## A Complex Dual Gaussian Fuzzy Number

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## Abstract

In this paper, a novel complex dual Gaussian fuzzy number (CDGFN) is proposed to more accurately model two-dimensional uncertainty, which serves as the medium to represent generalized quantum basic belief assignment (GQBBA).

*Keywords:* Generalized quantum evidence theory; Pattern classification; Generalized quantum basic belief assignment; Discrete Fourier transform; Gaussian fuzzy number; Complex fuzzy number

## 1. The proposed method

**Definition.** (CDGFN membership function)

Let  $|\mathcal{W}_p\rangle$  be a subclass in quantum frame of discernment  $|\Omega\rangle$ . Suppose there is an event x to be classified, which owns feature values  $\alpha_q$  and  $\theta_q$  in the frequency q for feature of magnitude  $\alpha$  and phase  $\theta$ . Then the degree of x for class  $|\mathcal{W}_p\rangle$  in frequency q is defined by a CDGFN membership function:

$$f_{|\mathcal{W}_p\rangle q}(x) = f^{\alpha}_{|\mathcal{W}_p\rangle q}(x) e^{if^{\theta}_{|\mathcal{W}_p\rangle q}(x)}$$

$$= e^{-\frac{(\alpha_q - \mu^{\alpha}_{|\mathcal{W}_p\rangle q})^2}{2\sigma^{\alpha}_{|\mathcal{W}_p\rangle q}^2}} e^{ie^{-\frac{(\theta_q - \mu^{\theta}_{|\mathcal{W}_p\rangle q})^2}{2\sigma^{\theta}_{|\mathcal{W}_p\rangle q}^2}},$$
(1)

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where  $\mu^{\alpha}_{|\mathcal{W}_p\rangle q}$  and  $\mu^{\theta}_{|\mathcal{W}_p\rangle q}$  denote the average value of magnitude and phase in frequency q for class  $|\mathcal{W}_p\rangle$  in the training set, while  $\sigma^{\alpha}_{|\mathcal{W}_p\rangle q}$  and  $\sigma^{\theta}_{|\mathcal{W}_p\rangle q}$  denote the standard deviation value of magnitude and pahse in frequency q for class  $|\mathcal{W}_p\rangle$ .

The modulus of  $f_{|\mathcal{W}_p\rangle q}(x)$  represents the degree of x in class  $|\mathcal{W}_p\rangle$ . Due to  $f^{\alpha}_{|\mathcal{W}_p\rangle q}(x)$ is in [0,1],  $|f_{|\mathcal{W}_p\rangle q}(x)|$  ( $|\cdot|$  denotes the modulus function) is constrained in the interval of [0,1], which obeys the rule of fuzzy number. Moreover, the greater the value of  $|f_{|\mathcal{W}_p\rangle q}(x)|$ , the higher the grade of membership of x in  $|\mathcal{W}_p\rangle$ .