Problems in the Theoretical Structure of Quantum Electrodynamics

Eliahu Comay†
Charactell Ltd.
PO Box 39019
Tel-Aviv, 61390, Israel

Abstract

A careful review of the literature reveals that electrodynamics is not free of theoretical problems. For instance, Peskin and Schroeder say in their book on Quantum Field Theory (QFT): "In fact, we will not discuss canonical quantization of the electromagnetic field at all in this book. It is an awkward subject, essentially because of gauge invariance". Additionally, although many texts treat the components of the electromagnetic potential as a 4-vector A_μ, Weinberg argues in his QFT textbook: "...there is no ordinary four-vector field for massless particles of helicity ± 1". The renormalization procedure is another problematic topic and Feynman called it in his QED book "a dippy process". These alarming quotations encourage rigorous examination of the mathematical framework of electrodynamics, that this work undertakes. It proves several quite unknown electromagnetic properties, and one of which explains why Weinberg's previous statement is right.

Keywords: Errors in QED; Radiation fields; Bound fields; The four potential; Gauge transformations.

†Email: elicomay@post.tau.ac.il

1 Introduction

A careful observation of the literature indicates quite unknown problematic aspects of the theoretical structure of quantum electrodynamics (QED). Contradictory assertions of textbooks illustrate this unfortunate situation. For example, Feynman stated: quantum electrodynamics is "the jewel of physics–our proudest possession" (see [1], p. 28). Similarly, Griffith says that "the quantum theory of electrodynamics was perfected by Tomonaga, Feynman, and Schwinger in the 19405" (see [2], p. 2). The plain meaning of these statements is that QED is regarded as a flawless theory.

In contrast, Peskin and Schroeder state in their textbook on Quantum Field Theory (QFT): "In fact, we will not discuss canonical quantization of the electromagnetic field at all in this book. It is an awkward subject, essentially because of gauge invariance" (see [3], p. 79). Obviously, Peskin and Schroeder point out that QED is not a perfect theory and that its *gauge invariance* element is a problematic QED issue. However, gauge invariance is regarded as a crucial part of the presently accepted physical theories. For example, the review article of Jackson and Ocun [4] is dedicated to this topic, and they say that "The principle of gauge invariance plays a key role in the standard model". Furthermore, while many texts treat the 4 components of the electromagnetic potential as a 4-vector A_μ (see e.g., [5], p. 48; [4]), Weinberg clearly negates this issue and states: "...there is no ordinary four-vector field for massless particles of helicity ± 1" (see [6], p. 251).

Another problem is the infinite energy that is associated with the field of a pointlike charge and the renormalization process that aims to settle it. Dirac said that renormalization has an "illogical character" [7] whereas Feynman derisively called it "a dippy process" (see [1], 127). An analogous opinion is put forward in Ryder's QFT textbook (see [8], p. 390). In this book, Ryder compares quantum divergences with classical ones and says: "In the quantum theory, these divergences do not disappear; on the contrary, they appear to get worse, and despite the comparative success of renormalisation theory the feeling remains that there ought to be a more satisfactory way of doing things."

These excerpts clearly indicate that the current theoretical structure of electrodynamics deserves a rigorous examination. This is a generally useful conclusion because rigorous examination can only improve the status of scientific theories. The primary purpose of this work is to examine well-established experimental data together with theoretical principles and show erroneous elements in the present mathematical structure of electrodynamics. Although many erroneous issues are pointed out below, this work does not claim that it shows *all* electromagnetic errors. The analysis uses fundamental physical theoretical elements, such as special relativity (SR), the least action principle, etc.

The discussion uses units where $\hbar = c = 1$. In these units, there is one type of dimension, and $[L^n]$ denotes the power n of the length unit that designates the dimension of the relevant variable. The metric is diagonal and its entries are (1,-1,-1,-1). The standard notation is used, and r sometimes denotes the four space-time coordinates $r = (t, x, y, z)$. The paper is organized in sections that are dedicated to specific electrodynamics issues. The second section discusses problematic points of electromagnetic fields, and the third section discusses problematic points of electromagnetic potentials. The fourth section analyzes gauge transformations. The fifth section discusses the relations between charges and photons. The sixth section shows how to construct a coherent relativistic form for the potential components of radiation fields. The seventh section shows how the Darwin Lagrangian (see [5], pp. 179-182, [9], pp. 593-595) and the corresponding Breit interaction treat bound fields in a form that is suitable for the least action principle. The eighth section discusses the self-field of a charge. The last section comprises concluding remarks.

