CLOCK AND RULERS IN QUANTUM MECHANICS

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Abstract. I did present a possible way of creating spacetime from just information from wave function.

Date: June 22, 2024.

Contents

1. Wave function

I will start by defining a wave function by using how coordinates are connected to basis vectors. Basis vectors are equal to wave-vector in given point of spacetime, where this wave vector consists of n vectors total in n dimensional spacetime. It means that I can write wave function:

$$\psi(\mathbf{x}) = \psi\left(\hat{\lambda}_\alpha x^\alpha\right) \tag{1.1}$$

Where each wave vector $\hat{\lambda}$ has directions α so it's denoted as $\hat{\lambda}_\alpha$. Those are basis vectors, there are connected to frequency basis vectors. Each frequency is a vector in given direction and their relation with wave vector can be expressed:

$$\hat{\omega}^\dagger_\alpha \hat{\lambda}_\alpha = \delta^\alpha_\alpha \tag{1.2}$$

$$\hat{\lambda}^\dagger_\alpha \hat{\omega}_\alpha = \delta^\alpha_\alpha \tag{1.3}$$

Where $\dagger$ operator means complex conjugate and transposition. Wave vector itself transforms under complex rotations that can be expressed using special unitary matrix [1] for each vector that will be denoted as $\hat{U}^\alpha_\beta$, so those rotations can be expressed:

$$\hat{\lambda}_\alpha \hat{U}^\alpha_\beta = \hat{\lambda}_\beta \tag{1.4}$$

$$\hat{\lambda^\dagger}_\alpha \hat{U^\dagger}^\alpha_\beta = \hat{\lambda}_\beta \tag{1.5}$$

Total energy density can be expressed as a scalar function of frequency vectors:

$$\rho(\mathbf{x}) = \hat{\omega}^\dagger_\alpha \hat{\omega}_\alpha \tag{1.6}$$

Now spacetime interval for given point is equal to wave vector times it's dagger operator part:

$$ds^2(\mathbf{x}) = \hat{\lambda}^\dagger_\alpha(\mathbf{x})\,\hat{\lambda}_\alpha(\mathbf{x}) \tag{1.7}$$

To find a geodesic [2] in this spacetime I need to take an integral over spacetime interval and varry it then make it equal to zero, but not for one path but all possible paths, where those paths are defined as starting from any point of space:

$$\sum_{\text{all paths}} \delta \int ds = 0 \tag{1.8}$$

Field equation for given wave function is given by:

$$-i\kappa\hat{\partial}_\alpha \psi(\mathbf{x}) = \hat{\omega}_\alpha \psi(\mathbf{x}) \tag{1.9}$$

Where κ is constant. Probability of traveling all possible paths that start at each point of space and each of them are geodesic is equal to one:

$$\sum_{\text{all paths}} \int_P \psi^*(\mathbf{x})\,\psi(\mathbf{x})\,ds = 1 \tag{1.10}$$

2. Many systems wave function

Now I can move to many systems wave function. Re-writing field equation for many system each of them will need to obey field equation, so only change is that frequency vector depends on all possible systems not just on one:

$$-i\kappa\hat{\partial}_{\alpha_n}\psi\left(\mathbf{x_1},...,\mathbf{x_n}\right)=\hat{\omega}_{\alpha_n}\left(\mathbf{x_1},...,\mathbf{x_n}\right)\psi\left(\mathbf{x_1},...,\mathbf{x_n}\right) \tag{2.1}$$

$$\hat{\partial}_{\alpha_1}=\begin{pmatrix}\hat{\partial}_{\alpha_1} & ... & \hat{\partial}_{\alpha_n}\end{pmatrix} \tag{2.2}$$

$$\hat{\omega}_{\alpha_n}\left(\mathbf{x_1},...,\mathbf{x_n}\right)=\begin{pmatrix}\hat{\omega}_{\alpha_1}\left(\mathbf{x_1}\right) & ... & \hat{\omega}_{\alpha_n}\left(\mathbf{x_n}\right)\end{pmatrix} \tag{2.3}$$

