CLOCK AND RULERS IN QUANTUM MECHANICS

TOMASZ KOBIERZYCKI

Abstract. I did present here a possible way of creating spacetime from just information from wave function.

Date: June 27, 2024.

Contents

1. Wave function

Wave function is basics of all quantum physics. I will instead of trying to add gravity effects to it, create a spacetime model out of wave function information alone. I will start be defining a wave function [1], as a sum of four complex vector fields:

$$\psi\left(\mathbf{x}\right)=\sum_{\alpha=1}^{n}\langle x_{\alpha}e^{\alpha}|e_{\alpha}\psi^{\alpha}\rangle=\sum_{\alpha=1}^{n}\langle x_{\alpha}|\psi^{\alpha}\rangle \tag{1.1}$$

Now it's easy to define it's complex conjugate function [2], that will be equal to:

$$\psi^{*}\left(\mathbf{x}\right)=\sum_{\alpha=1}^{n}\langle\psi_{\alpha}e^{\alpha}|e_{\alpha}x^{\alpha}\rangle=\sum_{\alpha=1}^{n}\langle\psi_{\alpha}|x^{\alpha}\rangle \tag{1.2}$$

Each vector here is a vector field but I skip writing that it depends on all coordinates and will reserve writing it just to wave function so scalar field. Now to explain basis vectors e^{α}, e_{α} are key in creating spacetime distance formula, as distance is fully dependent on them:

$$ds^{2}\left(\mathbf{x}\right)=\sum_{\alpha=1}^{n}\langle x_{\alpha}e^{\alpha}|e_{\alpha}x^{\alpha}\rangle=\sum_{\alpha=1}^{n}\langle x_{\alpha}|x^{\alpha}\rangle \tag{1.3}$$

It means that for four dimensional spacetime it's total sixteen equations that contribute to spacetime distance and wave function consists of sixteen independent parts. Now when taking probability of wave function following a given path I need to assume that it follows all geodesic paths that start at each point of space and probability of following given collection of paths is equal to sum of those paths. I can denote it:

$$\sum_{\text{all paths}}\int_{P}\psi^{*}\left(\mathbf{x}\right)\psi\left(\mathbf{x}\right)ds=1 \tag{1.4}$$

Where each path is a geodesic [3] in this spacetime so I can write path equation:

$$\sum_{\text{all paths}}\delta\int_{P}ds=0 \tag{1.5}$$

Where again I sum over all possible geodesic paths that start from all possible points of space. It means that wave function does not collapse after measurement, It just follows a given path, when I do measurement again it can follow a another path with probability given by this equation:

$$\int_{P}\psi^{*}\left(\mathbf{x}\right)\psi\left(\mathbf{x}\right)ds=\rho^{2}\left(\mathbf{x}\right) \tag{1.6}$$

2. Spacetime interval

I can use parametrized curve that is path in spacetime , to expand geodesic equation into integral [4], where I use parameter λ as it's path in spacetime and time is a dimension, where $\mathbf{x}(\lambda)$ denotes path in interval from a to b Now geodesic equation takes form:

$$\delta \int_a^b \sqrt{\sum_{\alpha=1}^{n} \left\langle \frac{dx_\alpha(\mathbf{x}(\lambda))}{d\lambda} \middle| \frac{dx^\alpha(\mathbf{x}(\lambda))}{d\lambda} \right\rangle} d\lambda = 0 \tag{2.1}$$

Now I can define proper path sum as all possible sums so all possible intervals:

$$\sum_{a\in\mathbb{C}} \sum_{b\in\mathbb{C}} \delta \int_a^b \sqrt{\sum_{\alpha=1}^{n} \left\langle \frac{dx_\alpha(\mathbf{x}(\lambda))}{d\lambda} \middle| \frac{dx^\alpha(\mathbf{x}(\lambda))}{d\lambda} \right\rangle} d\lambda = 0 \tag{2.2}$$

I can do same thing but now with probability of given path:

$$\sum_{a\in\mathbb{C}} \sum_{b\in\mathbb{C}} \int_a^b \psi^*(\mathbf{x}(\lambda))\, \psi(\mathbf{x}(\lambda)) \sqrt{\sum_{\alpha=1}^{n} \left\langle \frac{dx_\alpha(\mathbf{x}(\lambda))}{d\lambda} \middle| \frac{dx^\alpha(\mathbf{x}(\lambda))}{d\lambda} \right\rangle} d\lambda = 1 \tag{2.3}$$

Spacetime interval can be expressed same way, for given path that is solution to geodesic equation:

$$s(\mathbf{x}) = \int_a^b \sqrt{\sum_{\alpha=1}^{n} \left\langle \frac{dx_\alpha(\mathbf{x}(\lambda))}{d\lambda} \middle| \frac{dx^\alpha(\mathbf{x}(\lambda))}{d\lambda} \right\rangle} d\lambda \tag{2.4}$$

