The Equivalence Principle: Is it rocket science?

David L Berkahn, James M Chappell, Derek Abbott

School of Electrical and Electronic Engineering,
University of Adelaide, SA 5005
Australia

July 12, 2024

Abstract

Beginning with a standard application of the equivalence principle, we examine a simple localised experiment inside a rocket at constant thrust. We show in this case that it is possible to distinguish between this accelerating frame and the same frame sitting stationary on the surface of a source mass causing a gravitational field. We then discuss how this result relates to the equivalence principle. We also explore how the result can converge to a relative equivalence between both frames. Finally, we discuss how this relates to broader questions of relative and absolute motion.

1 Introduction

1.1 Historical review

The equivalence principle (EP) was the foundation of Einstein's theory of general relativity, asserting that the effects of gravity and acceleration are indistinguishable for observers in a local region of space-time. This principle led to a fundamental reinterpretation of the nature of gravity, space-time and the structure of the universe [TMW00]. Einstein first articulated the equivalence principle in 1907 [Ein07], refining the concept further in a series of publications [Ein11, Ein13, Ein16]. The equivalence principle has since undergone continuing theoretical development. Robert Dicke [Dic64] and others refined the concept and connected it with experimental verifications. Ohanian [Oha00] and others explored the philosophical and practical implications of the equivalence principle, providing a detailed comparison between observers in accelerating frames (like rockets) and those in gravitational fields. Experimental tests have continued to support the universal validity of the equivalence principle, such as Gravity Probe B [V$^+$80] and the MICROSCOPE space mission [TMR$^+$22], which have confirmed the EP to an extremely high precision of 10^{-15}.

1.2 The three forms of the equivalence principle

There are three formulations of the equivalence principle, of increasing generality, that are typically delineated [Wil18].

A) The Weak Equivalence Principle (WEP), as annunciated by Einstein, was that: *Local experiments confined to a sealed box cannot distinguish between a uniform gravitational field and an equivalent acceleration.* A modern statement of the WEP is *that the trajectory of a freely falling "test" body (one not acted upon by such forces as electromagnetism and too small to be affected by tidal gravitational forces) is independent of its internal structure and composition* [Wil18]. The Weak Equivalence Principle is also known as the universality of free fall, as verified by Galileo. Another way of stating this, is that the gravitational mass (the strength of the gravitational force it experiences) is equal to the inertial mass (its resistance to acceleration). That is, that the body's weight is proportional to its inertial mass.

B) The Einstein Equivalence Principle (EEP) begins with the WEP but then adds two further components: 1) *The outcome of any local non-gravitational experiment is independent of the velocity*

Table 1: Eva drops an apple inside a rocket with constant thrust T, while measuring her weight W standing on a balance. We can see that in the limit of a massless rocket, $m_R \to 0$, her weight reading on the scale does not change with $W_1 = W_2 = T$, even though she has reduced her inertial mass by m_A.

Holding apple	Apple dropped
$a_1 = \frac{T}{m_R+m_E+m_A}$ $W_1 = ma_1$ $= \frac{(m_E+m_A)T}{m_R+m_E+m_A}$	$a_2 = \frac{T}{m_R+m_E}$ $W_2 = ma_2$ $= \frac{m_E T}{m_R+m_E}$

of the freely-falling reference frame in which it is performed. This is also known as Local Lorentz Invariance LLI. 2) *The outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed* [Wil18]. This is also known as Local Position Invariance (LPI), and is verified with experiments such as those based on the gravitational redshift. The LLI implies that for objects in free fall, locally, the laws of physics are those of SR. The EEP is believed to imply a metric theory of gravity, such as general relativity (GR).

C) Strong Equivalence Principle (SEP) generalizes the EEP to include gravitating objects and not just test particles, and includes the effects of gravitational binding energy. This allows gravitational experiments to be undertaken as well as non-gravitational ones.

1.3 Motivation and segue to the thought experiment

If we consider the equation $F = ma$, the acceleration a is implicitly assumed to be measured with respect to an inertial observer. Additionally, Newton knew from Galileo that all objects fall at the same rate in gravity regardless of their mass. However, we need to reflect on what frame we are referring to when we say this. In fact, we typically mean the non-inertial frame that is stationary with respect to the surface of the Earth. However, this is not an inertial frame that we previously used to relate force and acceleration. Hence, if we want to equate the accelerations due to gravity and accelerations due to forces, we need to use equivalent frames of reference. That is, either use inertial for both or the non-inertial. Newton did not see this as a problem, partly because, as well as believing in absolute motion, he also conceived of the falling object in gravity as being due to the action of a force. Einstein was aware of this problem, but was also bothered about another related issue. That of the apparent absoluteness of acceleration while velocity was deemed relative. This, and the above incongruence mentioned regarding the mismatch of frames, was resolved by Einstein when he had the happiest thought of his life: *If a person falls freely, he will not feel his own weight.* From this he went on to formulate his famous equivalence principle (see Section 1.2 above). By working inside the frame of an accelerating rocket, in one stroke was able to equate the two accelerations above, and hence relativize acceleration. For example, in this frame it was natural to assume that all objects of differing mass all fall at the same rates in gravity, since they lie inside an accelerating rocket frame. Hence they were able to be considered as inertial or free fall objects in gravity. In the next section we wish to use these frames to explore more exactly their equivalence.

