SIMPLE SYMMETRY PROVES ALL THREE RIEMANN'S HYPOTHESES

DMITRI MARTILA

ABSTRACT. Suppose the Riemann Zeta function is multiplied by two arbitrary functions, and the resulting functions' values are equated at symmetrical points concerning the critical line $\Re s = 1/2$. In that case, the resulting system of four equations has to give the positions of the Zeta function's zeros. However, since the functions are arbitrary, the positions of the zero places are arbitrary, making a zero coincide with non-zero. Hence, the Riemann Hypothesis that the only zeroes are those on the critical line is true. This simple text is proof of the Riemann hypothesis, Generalized Riemann hypothesis and Extended Riemann hypothesis with according functions.

MSC Class: 11M26, 11M06.

There is a vivid interest in the Riemann Hypothesis, while there are no reasons to cast doubt on the validity of the Riemann Hypothesis [1]. This hypothesis was proposed by Bernhard Riemann (1859). Many colleagues consider it the most important unsolved problem in pure mathematics [2].

Functional equation of the Zeta function is $\xi(s) = \xi(1-s)$, where $s = x + i\,y$ is arbitrary number, whereas $\xi = u(s)\,\zeta(s)$ for certain $u(s)$ is Landau's xi function. The complex conjugate of $\xi(s) = 0$ is $\xi^*(s) = \xi(s^*) = 0$. This functional equation proves that if $s_1 = 1/2 - v + i\,y$ is a counter-example with $\zeta(s_1) = 0$, then there are must be a second counter-example $s_2 = 1/2 + v + i\,y$ with $\zeta(s_2) = 0$. This symmetry of potential counter-examples was shown already in Prof. Riemann's original paper. The Riemann, generalized Riemann, and Extended Riemann hypotheses use non-conceptually different expressions in which functional equations contain $f(s)$, $f(1-s)$ symmetry. Hence, symmetry explains all three of Riemann's hypotheses.

The first trillions of zeros of Riemann's Zeta function $\zeta(s) = 0$ have a real part equal to half: $\Re s = 1/2$. Prof. Riemann has shown that a hypothetical counter-example has a symmetric partner: $s_1 = 1/2 - v + i\,y$, $s_2 = 1/2 + v + i\,y$, $0 < v < 1/2$, $\zeta(s_1) = \zeta(s_2) = 0$. The s_1 and s_2 are

eestidima@gmail.com, Independent Researcher
J. V. Jannseni 6–7, Pärnu 80032, Estonia.

unknown positions now. Let's find a system of equations that produces zeroes of the Zeta function. Obviously, this is $A(s_1)\,\zeta(s_1) = A(s_2)\,\zeta(s_2)$, $B(s_1)\,\zeta(s_1) = B(s_2)\zeta(s_2)$, where $A(s)$, $B(s)$ are arbitrary functions. Any solution of this system is zero of the Zeta function. However, by the choice $A(s) = a(s)/\zeta(s)$, $B(s) = b(s)/\zeta(s)$ at $\zeta(s) \neq 0$ the system becomes satisfied without pointing to the Zeta function zero. I come to a contradiction.

Another way

Dr. Robin has shown that if sum of divisors $\sigma(n) < A(n)$, where $A(n)$ is in Ref. [3], for all $n > 5040$, the Riemann hypothesis is true.

Dr. Lagarias has shown that if sum of divisors $\sigma(n) < B(n)$, where $B(n)$ is in Ref. [4], for all $n > 1$, the Riemann hypothesis is true.

Holds $B(n) > A(n)$.

To every integer n, the $\sigma(n)$ is assigned. But the integers are being distributed on the two-dimensional space of possibilities $(n, \sigma(n))$. If we are shown that a particular place $(n, \sigma(n))$ satisfies the Riemann hypothesis and, simultaneously, dissatisfies, then this is a paradox. The solution to the paradox is that no points must dissatisfy the Riemann hypothesis. The assumption that there are such points leads us to a contradiction.

References

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[4] Jeffrey C. Lagarias. An elementary problem equivalent to the Riemann hypothesis. The American Mathematical Monthly 109(6): 534–543 (2002) https://doi.org/10.2307/2695443