For all possible paths this expression is a sum of all possible complex paths that are solution to geodesic equation:

$$s(\mathbf{x}) = \sum_{a\in\mathbb{C}} \sum_{b\in\mathbb{C}} \int_a^b \sqrt{\sum_{\alpha=1}^{n} \left\langle \frac{dx_\alpha(\mathbf{x}(\lambda))}{d\lambda} \middle| \frac{dx^\alpha(\mathbf{x}(\lambda))}{d\lambda} \right\rangle} d\lambda \tag{2.5}$$

Spacetime interval is a real, but it's expressed in form of complex numbers, to be more precise complex vector fields that are multiply by it's complex conjugate with transpose so they will always give a real number as result.

3. Field equation

Field equation will state that there is equality between change in wave function and it's energy. I can write field equation with two operators $\hat{D}^{\alpha}_{\alpha}$ that is differential operator [5] equal to:

$$\hat{D}^{\alpha}_{\alpha} = -\frac{Gl_P^2}{c^4}\sum_{\alpha=1}^{n} |\partial^{\alpha}\rangle \langle\partial_{\alpha}| = \kappa \sum_{\alpha=1}^{n} |\partial^{\alpha}\rangle \langle\partial_{\alpha}| \tag{3.1}$$

Now second operator is energy value operator, first I define operator itself then how it changes when adding index to it:

$$\hat{E} = \begin{bmatrix} E^1_1 & ... & E^1_n \\ ... & ... & ... \\ E^n_1 & ... & E^n_n \end{bmatrix} \tag{3.2}$$

So for four dimensional spacetime there is exactly sixty four functions of that energy operator, for each index α there are sixteen. Its components can be written as $E^{i_\alpha}_{j_\alpha} = \hat{E}^{\alpha}_{\alpha}$. I will write field equation:

$$\sum_{\alpha=1}^{n} \langle x_{\alpha}e^{\alpha}| \hat{D}^{\alpha}_{\alpha} |e_{\alpha}\psi^{\alpha}\rangle = \sum_{\alpha=1}^{n} \langle x_{\alpha}e^{\alpha}| \hat{E}^{\alpha}_{\alpha} |e_{\alpha}\psi^{\alpha}\rangle \tag{3.3}$$

$$\sum_{\alpha=1}^{n} \langle x_{\alpha}| \hat{D}^{\alpha}_{\alpha} |\psi^{\alpha}\rangle = \sum_{\alpha=1}^{n} \langle x_{\alpha}| \hat{E}^{\alpha}_{\alpha} |\psi^{\alpha}\rangle \tag{3.4}$$

That can be expanded components wise:

$$\sum_{\alpha=1}^{n} \overline{x}_{i_\alpha} D^{i_\alpha}_{j_\alpha}\psi^{j_\alpha} = \sum_{\alpha=1}^{n} \overline{x}_{i_\alpha} E^{i_\alpha}_{j_\alpha}\psi^{j_\alpha} \tag{3.5}$$

It's n^2 equations for n dimensional spacetime. It says how j component of wave function changes in i direction and it's equal to E^i_j energy component. Where there are n equations like this each representing one possible spacetime direction. Where overline means complex conjugate. Same equation for complex conjugate of wave function is equal to:

$$\sum_{\alpha=1}^{n} \langle \psi_{\alpha}e^{\alpha}| \hat{D}^{\dagger}{}^{\alpha}_{\alpha} |e_{\alpha}x^{\alpha}\rangle = \sum_{\alpha=1}^{n} \langle \psi_{\alpha}e^{\alpha}| \hat{E}^{\dagger}{}^{\alpha}_{\alpha} |e_{\alpha}x^{\alpha}\rangle \tag{3.6}$$

$$\sum_{\alpha=1}^{n} \langle \psi_{\alpha}| \hat{D}^{\dagger}{}^{\alpha}_{\alpha} |x^{\alpha}\rangle = \sum_{\alpha=1}^{n} \langle \psi_{\alpha}| \hat{E}^{\dagger}{}^{\alpha}_{\alpha} |x^{\alpha}\rangle \tag{3.7}$$

$$\sum_{\alpha=1}^{n} \overline{\psi}_{j_\alpha} \overline{D}^{j_\alpha}_{i_\alpha} x^{i_\alpha} = \sum_{\alpha=1}^{n} \overline{\psi}_{j_\alpha} \overline{E}^{j_\alpha}_{i_\alpha} x^{i_\alpha} \tag{3.8}$$

