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# Reducing approximate cosmological equations to exact equations

***Abstract:*** *At different times, famous scientists have proposed equations that demonstrate the relationship between cosmological parameters and fundamental physical constants. Some equations are approximate and the coincidences in them are estimated only by order of magnitude. The new large numbers on scales $10^{140}$, $10^{160}$, and $10^{180}$ derived from the scaling law allow us to bring the approximate cosmological equations to exact equations. The approximate Dirac, Teller, Eddington-Weinberg, and Rice equations are reduced to exact equations. The exact equations are obtained from the coincidence of large numbers on the scale $10^{60}$ and on the previously unknown scales $10^{140}$, $10^{160}$ and $10^{180}$.*

***Keywords:*** *large numbers, Dirac equation, Teller equation, Eddington-Weinberg equation, Rice equation, cosmological parameters, parameters of the observable Universe.*

## 1. Introduction

At different times, equations have been proposed by renowned scientists to show the relationship between cosmological parameters and fundamental physical constants. Cosmological equations have been derived from coincidences of large numbers of order $10^{40}$.

The cosmological equations linking the Hubble parameter **H** to the Newtonian constant of gravitation **G** are the Dirac equation [1, 2], the Stewart equation [3, 4], the Eddington-Weinberg equation [5], the Teller equation [6, 7], and the Valev equation [8, 9].

The equation relating the radius of the universe to the Newtonian constant of gravitation G is the Rice equation [10].

The equation relating the cosmological constant Λ to the Newtonian constant of gravitation G is the Nottale equation [11, 12].

The equations linking the three parameters of the Universe: radius, mass, and Newtonian constant of gravitation G are the equations of Milne [13, 14] and E.A, Bleksley [15].

The equations linking the three parameters of the Universe: mass, Hubble parameter and Newtonian constant of gravitation G are the equations of Hoyle [16] and Carvalho [22].

Some equations are approximate and coincidences in them are estimated only by order of magnitude. For a long time it was thought that several orders of magnitude would not matter much in expressions of scale $10^{40}$ and larger.

As a result, both exact equations and approximate cosmological equations are now known. Efforts have been made to reduce the approximate equations to exact equations [17, 18, 19, 20].

The large numbers obtained from the law of scaling [21] make it possible to bring the approximate cosmological equations to exact equations.

## 2. The law of scaling of large numbers

The regularities of the origin of large numbers, which follow from the relations of the dimensional parameters of the Universe, allowed us to derive the law of scaling of large numbers [21].

The law of scaling of large numbers has the form (Fig. 1):

$$D_i = (D_{20})^i = (\sqrt{\alpha D_0})^i$$
$$i = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 7, \pm 8, \pm 9$$

Fig. 1. The scaling law of large numbers. $D_0$ is a large Weyl number ($D_0$ = 4.16561...x $10^{42}$), $\alpha$ is the fine structure constant.

The scaling law provides a new method for calculating the values of large numbers.Its advantage is that large numbers are obtained from dimensionless constants. The scaling law generates large numbers up to scale $10^{180}$ with high accuracy. The large numbers obtained from the scaling law are close to the accuracy of the Newtonian constant of gravitation G. The values of the large numbers and the formulas for their calculation are given in Fig. 2.

$$(\sqrt{\alpha D_0})^0 = 1$$
$$D_{20} = (\sqrt{\alpha D_0})^1 = 1.74349...\bullet 10^{20}$$
$$D_{40} = (\sqrt{\alpha D_0})^2 = 3.03979...\bullet 10^{40}$$
$$D_{60} = (\sqrt{\alpha D_0})^3 = 5.29987...\bullet 10^{60}$$
$$D_{80} = (\sqrt{\alpha D_0})^4 = 9.24033...\bullet 10^{80}$$
$$D_{100} = (\sqrt{\alpha D_0})^5 = 16.1105...\bullet 10^{100}$$
$$D_{120} = (\sqrt{\alpha D_0})^6 = 28.088...\bullet 10^{120}$$
$$D_{140} = (\sqrt{\alpha D_0})^7 = 48.972...\bullet 10^{140}$$
$$D_{160} = (\sqrt{\alpha D_0})^8 = 85.383...\bullet 10^{160}$$
$$D_{180} = (\sqrt{\alpha D_0})^9 = 148.86...\bullet 10^{180}$$

FIG. 2. Large numbers and formulas for their calculation from the law of scaling large numbers.

