RANDOM FIELD THEORY FOR TESTING DIFFERENCES BETWEEN FREQUENCY RESPONSE FUNCTIONS IN POSTUROGRAPHY

Vittorio Lippi

1Institute of Digitalization in medicine, Faculty of Medicine and Medical Center - University of Freiburg, Freiburg im Breisgau, Germany

2Clinic of Neurology and Neurophysiology, Medical Centre-University of Freiburg, Faculty of Medicine, University of Freiburg, Breisacher Straße 64, 79106, Freiburg im Breisgau, Germany

Email: vittorio.lippi@uniklinik-freiburg.de

Introduction
The frequency response function (FRF) is an established way to describe the outcome of experiments in posture control literature. Specifically, the FRF is an empirical transfer function between an input stimulus and the induced body movement. By definition, the FRF is a complex function of frequency. When statistical analysis is performed to assess differences between groups of FRFs (e.g., obtained under different conditions or from a group of patients and a control group), the FRF’s structure should be considered. Usually, the statistics are performed defined a scalar variable to be studied, such as the norm of the difference between

Figure 1 Example of sets of FRFs and a one-dimensional statistical test. FRFs are plotted for body sway response to 1° peak-to-peak PRTS support surface tilt. In green, the eyes open (EO) condition responses are on the left. In red are the responses for eyes closed (EC) on the right. For each condition, the same 32 subjects are tested. Data are from [2]. On the right is the result of a paired Hotelling T² test performed on the continuum with [6]. The threshold for the interval of confidence in rejecting the null hypothesis is p<0.05, which is shown with the red dotted line. The statistics are expressed in the same space of the data (frequency). A side note on FRF plotting: note the average FRF plotted in black for the two sets. The average is performed in the complex domain; hence, plotting error bars with a measure of dispersion (e.g., STD) on magnitude and phase separately can be misleading (and ill-defined). The nature of the phase, i.e., an angle, may lead to jumps for values around 180°. This could be mitigated aesthetically by adding or removing 360° from the values to make the curves look “more continuous.” However, it is generally an arbitrary choice, and a good alignment for all the FRFs phases may not be possible. In the sample plot, they have been aligned to be as close as possible to the mean.
FRFs, or considering the components independently (that can be applied to real and complex components separately[1],[2]), in some cases both approaches are integrated, e.g., the comparison frequency-by-frequency is used as a post hoc test when the null hypothesis is rejected on the scalar value[3]. The two components of the complex values can be tested with multivariate methods such as Hotelling’s $T^2$ as done in [4] on the averages of the FRF over all the frequencies, where a further post hoc test is performed applying bootstrap on magnitude and phase separately. The problem with the definition of a scalar variable as the norm of the differences or the difference of the averages in the previous examples is that it introduces an arbitrary metric that, although reasonable, has no substantial connection with the experiment unless the scalar value is assumed a priori as the object of the study as in [5] where a human-likeness score for humanoid robots is defined on the basis of FRFs difference. On the other hand, testing frequencies (and components) separately does not consider that the FRF’s values are not independent, and applying corrections for multiple comparisons, e.g., Bonferroni can result in a too conservative approach destroying the power of the experiment. In order to properly consider the nature of the FRF, a method based on random field theory inspired by [6] is presented in the next section. A case study with data from posture control experiments [2] is presented. To take into account the two components (imaginary and real) as two independent variables, the fact that the same subject repeated the test in the two conditions, a 1-D implementation of the Hotelling $T^2$ is used as presented in[7] but applied in the frequency domain instead of the time domain.

**Case study with an example of results**

Consider the body sway induced by a movement of the support surface, where the support surface was tilted with a pseudorandom profile (PRTS, see [8] for the original formulation and [2] for a shorter version of the signal). It should be noted that the FRF can be obtained with other stimulus modalities (e.g., translation[1],[3],[9]) or input profiles (e.g., step or raised cosine). In the example shown in Fig. 1, the responses are compared (left and center). The Hotelling $T^2$ is shown (on the right), confirming the expected result that vision makes a significant difference in the response [1],[9],[10]. With the 1-D statistics, the difference is localized in frequency without requiring to define specific ranges of frequencies[9] or testing the single sample points independently[1],[3].

**Conclusions and future work**

Some advantages of applying 1-D statistics to FRFs for testing and visualizing the results have been presented. Future work will focus on making a set of standardized Matlab functions to analyze the FRFs available. In particular, nonparametric methods like the bootstrap presented in [3] will be implemented.

**References**