4. Quantum spacetime

What is key to build a quantum spacetime model is some invariant property of it. And in case of quantum spacetime it comes from field equation, field equation changes with respect to basis vectors , wave function and energy states but always gives same result. Let me consider two wave functions and their field equation energy solutions:

$$\psi(\mathbf{x}) = \sum_{\alpha=1}^{n} \langle x_\alpha | \psi^\alpha \rangle \quad \phi(\mathbf{x}) = \sum_{\alpha=1}^{n} \langle y_\alpha | \phi^\alpha \rangle \tag{4.1}$$

Now invariant property of those functions is that basis vectors cancel out with energy states, so more energy system has shorter it's basis vectors are and vice versa. It means that If I expand solutions to field equation I will arrive just at probability functions:

$$\sum_{\alpha=1}^{n} \langle x_\alpha e^\alpha | \hat{E}^\alpha_\alpha | \psi^\alpha e_\alpha \rangle = \sum_{\alpha=1}^{n} \langle x_\alpha | \hat{E}^\alpha_\alpha | \psi^\alpha \rangle \quad \sum_{\alpha=1}^{n} \langle y_\alpha e^\alpha | \hat{K}^\alpha_\alpha | \phi^\alpha e_\alpha \rangle = \sum_{\alpha=1}^{n} \langle x_\alpha | \hat{K}^\alpha_\alpha | \phi^\alpha \rangle \tag{4.2}$$

$$\sum_{\alpha=1}^{n} \langle x_\alpha n^\alpha | \hat{I}^\alpha_\alpha | \psi^\alpha n_\alpha \rangle = \sum_{\alpha=1}^{n} \langle x_\alpha | \hat{E}^\alpha_\alpha | \psi^\alpha \rangle \quad \sum_{\alpha=1}^{n} \langle y_\alpha n^\alpha | \hat{I}^\alpha_\alpha | \phi^\alpha n_\alpha \rangle = \sum_{\alpha=1}^{n} \langle x_\alpha | \hat{K}^\alpha_\alpha | \phi^\alpha \rangle \tag{4.3}$$

$$\sum_{\alpha=1}^{n} \langle n_\alpha | \overline{x}_{i_\alpha} \psi^{i_\alpha} | n^\alpha \rangle = \sum_{\alpha=1}^{n} \langle x_\alpha | \hat{E}^\alpha_\alpha | \psi^\alpha \rangle \quad \sum_{\alpha=1}^{n} \langle n_\alpha | \overline{y}_{i_\alpha} \phi^{i_\alpha} | n^\alpha \rangle = \sum_{\alpha=1}^{n} \langle x_\alpha | \hat{K}^\alpha_\alpha | \phi^\alpha \rangle \tag{4.4}$$

It means that wave function shrinks or expands it's coordinate vectors in agreement with it's energy. It means that action of energy operator or differential operator gives only raw probability functions:

$$\sum_{\alpha=1}^{n} \overline{x}_{i_\alpha} \psi^{i_\alpha} = \sum_{\alpha=1}^{n} \langle x_\alpha | \hat{D}^\alpha_\alpha | \psi^\alpha \rangle \quad \sum_{\alpha=1}^{n} \overline{y}_{i_\alpha} \phi^{i_\alpha} = \sum_{\alpha=1}^{n} \langle x_\alpha | \hat{D}^\alpha_\alpha | \phi^\alpha \rangle \tag{4.5}$$

$$\sum_{\alpha=1}^{n} \overline{x}_{i_\alpha} \psi^{i_\alpha} = \sum_{\alpha=1}^{n} \langle x_\alpha | \hat{E}^\alpha_\alpha | \psi^\alpha \rangle \quad \sum_{\alpha=1}^{n} \overline{y}_{i_\alpha} \phi^{i_\alpha} = \sum_{\alpha=1}^{n} \langle x_\alpha | \hat{K}^\alpha_\alpha | \phi^\alpha \rangle \tag{4.6}$$

This statement can be expressed other way using outer product of basis vectors being equal to energy operator:

$$\sum_{\alpha=1}^{n} | e^\alpha \rangle \langle e_\alpha | = \hat{E}^\alpha_\alpha \tag{4.7}$$

So I can re-write field equation solutions:

$$\sum_{\alpha=1}^{n} \langle x_\alpha | \hat{D}^\alpha_\alpha | \psi^\alpha \rangle = \sum_{\alpha=1}^{n} \langle x_\alpha | e^\alpha \rangle \langle e_\alpha | \psi^\alpha \rangle \quad \sum_{\alpha=1}^{n} \langle y_\alpha | \hat{D}^\alpha_\alpha | \psi^\alpha \rangle = \sum_{\alpha=1}^{n} \langle y_\alpha | \epsilon^\alpha \rangle \langle \epsilon_\alpha | \phi^\alpha \rangle \tag{4.8}$$