The table in Fig. 3 summarizes the relations of dimensional quantities that lead to large numbers. The many coincidences of large numbers make it possible to derive the cosmological equations, which include cosmological parameters and fundamental physical constants.

| Ratios of dimensional constants | Scale |
|---|---|
| $\frac{Gm_e^2}{r_e\alpha^2\hbar H}=\frac{G\hbar}{r_e^3Hc^2}=\frac{Gm_e}{r_e^2\alpha Hc}=\frac{Gm_e^3c}{\alpha^3\hbar^2H}=\frac{c^2}{M_UR_UG\Lambda}=\frac{c^3}{M_UGH}=\frac{c^2R_U}{M_UG}=\frac{l_{Pl}^4}{\Lambda r_e^6}=\frac{c^3T_U}{M_UG}=\frac{\Lambda c^2}{H^2}=\frac{c^3T_U^3}{R_U^3}=\frac{H^2}{M_UR_UG\Lambda^2}=.\frac{cr_e^3A_0}{G\hbar}=\frac{c^4}{M_UR_UH^2G}=\frac{M_UR_UHA_0G}{c^5}=1$ | $10^0$ |
| $D_{20}=\frac{r_e}{l_{Pl}}=\frac{t_0}{t_{Pl}}=\frac{\alpha\, m_{Pl}}{m_e}=\frac{cl_{Pl}}{r_e^2H}=\frac{l_{Pl}R_U}{r_e^2}=\frac{c^2l_{Pl}}{r_e^2A_0}=\sqrt{\alpha D_0}$ | $10^{20}$ |
| $D_{40}=\frac{T_U}{t_0}=\frac{R_U}{r_e}=\frac{m_ec^2}{\alpha\hbar H}=\frac{1}{t_0H}=\frac{r_e^2}{l_{Pl}^2}=\frac{t_0^2}{t_{Pl}^2}=\frac{\alpha^2m_{Pl}^2}{m_e^2}=\frac{c^2}{r_eA_0}=(\sqrt{\alpha D_0})^2$ | $10^{40}$ |
| $D_{60}=\frac{T_U}{t_{Pl}}=\frac{R_U}{l_{Pl}}=\frac{M_U}{m_{Pl}}=\frac{c}{l_{Pl}H}=\frac{r_e^3}{l_{Pl}^3}=\frac{t_0^3}{t_{Pl}^3}=\frac{c^3}{Gm_{Pl}H}=\frac{c^2}{l_{Pl}A_0}=(\sqrt{\alpha D_0})^3$ | $10^{60}$ |
| $D_{80}=\frac{R_U^2}{r_e^2}=\frac{HM_U^2\alpha G}{c^3m_e}=\frac{c^2}{r_e^2H^2}=\frac{cr_e}{Hl_{Pl}^2}=\frac{1}{r_e^2\Lambda}=(\sqrt{\alpha D_0})^4$ | $10^{80}$ |
| $D_{100}=\frac{m_ec^3}{l_{Pl}\alpha\hbar H^2}=\frac{r_e\alpha M_U}{l_{Pl}m_e}=\frac{HM_U^2\alpha Gr_e}{c^3m_el_{Pl}}=\frac{R_U^2}{r_el_{Pl}}=\frac{1}{r_el_{Pl}\Lambda}=(\sqrt{\alpha D_0})^5$ | $10^{100}$ |
| $D_{120}=\frac{T_U^2}{t_{Pl}^2}=\frac{R_U^2}{l_{Pl}^2}=\frac{M_U^2}{m_{Pl}^2}=\frac{c^2}{l_{Pl}^2H^2}=\frac{R_U^3}{r_e^3}=\frac{M_Uc^2}{\hbar H}=\frac{GM_U^2}{\hbar c}=\frac{c^5}{G\hbar H^2}=\frac{c^3}{G\hbar\Lambda}=\frac{1}{l_{Pl}^2\Lambda}=(\sqrt{\alpha D_0})^6$ | $10^{120}$ |
| $D_{140}=\frac{r_e^2m_ec^3}{l_{Pl}^3\alpha\hbar H^2}=\frac{r_e^3\alpha M_U}{l_{Pl}^3m_e}=\frac{R_U^3}{l_{Pl}r_e^2}=\frac{1}{t_{Pl}t_0^2H^3}=\frac{c}{l_{Pl}r_e^2H\Lambda}=\frac{c^2}{l_{Pl}r_e^2A_0\Lambda}=(\sqrt{\alpha D_0})^7$ | $10^{140}$ |
| $D_{160}=\frac{M_UR_Uc^2\alpha^2}{Gm_e^2}=\frac{M_U^2R_UG\alpha}{c^2r_e^2m_e}=\frac{1}{r_e^4\Lambda^2}=(\sqrt{\alpha D_0})^8$ | $10^{160}$ |
| $D_{180}=\frac{r_e^4m_ec^3}{l_{Pl}^5\alpha\hbar H^2}=\frac{r_e^5\alpha M_U}{l_{Pl}^5m_e}=\frac{R_U^3}{l_{Pl}^3}=\frac{c^3}{l_{Pl}^3H^3}=\frac{c}{l_{Pl}^3H\Lambda}=\frac{c^2}{l_{Pl}^3A_0\Lambda}=\frac{GM_UT_U^2l_{Pl}}{\Lambda r_e^6}=(\sqrt{\alpha D_0})^9$ | $10^{180}$ |

