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# EXTENDED LAW OF UNIVERSAL GRAVITATION

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***Abstract:*** *Besides the forces described by Newtonian dynamics, there is an additional cosmological force in the Universe, which is observed in experiments, but does not follow from Newton's law of gravitation. The additional cosmological force was not represented in Newtonian dynamics. The law of cosmological force $F_{cos}$ = (mc^2)√Λ is derived. The law of cosmological force makes it possible to obtain the extended law of universal gravitation. The extended law of universal gravitation includes two laws of gravitation: Newton's law of gravitation and the law of cosmological force. The extended law of universal gravitation does not have the limitations of Newton's law of gravitation. It describes the full force of gravitational interaction both on small scales and on the scale of the Universe. The coupling constants in the extended law of universal gravitation are two constants: the Newtonian constant of gravitation G and the cosmological constant Λ. The equation of the extended law of universal gravitation without using the gravitational constant G is given. The extended law of universal gravitation gives an explanation of the Galaxy rotation curve and the Pioneer anomaly. Extended Newtonian dynamics is able to provide solutions to the problems of astrophysics and cosmology without expecting (and instead of) quantum gravity.*

***Keywords****: The law of universal gravitation; The law of cosmological force; Parameters of the observable universe; Galaxy rotation curves; Pioneer anomaly.*

## 1. Introduction

The dominant force in the universe is gravity. Newton's law of gravity was a real breakthrough in science. Newton's law of gravitation (1) impresses with its simplicity and mathematical perfection:

$$F_N = G\frac{Mm}{r^2} \qquad (1)$$

Gravitational interaction has become the fourth fundamental interaction. Newton's law of gravitation makes it possible to explain and predict with great accuracy the motions of celestial bodies. The attractive thing about Newton's law of gravitation is the simple dependence of force on the parameters of interacting bodies.

For a long time it was thought that Newtonian dynamics was a complete description of all types of motion occurring in the Universe. Experimental observations of galaxy rotation show that this is not the case. Newtonian dynamics has been found to give predictions that do not agree with experimental observations [1, 2]. This has given reason to believe that Newtonian dynamics is only an approximation that is valid under certain conditions.

## 2. Galaxy rotation curve.

It is found that the velocities of stars around the centers of galaxies have a very large deviation from Newton's predictions. The rotation curve of galaxies (Fig. 1) demonstrates the presence of an additional force, which is observed in experiments but does not follow from Newton's law of gravitation.

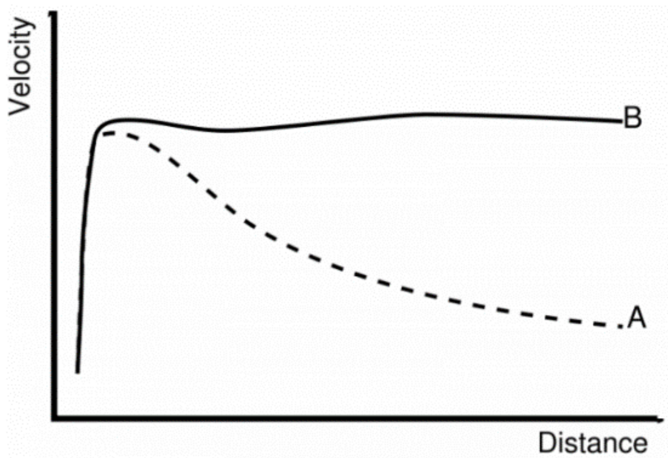


Fig. 1. Rotation curve of a typical spiral galaxy: predicted (A) and observed (B) [1, 3].

Newton's law of gravitation does not give an explanation of the dependence of the form (B). Newton's formula (1) "does not see" a significant fraction of the gravitational force on the scale of the Universe. Such a large discrepancy of theoretical and experimental value indicates that for the missing gravitational force there is an unknown law of gravitation, different from Newton's law.

Different hypotheses have been proposed to explain the rotation curve of galaxies. Some hypotheses retained Newton's law of gravity, while others questioned the correctness of Newton's law [1, 4, 5]. A hypothesis was proposed that preserves the law of gravitation as Newton proposed it, but introduces a new entity - "dark matter". All attempts to find "dark matter" have failed. Dark matter does not find experimental confirmation.

The impossibility to obtain an analytical solution of the three-body problem, the impossibility to extend Newton's law of gravitation to the Universe, the deviation of the velocity of stars from the predictions of Newton's law gave rise to distrust of the classical theory and even gave reason to question the correctness of Newton's law of gravitation. Numerous attempts were made to modify Newtonian dynamics [1,2 6,7,8].

No refinements and corrections of Newton's law of gravitation did not make it applicable in cosmology. The simple and mathematically perfect formula of Newton's law was not applicable in cosmology. The law of inverse squares and point masses are the main limiting factors in extending Newton's law to the Universe. Newton's law has limits of applicability. Beyond these limits another law of gravitation free from idealized point masses and the law of inverse squares is valid.