2 Analysis

2.1 A simple thought experiment inside a rocket

An observer Eva, inside a rocket, notes that she is experiencing a constant downward force. She knows that this experience is caused by either a constant acceleration generated by a constant rocket thrust out in free space or by standing on the surface of a planet. The only testing equipment she has available is an apple and a very precise set of weight scales. She tries to use these to see if she can distinguish these two situations. She stands on the scale with the apple in her hand and reads the scale. She then drops the apple and notes that her weight does not change (She ignores any short term transients effects, such as those due to the time it takes for the scale to react to her actions). After a quick calculation shown in Table 1, she concludes that she is in a rocket and not in gravity. How does she

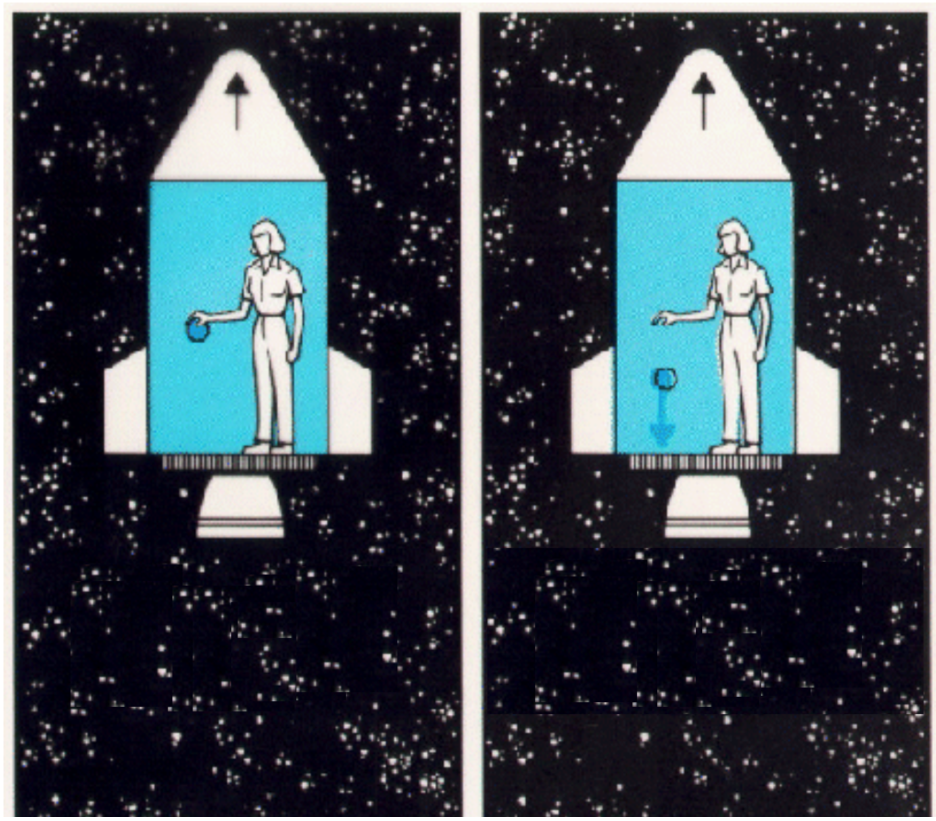

Figure 1: Eva standing on an electronic balance in a sealed room, drops an apple. She inspects the reading on the balance and notes that it has not changed. From this she immediately concludes that she is actually in a rocket at constant thrust and not in a gravitational field.

know this? Eva realizes that in a rocket with constant thrust, that even though she drops an object and is apparently losing mass, the reading on the balance will not change, because the acceleration of the rocket increases to retain a constant reaction force. This actually being required by Newton's third law, that is, the reaction force of her weight must equal the thrust force. She also notes that $F = ma$ implies that if m decreases then acceleration must increase because F is the same. This implies that her acceleration has slightly increased.

In gravity however, she understands that her weight will get lighter in proportion to the mass of the apple. That is, she has an initial weight of $W_1 = (m_E + m_A) \times 9.81N$, which will reduce to $W_2 = m_E \times 9.81N$, after dropping the apple.

