Causal effect vector and multiple correlation

Ait-taleb Nabil *

Abstract

In this article, we will describe the mechanism that links the notion of causality to correlations. This article answers yes to the following question: Can we deduce a causal relationship from correlations?

*Corresponding author: nabiltravail1982@gmail.com

1 Introduction

In this paper, we will understand from a proof how to relate the notion of causality to the correlation. For this, we will have to introduce the causal effect vector $X - E[X|\Omega]$ which corresponds to the signal obtained when Ω acts on X.

2 Correlation and causality

The relationship which links the causality to the correlations can be written as follows:

$$\boxed{\sqrt{1 - K_{X,\Omega}.K_{\Omega^2}^{-1}.K_{\Omega,X}} = \sqrt{\frac{Var(X - E[X|\Omega])}{Var(X)}}}$$

where $0 \leq \sqrt{1 - K_{X,\Omega}.K_{\Omega^2}^{-1}.K_{\Omega,X}} \leq 1$ and $E(.|.)$ is the conditional average and $Var(.)$ is the variance.

Where $X - E[X|\Omega]$ is the causal effect vector corresponding to the signal obtained when the causes Ω act on X.

$K_{X,\Omega}.K_{\Omega^2}^{-1}.K_{\Omega,X}$ corresponds to the square multiple correlation.

Proof:

In what follows, we will factorize the variance Σ_{X^2} of the conditional variance $\Sigma_{X^2|\Omega}$ to show the correlations K:

$\Sigma_{X^2|\Omega} = \Sigma_{X^2} - \Sigma_{X,\Omega}.\Sigma_{\Omega^2}^{-1}.\Sigma_{X,\Omega}$

$\Sigma_{X^2|\Omega} = \Sigma_{X^2} - \Sigma_{X,\Omega}.(diag^{-1}(\Sigma_{\Omega^2}))^{\frac{1}{2}}.K_{\Omega^2}^{-1}.(diag^{-1}(\Sigma_{\Omega^2}))^{\frac{1}{2}}.\Sigma_{\Omega,X}$

$\Sigma_{X^2|\Omega} = \Sigma_{X^2} - \Sigma_{X^2}^{\frac{1}{2}}.K_{X,\Omega}.K_{\Omega^2}^{-1}.\Sigma_{X^2}^{\frac{1}{2}}.K_{\Omega,X}$

$\Sigma_{X^2|\Omega} = \Sigma_{X^2}.(1 - K_{X,\Omega}.K_{\Omega^2}^{-1}.K_{\Omega,X})$

The relationship can also be written:

$$\frac{\Sigma_{X^2|\Omega}}{\Sigma_{X^2}} = 1 - K_{X,\Omega}.K_{\Omega^2}^{-1}.K_{\Omega,X} = \frac{||X - E(X|\Omega)||^2}{||X - E(X)||^2} = \frac{\frac{||X-E(X|\Omega)||^2}{N}}{\frac{||X-E(X)||^2}{N}}$$

As we have:$E_\Omega(E(X|\Omega)) = \frac{1}{N}\sum_{\Omega} E(X|\Omega) = E(X)$, we obtain:

$$1 - K_{X,\Omega}.K_{\Omega^2}^{-1}.K_{\Omega,X} = \frac{Var(X - E[X|\Omega])}{Var(X)}$$

By taking the square root we obtain the relationship.

Note that the entropy of X gives $h(X)$ and that the entropy of the impacted signal $X - E[X|\Omega]$ gives the following conditional entropy:

$$h(X - E[X|\Omega]) = \frac{1}{2}\ln(2.\pi.e.Var(X - E[X|\Omega])) = \frac{1}{2}\ln(2.\pi.e.\Sigma_{X^2|\Omega}) = h(X|\Omega)$$

.

The signal X therefore becomes the signal $X - E[X|\Omega]$ when the causes Ω have acted on the variable X.

3 Conclusion

In this paper, we have shown mathematically the steps to follow to obtain a relationship relating the notion of causality and correlation.

[1]Optimal stastical decisions. Author: Morris H.DeGroot. Copyright 1970-2004 John Wiley and sons.

[2]Matrix Analysis. Author: Roger A.Horn and Charles R.Johnson. Copyright 2012, Cambridge university press.