2 The Electromagnetic Fields

Let us examine the radiation electromagnetic fields and the bound electromagnetic fields. The hydrogen atom is useful for this end and the Schroedinger and the Dirac equations adequately describe the states of this atom. Consider the ground state $1s$ of this atom and an incoming photon whose energy equals the difference between the $2p$ atomic state and its $1s$ state. The transition between these states is an *allowed transition* (see e.g. [10], p. 254). This process proves that the spin (namely, helicity) and the parity of the photon are 1^-. The photon part of the report of the *particle data group* (PDG) [11] confirms these attributes. Moreover, the Dirac and the Schroedinger theories use the electronic states of the hydrogen atom and define the spin and parity of each energy level of this atom. The definitions of these successful theories in terms of the electronic state completely ignore the atomic bound electromagnetic field. Therefore, either this bound field represents no particle or the spin and parity of this particle are 0^+. The Wigner's work [12] is relevant to this issue because it proves that the state of a quantum system takes a definite spin. Hence, it supports the following outcome:

> Conclusion: Radiation electromagnetic fields and bound electromagnetic fields are different physical objects. Hence, a coherent electromagnetic theory should treat them separately.

The present structure of electrodynamics relies on the variational principle, that uses the least action of the system. The action is the appropriate integral of a given Lagrangian/Lagrangian density. Thus, Landau and Lifshitz utilize this principle in their book on classical electrodynamics [5], and this principle is also the basis of the present QFT structure [3,6]. The electromagnetic field term of the Lagrangian density of these theories is

$$\mathcal{L}_{EM} = \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} \tag{1}$$

(see [5], p. 73; [3], p. 78). Here the electromagnetic fields $F^{\mu\mu}$ is the sum of radiation fields $F_R^{\mu\nu}$ and bound fields $F_B^{\mu\nu}$

$$F^{\mu\nu} = F_R^{\mu\nu} + F_B^{\mu\nu} \tag{2}$$

However, the foregoing discussion proves that these fields represent different physical objects.

Conclusion: the electromagnetic fields term (1) *of the Lagrangian density of the present form of electrodynamics is an erroneous expression.*

3 The Electromagnetic Potentials

Landau and Lifshits show the Lienard-Wiechert 4-potential of a given charge e

$$A_\mu = ev_\mu/(R_\nu v^\nu) \tag{3}$$

(see [5], p. 174). Here R_ν and v^ν are respectively the 4-vector from the retarded position of the radiating charge to a given point at the laboratory, and the retarded velocity of the charge. The expression (3) is a 4-vector because it divides the velocity 4-vector by the scalar product of two 4-vectors.

The electromagnetic fields that are derived from (3) are (see [9], p. 657; [5], p. 175)

$$\boldsymbol{E} = e\frac{1-v^2}{(R-\boldsymbol{R}\cdot\boldsymbol{v})^3}(\boldsymbol{R}-\boldsymbol{v}R) + \frac{e}{(R-\boldsymbol{R}\cdot\boldsymbol{v})^3}\boldsymbol{R}\times[(\boldsymbol{R}-\boldsymbol{v}R)\times\boldsymbol{a}] \tag{4}$$

and

$$\boldsymbol{B} = \boldsymbol{R}\times\boldsymbol{E}/R. \tag{5}$$

Here $\boldsymbol{R}$ is the 3-vector that is related to the retarded 4-vector R^μ, and $\boldsymbol{v}$ and $\boldsymbol{a}$ denote the charge's retarded velocity and acceleration, respectively. The first term of (4) and that of (5) are called velocity fields and the second ones are called acceleration fields. At a large enough distance (called the far zone) the velocity fields decrease like R^{-2} and they are bound fields, whereas the acceleration fields decrease like R^{-1} and they are radiation fields. Therefore, velocity fields can be ignored at the far zone.