Same can be done with geodesic equation, I will sum all possible paths of all possible systems:

$$\sum_{i=1}^{n}\sum_{\text{all paths}}\delta\int ds_i\left(\mathbf{x_i}\right)=0 \tag{2.4}$$

$$ds_i\left(\mathbf{x_i}\right)=\sqrt{\hat{\lambda}^{\dagger}_{\alpha_i}\left(\mathbf{x_i}\right)\hat{\lambda}_{\alpha_i}\left(\mathbf{x_i}\right)} \tag{2.5}$$

It means that total energy density is defined as:

$$\rho\left(\mathbf{x_1},..,\mathbf{x_n}\right)=\hat{\omega}^{\dagger}_{\alpha_n}\left(\mathbf{x_1},..,\mathbf{x_n}\right)\hat{\omega}_{\alpha_n}\left(\mathbf{x_1},..,\mathbf{x_n}\right) \tag{2.6}$$

Now probability of all wave functions following given geodesic paths is equal to:

$$\sum_{\text{all paths}}\int_{P_1}...\int_{P_n}\psi^{*}\left(\mathbf{x_1},..,\mathbf{x_n}\right)\psi\left(\mathbf{x_1},..,\mathbf{x_n}\right)ds_1...ds_n=1 \tag{2.7}$$

Wave function of many systems is defined same wave as before but for many systems:

$$\psi\left(\mathbf{x_1},..,\mathbf{x_n}\right)=\psi\left(\hat{\lambda}_{\alpha_1}x^{\alpha_1},...,\hat{\lambda}_{\alpha_n}x^{\alpha_n}\right) \tag{2.8}$$

From it follows that wave vectors are connected to frequency vectors by same relations:

$$\hat{\omega}^{\dagger}_{\alpha_n}\left(\mathbf{x_1},..,\mathbf{x_n}\right)\hat{\lambda}_{\alpha_n}\left(\mathbf{x_1},..,\mathbf{x_n}\right)=n\delta^{\alpha}_{\alpha} \tag{2.9}$$

$$\hat{\lambda}^{\dagger}_{\alpha_n}\left(\mathbf{x_1},..,\mathbf{x_n}\right)\hat{\omega}_{\alpha_n}\left(\mathbf{x_1},..,\mathbf{x_n}\right)=n\delta^{\alpha}_{\alpha} \tag{2.10}$$

They rotate by special unitary matrix but now it depends on all possible systems:

$$\hat{\lambda}_{\alpha}\left(\mathbf{x_1},..,\mathbf{x_n}\right)\hat{U}^{\alpha}_{\beta}\left(\mathbf{x_1},..,\mathbf{x_n}\right)=\hat{\lambda}_{\beta}\left(\mathbf{x_1},..,\mathbf{x_n}\right) \tag{2.11}$$

$$\hat{\lambda^{\dagger}}_{\alpha}\left(\mathbf{x_1},..,\mathbf{x_n}\right)\hat{U^{\dagger}}^{\alpha}_{\beta}\left(\mathbf{x_1},..,\mathbf{x_n}\right)=\hat{\lambda}_{\beta}\left(\mathbf{x_1},..,\mathbf{x_n}\right) \tag{2.12}$$

$$\hat{U}^{\alpha}_{\beta}\left(\mathbf{x_1},..,\mathbf{x_n}\right)=\begin{pmatrix}\hat{U}^{\alpha_n}_{\beta_1}\left(\mathbf{x_1}\right) & ... & \hat{U}^{\alpha_n}_{\beta_n}\left(\mathbf{x_n}\right)\end{pmatrix} \tag{2.13}$$

$$\hat{U^{\dagger}}^{\alpha}_{\beta}\left(\mathbf{x_1},..,\mathbf{x_n}\right)=\begin{pmatrix}\hat{U^{\dagger}}^{\alpha_n}_{\beta_1}\left(\mathbf{x_1}\right) & ... & \hat{U^{\dagger}}^{\alpha_n}_{\beta_n}\left(\mathbf{x_n}\right)\end{pmatrix} \tag{2.14}$$

References

[1] https://mathworld.wolfram.com/SpecialUnitaryGroup.html

[2] https://mathworld.wolfram.com/Geodesic.html

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