Fig. 3. Set of coincidences of large numbers. MU is the mass of the observable Universe, α is the fine structure constant, ħ is Planck's constant, G is the Newtonian gravitational constant, Λ is the cosmological constant, $R_U$ is the radius of the observable Universe, $T_U$ is the time of the Universe, H is the Hubble constant, $A_0$ is the cosmological acceleration, $r_e$ is the classical radius of the electron; c - speed of light in vacuum; $t_0 = r_e/c$, $m_e$ - electron mass, $D_0$ - large Weyl number, $t_{pl}$ - Planck time, $l_{pl}$ - Planck length, $m_{pl}$ - Planck mass.

## 3. Exact cosmological equations.

Among the known cosmological equations, the exact equations are the Stewart equation [3, 4], the Nottale equation [11, 12] , the E. A. Milne equation [13] , the A. A. Milne equation. E.H. Bleksley [15], the equation of J. C. Carvalho [22].

The **GH**-equation of Stewart [3, 4] establishes the relationship between the two parameters of the universe G and H. The formula is of the form

$$Gm_e^2 / r_e = \alpha^2 \hbar H \qquad (1)$$

**GΛ**-equation of Nottale [11, 12] establishes the relationship between the two parameters of the universe G, Λ. The formula is of the form:

$$c^3 / G\hbar\Lambda \qquad (2)$$

**GMT**-equation Milne [13] establishes the relationship between the three parameters of the universe G, $M_U$, $T_U$. The formula is of the form:

$$\mathbf{M_U = c^3 T_U / G} \qquad (3)$$

**GMR**-equation Bleksley [15] establishes the relationship between the three parameters of the Universe G, $M_U$, $R_U$. The formula is of the form:

$$\mathbf{M_U = c^2 R_U / G} \qquad (4)$$

The **GMH**-equation of Carvalho [22] establishes the relation between the three parameters of the Universe G, $M_U$, H. The formula is of the form:

$$M_U = c^3 / GH \qquad (5)$$

## 4. Approximate cosmological equations that combine fundamental physical constants and parameters of the Universe.

The approximate equations are the Dirac equation [1, 2], the Eddington-Weinberg equation [5], the Teller equation [6, 7], and the James Rice equation [10].

The **GH**-Dirac equation [1, 2] is of the form:

$$m_e c^3 /(He^2) \approx e^2 /(Gm_e^2) \qquad (6)$$

The **GH**-equation of Eddington-Weinberg [5] has the form:

$$\hbar^2 H \approx Gcm_p^3 \qquad (7)$$

The **GH**-Teller equation [6, 7] has the form:

$$2\frac{G\hbar H}{c^4 l_{Pl}} = 2t_{Pl}H = 2\frac{Gm_{Pl}H}{c^3} \cong \exp(-1/\alpha) \qquad (8)$$

The **GR**-equation of James Rice [10] has the form:

$$\frac{4\pi}{\alpha} \approx \frac{r_e^2 c^2}{6R_U G m_e} \qquad (9)$$

## 5. Reducing approximate cosmological equations to exact equations

The law of scaling of large numbers allows us to reduce the approximate cosmological equations of Dirac, Eddington-Weinberg, Teller, and Rice to exact equations.

The Dirac **GH**-equation [1, 2] is an approximate equation. Earlier attempts were made to bring the Dirac equation to exact equality of the left and right parts [17, 18]. The large numbers obtained from the scaling law (Fig. 1) allow us to obtain the exact equation (Fig. 4).

| Approximate Dirac equation | Exact equation |
|---|---|
| $m_e c^3/(He^2) \approx e^2/(Gm_e^2)$ | $m_e c^3/(He^2) = \alpha e^2/(Gm_e^2)$ |

Fig. 4. Dirac's equation reduced to exact equality.

It should be borne in mind that Dirac proposed his equation before the SI system was introduced. Therefore, when using a charge value of $1.602\ldots x\ 10^{-19}$ C in the formula, ($k = c^2 10^{-7}$) must be considered.

The **GH**-equation of Eddington-Weinberg [5] does not take into account the electron mass and the fine structure constant alpha. An exact equation is obtained on a scale of $10^{40}$ (Fig. 5). The same exact equation was obtained in [19, 20] .

| Approximate Eddington-Weinberg equation | Exact equation |
|---|---|
| $\hbar^2 H \approx Gcm_p^3$ | $\hbar^2 H \alpha^3 = Gcm_e^3$ |

Fig. 5. Eddington-Weinberg equation reduced to exact equality.