These expressions are mathematically correct descriptions of the potentials that yields the sum of radiation electromagnetic fields and bound electromagnetic fields. However, the previous analysis proves that bound fields and radiation fields are different physical entities. Therefore, although (3) is a mathematically correct expression of the potentials, it is physically unacceptable.

This is an inherent problem of the 4-potential: As a physical expression, it requires a coherent mathematical basis. However, the correct mathematical expression is unacceptable.

This is a special aspect of theoretical physics. It shows that a correct mathematical expression can be physically unacceptable. In mathematical parlance, this example shows that a correct mathematical form is just a *necessary condition* for an acceptable physical expression. (However, this condition is not sufficient for this issue.)

Remark: The mathematical correctness of the Lienard-Wiechert 4-potential justifies its utilization in calculations. This work sometimes takes advantage of this matter.

Another problematic feature of the 4-potential (3) is that it depends on the coordinates of a charge that may be inaccessible to the laboratory coordinates where the action is calculated. For example, charges of the Andromeda galaxy that emit radiation are inaccessible to persons working in a laboratory on planet Earth. Therefore, it is unuseful for the calculation of the action, which is a crucial theoretical requirement.

Unfortunately, contrary to the previous result stating that there is no 4-vector potential for radiation fields, many texts use the 4-potential as a 4-vector (see e.g., [4]). As a matter of fact, the inexistence of this 4-vector is already stated above in the Introduction section: "...there is no ordinary four-vector field for massless particles of helicity ± 1" (see [6], p. 251).

Conclusion: A coherent electromagnetic theory should not use the four potentials as components of a 4-vector.

Please note that section 6 shows how one can construct an acceptable 4-potential for radiation fields.

4 The Electromagnetic Gauge Transformations

The electromagnetic gauge transformation takes this form

$$A'_\mu = A_\mu - \chi_{,\mu} \tag{6}$$

(see [5], p. 52; [9], p. 220), where the gauge function χ is an *arbitrary function of the space and time coordinates.* However, it is proven above that the 4-potential is unacceptable as a 4-vector. Hence, the electromagnetic gauge transformation suffers from inherent problems.

The following arguments substantiate this assertion.

Ar.1 A fundamental physical requirement says that all terms of a sum must have the *same dimension.* The dimension of the potentials A_μ is $[L^{-1}]$. Hence, the gauge function χ cannot be an arbitrary function of the space-time coordinates.

Ar.2 SR says that an electromagnetic quantity must have *a definite tensorial structure.* Hence, the gauge function χ cannot be an arbitrary function of the space-time coordinates.

It turns out that the examination of the full form of the QED gauge transformation yields another attribute of the gauge function. Consider the full form of the gauge transformation (see [3], p. 78; [6], p. 345):

$$A_\mu(x) \to A_\mu(x) + \chi_{,\mu}; \quad \psi(x) \to \exp(-ie\chi(x))\psi(x), \tag{7}$$

where the symbol e of the exponent denotes the electronic charge. As stated above, a significant attribute of the gauge function is that it is an *arbitrary* function of the space-time coordinates (see e.g., [6], p. 342; [5], p. 52; [13] p. 70). The change of the phase $\chi(x)$ of the Dirac function $\psi(x)$ of (7) is called a *local phase rotation* (see [3], p. 78). The power series expansion of the exponent of (7)

$$e^{-ie\chi(x)} = 1 - ie\chi(x) + ... \tag{8}$$

proves that χ must be a mathematically real dimensionless Lorentz scalar. Indeed, the pure number 1 of the first term of (8) is a dimensionless Lorentz scalar. The same is true with the pure imaginary number i and with the electric charge e. Hence, the assertion stating that $\chi(x)$ must be a dimensionless Lorentz scalar holds.

This structure of $\chi(x)$ yields far-reaching results. Consider the power series expansion of the gauge function $\chi(x)$

$$\chi(t, x, y, z) = a_0 + ax^{m_1}y^{m_2}z^{m_3}t^{m_4} + ... \tag{9}$$

Here each m_i of the power series (9) is a non-negative integer. Let us examine the case where $a \neq 0$ for a given term where one $m_i > 0$. The dimension of this term is $[L^N]$, where $N \geq m_i > 0$. Therefore, the dimensionless of the gauge function $\chi(x)$ proves that $a = 0$, and the gauge function reduces to the constant a_0.

