].

13. Is the Newtonian constant of gravitation G independent?

The Newtonian constant of gravitation G is traditionally considered an independent fundamental physical constant. It has the status of a universal constant on a par with the speed of light and Planck's constant [17]. An unexpected result follows from the Universal formula of force, which does not confirm the independent status of the constant G. From the universal formula of force follows its composite essence. The Newtonian constant of gravitation G is a combination of the most important fundamental physical constants. The formula for G includes the speed of light, the Planck constant, the finite-structure constant, and the electron constant. The loss of independent status is not a great loss for this constant. On the contrary, the dependent status of the G constant opens up new possibilities. In addition to experimental methods, it is possible to determine its value by calculation. Combining the two approaches - experimental and computational methods - will make it possible to obtain its value with greater accuracy.

14. The need to find a method for a more accurate determination of the Newtonian constant of gravitation G.

In the family of universal fundamental physical constants, the accuracy of the Newtonian constant of gravitation G is the lowest. More than three hundred experiments to measure the constant G have been conducted [18]. But it is impossible to improve the accuracy. This is a big problem of the constant G. For 220 years, its value has not gone far from the value obtained by Cavendish. Here are the results of some of the experiments that show how the values of constant G have changed over 220 years:

G=6.754 x 10^{-11} m^3 kg^{-1} s^{-2} (Cavendish, 1798) [19],
G=6.70(4) x 10^{-11} m^3 kg^{-1} s^{-2} (F. Reich, 1837-1852) [19],
G=6.670 ± 0.005 x 10^{-11} m^3 kg^{-1} s^{-2} (Heyl, P R, 1930) [19],
G=6.67260(50) x 10^{-11} m^3 kg^{-1} s^{-2} (Luther, Towler 1982) [19, 20],
G=6.67259(85) x 10^{-11} m^3 kg^{-1} s^{-2} (CODATA 1986),
G=6.673(10) x 10-11 10^{-11} m^3 kg^{-1} s^{-2} (CODATA 1998),
G=6.67390 x 10^{-11} m^3 kg^{-1} s^{-2} (University of Washington in Seattle, 2000) [21],
G=6.67191(99) x 10-11 10^{-11} m^3 kg^{-1} s^{-2} (Rosi G. 2014г.) [22],
G=6.67430(15) x 10^{-11} m^3 kg^{-1} s^{-2} (CODATA 2018) [17].

No modern methods of measuring the Newtonian constant of gravitation G allow us to obtain its value with an accuracy close to other fundamental physical constants. It remains the way to calculate the value of constant G analytically instead of experimentally. Perhaps this is the only way to solve the problem of the accuracy of constant G.

The possibility of calculating the value of constant G appeared due to the revealed connection of this constant with other fundamental constants. The dependence of Newtonian constant of gravitation G on fundamental physical constants opens up new possibilities. There is a hope to approach the accuracy of the value of constant G to other fundamental constants. The impossibility of obtaining an exact value in experiments makes the calculation method the only real way to solve the problem of the accuracy of the G constant.

From the equivalent formulas (Fig. 15) it follows that the accuracy of the constant G is limited by the Planck constants and a large Dirac number D_0. The Planck constants in turn depend on the large Dirac number [16, 23, 24]:

$$m_{pl} = m_e \bullet \sqrt{\frac{D_0}{\alpha}} \ ; \quad l_{pl} = \frac{r_e}{\sqrt{D_0 \bullet \alpha}} \ ; \quad t_{pl} = \frac{r_e}{c \bullet \sqrt{D_0 \bullet \alpha}} \ . \qquad (17)$$

As a result, the large Dirac number D_0 is the only limiting factor on the way of increasing the accuracy of the Newtonian constant of gravitation G. Therefore a more exact value of the large Dirac number D_0 is the key to solve the problem of the constant G. The Dirac large number D_0 is a dimensionless constant. It is the electron constant. The value D0 = 4.16561...x 10^{42} is obtained using the constant G. It is necessary to identify a new way to calculate a large Dirac number D_0. In [9 - 16, 23, 24] an assumption was made that the number D_0 can be obtained from primary dimensionless constants. The primary dimensionless constants can be the fine structure constant, the photon-electron mass ratio, and the π number [9 - 16, 23, 24]. Potentially, the primary dimensionless constants can provide for a large Dirac number an accuracy close to 10^{-10}. This will allow to calculate the gravitational constant G with accuracy close to 10^{-8} - 10^{-9}.