Newton's formula $F_N$ = GMm/r^2 gives the force of gravitational interaction between two bodies. Accordingly, Newton's law formula gives only part of the force of universal gravitation. In addition to the forces described by Newtonian dynamics, there is a gravitational force in the universe that is observed in experiments but does not follow from Newton's law of gravitation. Newtonian dynamics "does not see" the additional cosmological force of gravitational interaction of bodies with the mass of the

Universe. This means that the additional cosmological force must be represented by a new law of gravitation, different from Newton's law. Supplementing Newtonian dynamics with a new law of gravitation will make the classical theory of gravitation complete, consistent and complete.

## 3. The law of gravitation of two bodies without using the gravitational constant G.

Newton's formula (1) is not the only possible formula for the law of gravitational interaction between two bodies. To apply formula (1), it is necessary to know the masses of the interacting bodies. For large distances this is a difficult task. It is much more accurate to know parameters such as distances and periods. In addition to the known Newton's formula, the second formula of the law of gravitational interaction between two bodies is proposed:

$$F_K = \frac{R^3 m}{T^2 r^2} \qquad (2)$$

where: m is the mass of the body, R and T are orbit parameters, r is the distance.

The formula (2) does not include the gravitational constant G and the large mass M. The formula includes Kepler's relation. Formula (2), like formula (1), includes the law of inverse squares. Formula (2) is equivalent to Newton's formula (1).

Since the law of gravitation (2) gives the force of gravitational interaction between two bodies, formula (2), like formula (1), gives only part of the force of universal gravitation. Formula (2) also "does not see" the additional cosmological force of gravitational interaction between bodies and the mass of the Universe. This means that the additional cosmological force must be represented by a new law of gravitation.

## 4. Cosmological constant Λ and cosmological acceleration $A_0$.

The solution of the systems of algebraic equations of the Universe [9] gives two important parameters: the cosmological constant Λ ($\Lambda = 1.36285... \bullet 10^{-52}$ $m^{-2}$) and the cosmological acceleration $A_0$ ($A_0 = 10.4922... \bullet 10^{-10}$ $ms^{-2}$). These parameters of the Universe are unknown in Newtonian dynamics. For this reason, Newton's classical gavitation has limitations and does not apply to cosmology. The values of constants Λ and A0 are obtained with an accuracy close to that of the Newtonian constant of gravitation G. The value of the cosmological acceleration $A_0 = 10.4922... \bullet 10^{-10}$ $ms^{-2}$ is close to the critical acceleration proposed in MOND [1] and very close to the Pioneer anomaly.

Using Newton's second law, we obtain the value of the cosmological force Fcos, which acts on all bodies in the universe and causes acceleration $A_0$.

$$F_{Cos} = mA_0 \qquad (3)$$

The cosmological acceleration constant $A_0$ is related to the cosmological constant Λ by the following relation:

$$\mathbf{A_0 = c^2\sqrt{\Lambda}} \qquad (4)$$

Where: c - speed of light in vacuum.

## 5. A new law of gravitation beyond the applicability of Newton's law.

In addition to Newton's law of gravitation, a new law of gravitation has been proposed [10]. It is the law of cosmological force (Fig. 2).

$$F_{Cos} = mc^2\sqrt{\Lambda}$$

Fig. 2. Formula of the law of cosmological force.

The law of cosmological force is presented using the cosmological constant Λ: $F_{Cos}$ = (mc^2)√Λ. The new law of gravitation shows that any body of mass m is subject to a cosmological force proportional to the mass of the body and to the cosmological constant Λ. This law operates beyond the applicability of Newton's law of gravitation and applies to the gravitational interaction of the universe. The cosmological force has a linear dependence on the mass of the body and does not obey the law of inverse squares. On small scales, the additional cosmological force is much smaller than the Newtonian force. For example, on the Earth, $F_{Cos}$ is 10^9 times smaller than the Newtonian force. On the scale of the universe, the cosmological force exceeds the Newtonian force and has a theoretical limit equal to the Planck force $F_P$ = c^4/G = 1.21027•10^44 N. As we can see, Newton's law of gravitation $F_N$ = GMm/r^2 does not include the very significant cosmological force $F_{Cos}$ = (mc^2)√Λ.

## 6. The theoretical limit of the cosmological force is equal to the Planck force $c^4$/G.

The study of the equivalent equations of the new law of the cosmological force shows that the value of the cosmological force in the limit is equal to the Planck force:

$$\lim_{m \to M_U} F_{Cos} = \lim_{m \to M_U} mc^2\sqrt{\Lambda} = 1.21027 \bullet 10^{44}\,N = \frac{c^4}{G} \qquad (5)$$

The theoretical limit of the cosmological force at m → $M_U$ reaches the enormous value $c^4/G$ = 1.21027 x $10^{44}$ N.