2.2 Further considerations of the rocket observer Eva

How is Eva to further understand her results? Considering the Newtonian framework, she begins by comparing how the weight and acceleration vary in the rocket frame in comparison to gravity, Her result of a constant weight W, even though she reduced her mass, must be due to a varying, given by $a = W/m$. This then implies qualitatively that $W \neq f(m)$, or the weight is not actually related to the inertial mass. This means that when an object is dropped the acceleration of the remaining mass increases with respect to the object. Hence the observer inside the rocket records this as faster free fall of the object. However she reasons that in gravity, when the apple is dropped the weight W on the scale decreases. Now, since $W = f(m)$ in gravity, then applied to the apple we have $g = W/m$ or no change in acceleration of the the apple. So a longer time is recorded for the apple to fall in gravity compared to in the rocket.

Invariants are very important quantities in physics, and we note that within the rocket frame, we have an invariant, that of the weight of an object, as it does not change even if the mass decreases. That is, if we reduce the mass, the acceleration will increase, keeping the weight constant. We have

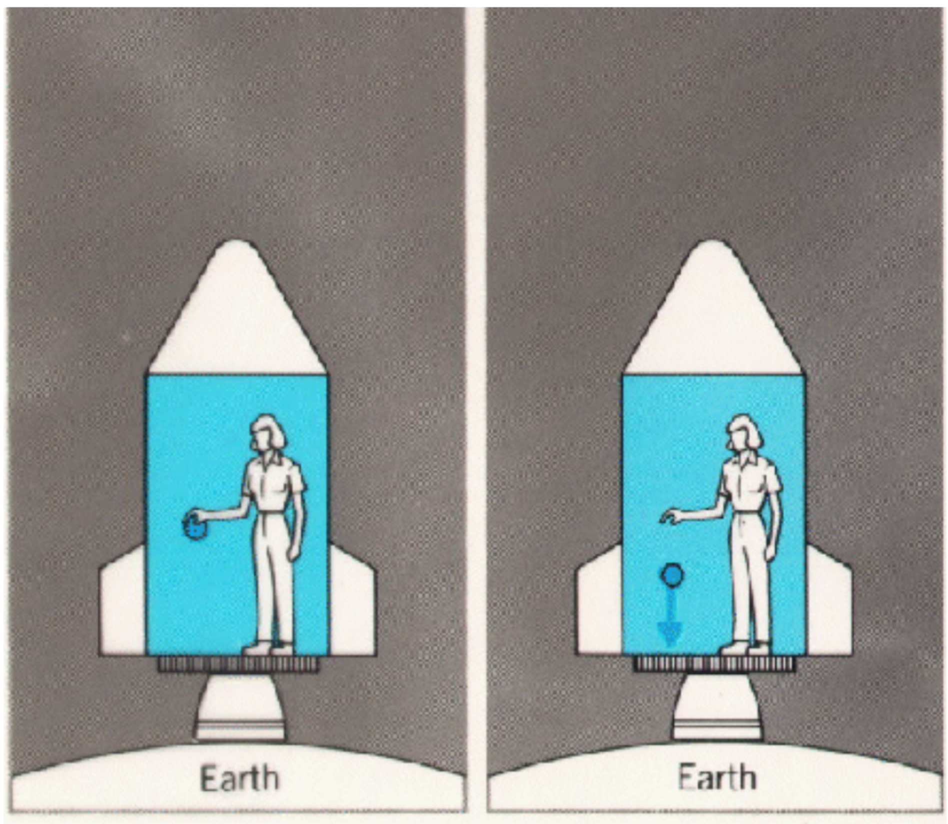

Figure 2: Eva in a rocket at rest on the surface of the Earth, drops an apple. She inspects the balance and notes that the reading reduces by an amount equal to the weight of the apple, as expected for gravity.

the weight $W = (m_E + m_A)a_0 = m_E a$. Therefore, the new acceleration a observed in the rocket frame, for a person of mass m_E dropping a mass m_A, will be

$$\frac{a}{a_0} = 1 + \frac{m_A}{m_E}. \tag{1}$$

We can see that for the rocket observer the result converges to the gravitational case in the limit of a vanishingly small dropped mass $m_A \to 0$. Furthermore we see that since the act of dropping the apple changes the acceleration, therefore the weight of the apple does not correspond to the expected acceleration. This is analogous to the idea of the uncertainty principle, where the act of measurement affects the outcome. This is implied also by Eq. (1), where the larger the mass of the apple, the larger the effect on the system. Therefore by this simple local experiment inside the box we can distinguish the rocket observer frame from gravity. Therefore, Eva can also confirm the conclusion of being in a rocket by dropping different sized masses and finding that larger masses fall faster, which is inconsistent with the WEP under gravity, as shown in Eq. (1). Ironically, this is consistent with Aristotle's rather discredited view that heavy objects fall faster than light ones! There is also a known effect, that of a larger mass creating a reciprocal force back on the larger mass in gravity, so that even in Newtonian gravity, larger masses do actually fall slightly faster.