5. Probability and many system field equation

Probability of following all possible paths in spacetime is equal to one. I can rewrite probability equation from before:

$$\sum_{a\in\mathbb{C}}\sum_{b\in\mathbb{C}}\int_a^b \psi^*\left(\mathbf{x}\left(\lambda\right)\right)\psi\left(\mathbf{x}\left(\lambda\right)\right)\sqrt{\sum_{\alpha=1}^{n}\left\langle\frac{dx_\alpha\left(\mathbf{x}\left(\lambda\right)\right)}{d\lambda}\middle|\frac{dx^\alpha\left(\mathbf{x}\left(\lambda\right)\right)}{d\lambda}\right\rangle}d\lambda=1 \quad (5.1)$$

Probability of given paths collection will be define same way but with using of some set of complex numbers that represent starting and ending point of that path, where I denote that set as $\mathbb{A}$:

$$\sum_{a\in\mathbb{A}}\sum_{b\in\mathbb{A}}\int_a^b \psi^*\left(\mathbf{x}\left(\lambda\right)\right)\psi\left(\mathbf{x}\left(\lambda\right)\right)\sqrt{\sum_{\alpha=1}^{n}\left\langle\frac{dx_\alpha\left(\mathbf{x}\left(\lambda\right)\right)}{d\lambda}\middle|\frac{dx^\alpha\left(\mathbf{x}\left(\lambda\right)\right)}{d\lambda}\right\rangle}d\lambda=\rho(\mathbf{x}) \quad (5.2)$$

When I do measurement it means that wave function had to follow a given path in spacetime. But it does not change quantum state. It has to work this way in order to keep gravity field in place and not vanish with measurement. I will start by many systems wave function, that is just a tensor product of many wave functions:

$$\psi\left(\mathbf{x}_1,...,\mathbf{x}_k\right)=\sum_{\alpha=1}^{n}\left\langle x_{\alpha_1}\right|...\left\langle x_{\alpha_k}\middle|\psi_1^\alpha\right\rangle...\left|\psi_k^\alpha\right\rangle=\sum_{\alpha=1}^{n}\left\langle x_{\alpha_1},...,x_{\alpha_k}\middle|\psi_1^\alpha,...,\psi_k^\alpha\right\rangle \quad (5.3)$$

This leads to rather straight forward expression of field equation:

$$\sum_{\alpha=1}^{n}\left\langle x_{\alpha_1},...,x_{\alpha_k}\right|\hat{D}_{\alpha_k}^{\alpha_k}\left|\psi_1^\alpha,...,\psi_k^\alpha\right\rangle=\sum_{\alpha=1}^{n}\left\langle x_{\alpha_1},...,x_{\alpha_k}\middle|e_1^\alpha,...,e_k^\alpha\right\rangle\left\langle e_{\alpha_1},...,e_{\alpha_k}\middle|\psi_1^\alpha,...,\psi_k^\alpha\right\rangle \quad (5.4)$$

Where differential operator is now defined as tensor product:

$$\hat{D}_{\alpha_k}^{\alpha_k}=\left(-\frac{Gl_P^2}{c^4}\right)^k\sum_{\alpha=1}^{n}\left|\partial_1^\alpha,...,\partial_k^\alpha\right\rangle\left\langle\partial_{\alpha_1},..,\partial_{\alpha_n}\right|=\kappa^k\sum_{\alpha=1}^{n}\left|\partial_1^\alpha,...,\partial_k^\alpha\right\rangle\left\langle\partial_{\alpha_1},..,\partial_{\alpha_n}\right| \quad (5.5)$$

From it follows definition of energy operator:

$$\hat{E}_{\alpha_k}^{\alpha_k}=\sum_{\alpha=1}^{n}\left|e_1^\alpha,...,e_k^\alpha\right\rangle\left\langle e_{\alpha_1},...,e_{\alpha_k}\right| \quad (5.6)$$

That re-writes field equation:

$$\sum_{\alpha=1}^{n}\left\langle x_{\alpha_1},...,x_{\alpha_k}\right|\hat{D}_{\alpha_k}^{\alpha_k}\left|\psi_1^\alpha,...,\psi_k^\alpha\right\rangle=\sum_{\alpha=1}^{n}\left\langle x_{\alpha_1},...,x_{\alpha_k}\right|\hat{E}_{\alpha_k}^{\alpha_k}\left|\psi_1^\alpha,...,\psi_k^\alpha\right\rangle \quad (5.7)$$

References

[1] https://mathworld.wolfram.com/Ket.html

[2] https://mathworld.wolfram.com/Bra.html

[3] https://mathworld.wolfram.com/Geodesic.html

[4] https://mathworld.wolfram.com/LineIntegral.html

[5] https://mathworld.wolfram.com/HermitianOperator.html

Email address: kobierzyckitomasz@gmail.com