Conclusion: The gauge function is a trivial uniform numerical constant and it means that its derivative vanishes. Therefore, it adds a null quantity to the potentials. Its quantum mechanical form is the well-known global phase transformation (see e.g. [14], p. 314; [15], p. 121). Therefore, the electromagnetic version of the quantum concept of *local phase transformation* does not hold.

5 Photons and Charges

An electromagnetic radiation wave carries energy. Therefore, due to the energy conservation law (see [5], pp. 88, 89), this wave must have a source of electric charges that has supplied its energy. The location of the radiation source and the field point where the radiation is measured define the direction of the wave propagation. The retarded position of a charge at the radiation source is determined by the following equation

$$t' + R(t')/c = t, \tag{10}$$

where t, t' denote the measurement time and the retarded time, respectively, R is the distance from the retarded position of the charge to the field point where the measurement is carried out and the speed of light c is written explicitly (see [5], p. 174). The retarded position and the retarded velocity of a specific charge at the source define the retarded 4-potential (3) of this charge. However, the overall radiation that is emitted from a given source is determined by the interference of the fields of all charges of the radiating system. It means that the actual radiation is a multi-charge effect. This conclusion is obvious because a charge that belongs to a system that comprises just one charge does not accelerate, and such a system does not emit radiation.

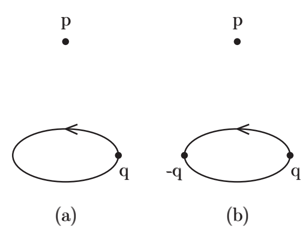

Figure 1: Two radiating systems. (a) A charge q moves uniformly along a circle. (b) Two charges, $\pm q$ move uniformly along a circle. (See text.)

The following example illustrates this issue. Consider the two radiating systems of Fig. 1. Fig. 1(a) comprises a single charge q that moves uniformly along a circle that is embedded in the (x, y) plane and its center coincides with the origin of the coordinates. The point p lies on the z-axis at the radiation zone. The circular motion of the charge q proves that it accelerates towards the center of the circle. The Lienard-Wiechert fields (4), (5) show that due to the acceleration $\boldsymbol{a}$, a nonvanishing amount of radiation streams at the vicinity of point p (see [5], p. 175; [9], p. 657).

Let us examine the radiation fields $\boldsymbol{E}$, $\boldsymbol{B}$ at point p and compare the fields of Fig. 1(a) with those of Fig. 1(b). In Fig. 1(b) there are two charges $\pm q$, that are located at two antipodal points of this circle. These charges move along the circle with the same velocity as that of the charge q of Fig. 1(a). For point p, the retarded time of the charge $+q$ is the same as that of the charge $-q$. The radiation fields parts of formulas (4) and (5) prove that, at point p, the electric field and the magnetic field of the charge $+q$ are the same as those of the charge $-q$. It means that at point p, the radiation fields $\boldsymbol{E}$, $\boldsymbol{B}$ of Fig. 1(b) are twice as strong as those of Fig. 1(a). Now, the Poynting vector (see [5], p. 81 or [9], p. 237)

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{B}/4\pi \tag{11}$$

shows the energy current of electromagnetic fields. This vector proves that *at point p of Fig.* 1(b) *the energy current is four times greater than that of Fig.* 1(a). Furthermore, since the same frequency holds for the fields of the two cases, one concludes that at the vicinity of point p, *the number of photons of Fig.* 1(b) *is four times greater than the number of photons of Fig.* 1(a). This example proves the following important conclusion:

The radiation emitted from a system of charges is a multi-charge effect. Moreover, generally one cannot associate a given photon with a specific charge.