We also note that for the rocket observer, the acceleration depends on the order you undertake the experiment. That is, for two different masses m_1 and m_1, then if we drop m_1 first, then the acceleration will be less than if we drop it after m_2. This adds an element of non-commutativity into experiments.

2.3 Implications for the Equivalence principle and Absolute motion

We are now in a position to ask what does all this imply for the equivalence principle? For the particular aspect of the Weak equivalence where it is asserted that no *local* experiment performed inside a sealed room can distinguish between gravity and acceleration, this particular experiment indicates otherwise both kinematically and dynamically. However, the modern version of the WEP, presented above does survive, only if both masses are dropped at the same time as we already noted, but not if dropped sequentially.

We now come to a difficult issue. Does this experiment show that some aspects of the equality of gravitational and inertial mass need clarifying? Put simply in this rocket frame there is no relationship between weight and inertial mass. Therefore in this frame we are unable to make any connection with the idea of inertial and gravitational mass equivalence. We can therefore make more sense of the issue if we step outside the rocket and apply Newton's laws. This is easily understood when we apply the reasoning already applied in Section 2.2.

While our result appears not to contradict Local LLI it would appear to violate LPI, due to the varying acceleration in this frame.

Finally, we want to make a comment on the possible implications this result has on detecting absolute motion. We have shown a method to distinguish acceleration from gravity within the rocket with a local experiment. Similarly one might argue that another example is the bending of light in both frames. Even though twice the bending value occurs in gravity compared to acceleration, and perhaps could also be used to distinguish gravity and acceleration, the equivalence principle in this specific relativistic comparison, was instrumental in informing how gravity behaved.

3 Conclusion

We have explored the principle of equivalence as it is conventionally defined, as shown in Section 1.2, with a thought experiment of an observer in a rocket, refer Fig. 1 and Fig. 2. This allows us to develop a locally performed experiment which provides a distinction with respect to the weak equivalence principle. In particular we show that objects do not fall at the same rate within this specific rocket frame, as shown in Fig. 1. Thence, Eq. (1) shows that we have a smooth transition from the accelerating frame to the rocket observer frame, and so provides a generalized viewpoint. That is, for the special case where the mass of the dropped object goes to zero, we reproduce the accelerating frame.

A useful application of this approach, is for simple student problems, such as a block down a ramp, clarifying the nature of reaction forces, analogous to Eva dropping the apple in the rocket. For a observer measuring their weight on the block as they go down the ramp, they will not detect weight changes as they transition off the end of the ramp, like they would if in gravity.

While the WEP has been verified to a very high precision, confirming the effective equivalence of gravity and acceleration, nevertheless, the rocket frame described above with its many interesting properties, could have novel application in other areas of physics, such as large scale cosmic questions, or situations in the quantum regime. We also note that if we follow the Einsteinian view that there should be no preferred frame, then the rocket observer frame described in this paper should also be considered as a valid viewpoint. This investigation of the rocket observer thus casts new light on the equivalence principle, as it relates the absoluteness of acceleration and gravity.

References

[Dic64] Robert Dicke. Principle of equivalence and the weak equivalence principle. 1964.

[Ein07] Albert Einstein. Über das relativitätsprinzip und die aus demselben gezogenen folgerungen. 1907.

[Ein11] Albert Einstein. On the influence of gravitation on the propagation of light. 1911.

[Ein13] Albert Einstein. Über den einfluß der schwerkraft auf die ausbreitung des lichtes. 1913.

[Ein16] Albert Einstein. Die grundlage der allgemeinen relativitätstheorie. 1916.

[Oha00] Hans C. Ohanian. What is the principle of equivalence? 2000.

[TMR+22] P. Touboul, G. Métris, M. Rodrigues, J. Bergé, A. Robert, Q. Baghi, Y. André, J. Bedouet, D. Boulanger, S. Bremer, et al. MICROSCOPE Mission: Final results of the test of the equivalence principle. *Phys. Rev. Lett.*, 129(12):121102, 2022.

[TMW00] K. S. Thorne, C. W. Misner, and J. A. Wheeler. *Gravitation*. Freeman, San Francisco, CA, 2000.

[V+80] R.F.C. Vessot et al. Test of relativistic gravitation with a space-borne hydrogen maser. 1980.

[Wil18] Clifford M Will. *Theory and experiment in gravitational physics*. Cambridge university press, 2018.