In recent years the interest in the analytical calculation of physical constants has increased considerably [25 - 30]. Dimensionless constants play an important role in the methods of calculating physical constants. Pellis, Stergios [27 - 29] actively develops methods of calculating the constant G in their works.

15. Conclusion

Thus, the similarity of the formulas of Newton's law of gravity and Coulomb's law is not a coincidence. The reason for the similarity is that these laws derive from a single law of force. The general law of force interaction is represented by the Universal formula of force. Moreover, other laws of force interaction in mechanics and in electromagnetism derive from a single law of force represented by the Universal Formula of Force. The profound unity of the laws extends to Newton's second law, Ampere's law, the Lorentz force, and the centripetal force. The possibility of presenting the laws of force interaction by a single universal formula of force points to the fundamental unity of the forces of inertia, gravitation and electromagnetism.

An unexpected result follows from the universal formula of force. It affects the status of Newtonian constant of gravitation G. It means its compound essence and dependence on the most important fundamental physical constants: the speed of light, Planck constant, finite-structure constant and electron constants. On the other hand it is a desirable result. The dependence of Newtonian constant of gravitation G on fundamental physical constants opens a new way to obtain the exact value of constant G. It is to obtain its value by calculation. The only limiting factor on this way is a large

Dirac number D_0. Therefore, the key to solving the problem of the accuracy of the constant G is a more accurate value of the large Dirac number D_0. Combination of two approaches - experimental and method of calculation of Newtonian constant of gravitation G will allow to obtain its value with greater accuracy.

16. Conclusions

1. The similarity of the formulas of Newton's law of gravitation and Coulomb's law is not formal and is not an accidental coincidence. The similarity of the formulas stems from the single origin of all laws of force interaction in mechanics and in electromagnetism from a single law of force. This is the general law for the forces of inertia, gravitation, electric force, and magnetic force. This law is represented by the Universal formula of force.

2 From the Universal formula of force, as particular results, follow Newton's law of gravitation, Newton's second law, Coulomb's law of electrostatics, Ampere's law, Lorentz's magnetic force, Lorentz's electric force, centripetal force.

3. The interaction constant in the Universal formula of force is the fundamental constant of force F_0 = 29.0535101 N. This is the electromagnetic interaction force of two electrons. In spite of the electromagnetic status of this constant, it enters into both formulas of Newton's laws of mechanics and the laws of electromagnetism.

4 From the Universal formula of force directly follows the equation for calculating the Newtonian constant of gravitation G using the fundamental physical constants.

5. The formulas for calculating the Newtonian constant of gravitation G include Planck's constant, Sommerfeld's constant, and the fundamental constants of the electron.

6. The constant G is not an independent constant. It is a compound constant and is related to the fundamental physical constants.

7. Dependence of Newtonian constant of gravitation G on fundamental physical constants opens a new way to obtain exact value of constant G by analytical calculation. The combination of two approaches - experimental and method of calculating the Newtonian constant of gravitation G will make it possible to obtain its value with a greater accuracy.

8. To obtain exact value of constant G it is necessary to obtain value of large Dirac number D_0 with accuracy close to accuracy of dimensionless fundamental physical constants.

9. The Universal formula of force allows, based on the dimensions of physical quantities, to easily obtain the equation of any law of force interaction in mechanics and in electromagnetism.

10. The Universal formula of force will facilitate the study and understanding of the laws of mechanics and the laws of electromagnetism in the educational process

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