6 Components of the Potential of Radiation Fields

It is shown above that the 4-potential of electromagnetic fields cannot take a coherent form of a 4-vector (see section 3). However, it is explained here that a relativistically coherent expression for the 4 components of a potential of radiation fields can be constructed for every inertial frame. Consider the two invariants of the electromagnetic fields

$$\boldsymbol{B}^2 - \boldsymbol{E}^2; \quad \boldsymbol{E} \cdot \boldsymbol{B} \tag{12}$$

(see [5], p. 68). These invariants vanish for radiation fields that are emitted from a given source. Let us use specific axes and examine radiation fields that move in the z-direction and the radiation is linearly polarized in the x-direction. Here the non-vanishing fields' components at a given (t, z) are

$$E_x = W, \quad B_y = W, \tag{13}$$

where W takes a specific value. These fields satisfy the two invariants (12), and they are derived from the vector potential

$$\boldsymbol{A}(z,t) = (W/k\sin(kz-\omega t),0,0), \tag{14}$$

where $k = \omega$ in the vacuum. Relation (14) means that if the radiation fields are known then the 4 components of the potential can be constructed (the 0-component obviously vanishes). This expression is consistent with SR because it is based on the Maxwellian fields $F^{\mu\nu}$ that transform relativistically. It means that although the 4-potentials of radiation fields are not components of a 4-vector, they undergo a relativistically coherent transformation. This analysis was published in [16].

The 4-potential of (14) enables to use the radiation fields in the QED Lagrangian density, which is the primary theoretical expression

$$\mathcal{L}_{QED} = \bar{\psi}[\gamma^\mu(i\partial_\mu - eA_\mu) - m]\psi - \frac{1}{16\pi}F_{R\mu\nu}F_R^{\mu\nu} \tag{15}$$

(see e.g., [3], p. 78; [13], p. 84). Here the subscript $_R$ denotes radiation fields.

> These expressions show how one can use the radiation fields in the standard QED theoretical structure, namely, in the theory's Lagrangian density.

7 The Action of Bound Fields

Landau and Lifshitz show the Darwin Lagrangian of a system of charges. The electromagnetic part of this Lagrangian is

$$L_{Darwin} = -\sum_j\sum_{i>j}\frac{e_je_i}{R_{ij}} + \sum_j\sum_{i>j}\frac{e_je_i}{2R_{ij}}[\boldsymbol{v}_j\cdot\boldsymbol{v}_i + (\boldsymbol{v}_j\cdot\boldsymbol{n}_{ij})(\boldsymbol{v}_i\cdot\boldsymbol{n}_{ij})] \tag{16}$$

(see [5], pp. 179-182, [9], pp. 593-595). Here $\boldsymbol{v}_i$ denotes the velocity of the ith charge and $\boldsymbol{n}_{ij}$ is the unit vector from the ith charge to the jth charge. This Lagrangian takes the classical structure where the position and velocity of all charged particles are the values of the laboratory time t. The derivation of this Lagrangian shows that it is correct up to the second power v^2 of the particles' velocity. Hence, it pertains to bound fields, because "acceleration fields are typical radiation fields" ([9], p. 657). This form of the Lagrangian is suitable for the quantum domain because here the time-independent Heisenberg picture can be utilized (see [17], p. 352). Thus, the quantum version of the Darwin Lagrangian is called the Breit interaction (see [18], pp. 170, 195). In other words, the bound fields are treated in the classical and quantum domains that belong to the framework of the least action principle.

8 Fields and Charges

Consider Jackson's definition of the electric field. He uses the limit of the force that is exerted on a test particle whose charge tends to zero (see [9], p. 28). Hence, one may argue that the exclusion of the self-interactions of the field of a point charge is a consistent concept. Note that the Darwin Lagrangian (16) describes a 2-body interaction and that it ignores the self-field interaction. This approach also has a favorable virtue because it removes the infinite energy of the self-field interaction of an elementary pointlike charge.

Let us consider the theoretical and experimental properties of elementary pointlike particles. Thus, Landau and Lifshitz use SR and prove that "elementary particles must be treated as points." (see [5], p. 47). The general form of the QFT Lagrangian density yields the same outcome. The form of this Lagrangian density is

$$\mathcal{L}(\psi(x), \psi(x)_{,\mu}), \tag{17}$$

where $x \equiv (t, \boldsymbol{x})$ denotes the four space-time coordinates. QFT textbooks support this approach: "All field theories used in current theories of elementary particles have Lagrangians of this form" (see [6], p. 300).