The GH-equation of Teller [6] is an approximate equation.From the coincidence of large numbers on scales $10^{60}$, $10^{120}$ and $10^{180}$, the exact equation is obtained (Fig. 6).

| Approximate Teller's equation | Exact equation |
|---|---|
| $2\dfrac{G\hbar H}{c^4 l_{Pl}} = 2t_{Pl}H = 2\dfrac{Gm_{Pl}H}{c^3} \cong \exp(-1/\alpha)$ | $\dfrac{G\hbar H}{c^4 l_{Pl}} = t_{Pl}H = \dfrac{Gm_{Pl}H}{c^3} = (\sqrt{\alpha D_0})^{-3}$ |

Fig. 6. E. Teller equation reduced to exact equality.

The GR-equation of Rice [10] is an approximate equation. At a scale of $10^{60}$, the exact equation is obtained (Fig. 7).

| Approximate equation Rice | Exact equation |
|---|---|
| $\dfrac{2\pi}{\alpha} = \dfrac{r_e^2 c^2}{6R_U 2Gm_e}$ | $\dfrac{R_U G m_e}{r_e^2 c^2} = \alpha$ |

Fig. 7. James Rice equation reduced to exact equality.

## References


1. Dirac, P. A. M., *Nature*, 139, 323 (**1937**).
2. Dirac, P. A. M., *Proc. R. Soc.*, **A 165**, 199 (1938)
3. Stewart J. O. “Phys. Rev.”, 1931, v. 38, p. 2071.

4. Muradyan, R. M. Universal constants. Physical and astrophysical constants and their dimensional and dimensionless combinations. Fiz. ehlementar. chastits i atom. yadra, Vol. 8, No. 1, p. 175 - 192. 1977.
5. S. Weinberg, “Gravitation and Cosmology,” Wiley, New York, 1972.
6. E. Teller, Phys. Rev. 73, 801(1948) E. Teller (1948). On the change of physical constants. Physical Review, vol.73 pp. 801—802. DOI:10.1103/PhysRev.73.801
7. Kuhne, Rainer W.. “TIME-VARYING FINE-STRUCTURE CONSTANT REQUIRES COSMOLOGICAL CONSTANT.” Modern Physics Letters A 14 (1999): 1917-1922.
8. Valev, D. (2013). Three Fundamental Masses Derived by Dimensional Analysis. Space Science International, 1(2), 145-149. https://doi.org/10.3844/ajssp.2013.145.149).
9. Forsythe, C. J. & Valev, D. T. (2014). EXTENDED MASS RELATION FOR SEVEN FUNDAMENTAL MASSES AND NEW EVIDENCE OF LARGE NUMBERS HYPOTHESIS. Physics International, 5(2), 152-158. https://doi.org/10.3844/pisp.2014.152.158
10. Rice J. On Eddington's natural unit of the field // Philosophical Magazine Series 6. — 1925. — Vol. 49. — P. 1056—1057.
11. Nottale, L. (1993) Mach’s Principle, Dirac’s Large Numbers and the Cosmological Constant Problem. http://www.luth.obspm.fr/%7Eluthier/nottale/arlambda.pdf
12. Laurent Nottale. Solution to the cosmological constant problem from dark energy of the quark vacuum. 2019. ⟨hal-02133255⟩.
13. Milne, E. A. On the foundations of dynamics. *Proc. R. Soc. A* **1936**, *154*, 22–52.
14. Espen Gaarder Haug. Progress in the Composite View of the Newton Gravitational Constant and Its Link to the Planck Scale. Universe 2022, 8(9), 454; https://doi.org/10.3390/universe8090454
15. A. E. H. Bleksley. A new approach to cosmology: Iii. South African Journal of Science, 48 (1), 1951. URL https://hdl.handle.net/10520/AJA00382353_3467
16. F. Hoyle, in Proc. 11th Solvay Conference in Physics, Ed. R. Stoops, Brussels, Belgium (1958).
17. CAVALLO, G. Interpretation of the Dirac Relationship between Fundamental Constants. Nature 245, 313–314 (1973). https://doi.org/10.1038/245313a0
18. NARLIKAR, J. Lepton Creation and the Dirac Relationship between Fundamental Constants. *Nature* 247, 99–100 (1974). https://doi.org/10.1038/247099a0
19. L. Nottale, Fractal Space-Time and Microphysics (World Scientific, London, (1993) pp. 283-307. (misprints corrected and notes added, 27 March 2006.)
20. P. H. Chavanis, Phys. Dark Univ. 24, 100271 (2019).
21. Mykola Kosinov. THE LAW OF SCALING FOR LARGE NUMBERS: origin of large numbers from the primary large number D20 = 1.74349...x 10^20. January 2024. DOI: 10.13140/RG.2.2.33664.20480
22. Carvalho, J.C. Derivation of the mass of the observable universe. Int J Theor Phys 34, 2507–2509 (1995). https://doi.org/10.1007/BF00670782