Let us examine the pointlike properties of the quantum function $\psi(x)$ of the Lagrangian density (17). This function depends on a single set of four space-time coordinates. It means that this function can describe the probability of finding the particle at the space-time point $x = (t, \boldsymbol{x})$, but it *cannot* describe the distribution of the particle around x. Hence, the form of the QFT Lagrangian density (17) applies to elementary pointlike particles. The experimental side provides amazing support for this issue. Indeed, the measured upper bound of the electron's radius is about 7 orders of magnitude smaller than the proton's radius [19]. This is an example of the pointlike attribute of an elementary quantum particle and of the suitability of the above-mentioned form of its Lagrangian density.

Experimental data of the hydrogen atom strongly support the exclusion of the self-interaction of the field of a pointlike particle. Indeed, the quantum calculations that ignore the self-interaction of the electron's field yield very good results (see e.g., [20] section 4.4; [17], pp. 202-211). Another favorable issue of the exclusion of the self-interaction of the field of a pointlike charge is the removal of the 4/3 problem of the relativistic transformation of the energy and momentum of a point charge [21].

9 Concluding Remarks

This work points out erroneous issues of electrodynamics. The contradictory quotations from the literature indicate the need for this publication. In particular, this work proves that the electromagnetic radiation fields $F_R^{\mu\nu}$ and the bound fields $F_B^{\mu\nu}$ are different physical entities. Hence, they should be treated separately. Moreover, contrary to the assertion of many publications, the electromagnetic potential components are not entries of a 4-vector. This conclusion agrees with Weinberg's assertion (see [6], p. 251). However, the analysis proves that one can construct the 4 components of the potential of radiation fields that are consistent with SR. This construction uses the radiation fields $F_R^{\mu\mu}$ that undergo a Lorentz transformation. The discussion also proves that there is no freedom for the gauge function – it is just a uniform constant and its derivative vanishes. The analysis proves that it is impossible to assign a given photon to a specific charge. This work proves that the Darwin Lagrangian and the Breit interaction adequately describe bound fields. Furthermore, the discussion explains why one can remove the self-interaction of the field of a pointlike particle as well as its infinities.

References

[1] R. P. Feynman, *QED, The Strange Theory of Light and Matter* (Princeton University Press, Princeton, 1985).

[2] D. Griffiths, *Introduction to Elementary Particles*, 2nd edition, Weinheim, Wiley-VCH, 2008.

[3] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading Mass, 1995).

[4] J. D. Jackson and L. B. Okun, Rev. Mod. Phys., **73**, 663 (2001).

[5] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Amsterdam, Elsevier, 2005).

[6] S. Weinberg, *The Quantum Theory of Fields*, Vol. I, Cambridge University Press, Cambridge, 1995.

[7] Dirac, P.A.M. (1963) The Evolution of the Physicists Picture of Nature. Scientific American, 208, 45-53.

[8] L. H. Ryder, *Quantum Field Theory.* (Cambridge University Press, Cambridge, 1997).

[9] J. D. Jackson, *Classical Electrodynamics* (John Wiley, New York, 1975).

[10] Schiff L. I. Quantum mechanics. New York, McGraw-Hill; 1955.

[11] `https://pdg.lbl.gov/index.html`

[12] E. Wigner, Ann. Math., **40**, 149 (1939).

[13] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[14] F. Halzen and A. D. Martin, Quarks and Leptons, An Introductory Course in Modern Particle Physics (John Wiley, New York,1984).

[15] A. Messiah, Quantum Mechanics, V2 (North Holland, Amsterdam, 1965).

[16] E. Comay, *Acta Physica Polonica A,* **133**, 1294 (2018). `http://przyrbwn.icm.edu.pl/APP/PDF/133/app133z5p26.pdf`.

[17] E. Merzbacher, Quantum Mechanics. John Wiley, New York, 1970.

[18] H. A. Bethe and E. E. Salpeter *Quantum Mechanics of One- and Two-Electron Atoms* (Springer, Berlin, 1957).

[19] H. Dehmelt, Physica Scripta, **T22**, 102 (1988).

[20] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics*, New York, McGraw-Hill, 1964.

[21] F. Rohrlich, *Am. J. Phys.*, **65**, 1051 (1997).