## 7. The law of cosmological force explains Galaxy rotation curve

Newton's law does not reveal the presence of cosmological force of gravitational interaction of bodies with the mass of the Universe. The coupling constant in Newton's law of gravitation is the constant G. The cosmological force of gravitational interaction is an additional force to the Newtonian force of gravitation of two bodies. The additional cosmological force is represented by a different law of gravitation from Newton's law. The law of cosmological force (Fig. 2) is represented using the cosmological constant Λ. The cosmological force has a linear dependence on the mass of the body (Fig. 3) and does not obey the law of inverse squares. Fig. 3 conventionally shows the contribution of the cosmological force to the Galaxy rotation curve.

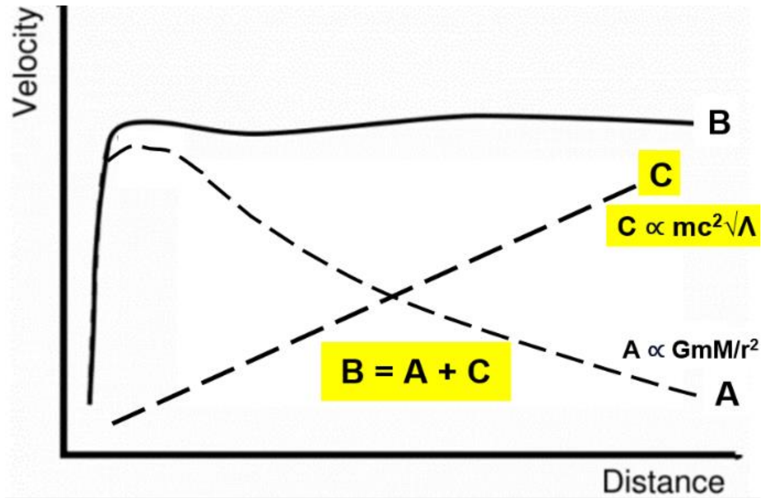

Fig. 3. Galaxy rotation curve (B) as a result of two forces: the contribution of the Newtonian force (A) and the contribution of the cosmological force (C).

The total force of universal gravitation, which acts on a body, consists of two forces. This is the Newtonian force of gravitational interaction of two bodies and the additional cosmological force of gravitational interaction of a body with the mass of the Universe. Accordingly, the Galaxy rotation curve (B) (Fig. 3) is represented by the sum B = A + C. Combining the two laws of forces $F_N$ and $F_{Cos}$ gives the law of universal gravitation. At small distances, the main share of the force of universal gravitation is represented by the Newtonian force $F_N$. At large distances, the cosmological force $F_{Cos}$ represents the major fraction of the universal gravitational force. As a result of the two forces, the velocity in the graph (Fig. 3) is represented by curve (B).

The assumption about the influence of the rest of the visible Universe on the rotation curves of galaxies was first made by Philip D. Mannheim [11].

## 8. Extended law of universal gravitation

The law of cosmological force allows to present the law of universal gravitation in a new form. The law of universal gravitation should include not only the Newtonian force of interaction between two bodies, but also an additional cosmological force.

The most simple and mathematically perfect is the following form of the Law of universal gravitation:

$$\begin{cases} F_N = G\dfrac{Mm}{r^2} \\ F_{Cos} = mc^2\sqrt{\Lambda} \end{cases}$$

Fig. 4. Extended Law of universal gravitation.

The law of universal gravitation contains two components. The first formula in the law of universal gravitation is Newton's well-known law of gravitation, and the second formula is the new law of cosmological force. Together the two laws of gravitation give the law of universal gravitation. The coupling constants in the law of universal gravitation are two constants: the gravitational constant G and the cosmological constant Λ.

The contribution of the Newtonian force and the cosmological force to the total gravitational force depends on the distance and mass of the interacting bodies. For small masses and small distances, Newton's law of gravitation makes a significant contribution. The fraction of the cosmological force for small masses is much smaller than the Newtonian force. The reason is the small value of the cosmological constant Λ ($\Lambda = 1.36285... \times 10^{-52}$ $m^{-2}$). As the mass of the interacting bodies increases and the distance increases, the Newton force's fraction of the law of universal gravitation decreases and the cosmological force's fraction increases. The total gravitational force acting on a body is a vector sum of two forces: the Newton force and the cosmological gravitational force of the Universe.

$$\vec{F}_U = \vec{F}_N + \vec{F}_{Cos} \qquad (6)$$

Thus, the law of universal gravitation is presented in a new form (Fig. 5):

**LAW OF UNIVERSAL GRAVITATION** $= \boxed{F_N = G\dfrac{Mm}{r^2}} + \boxed{F_{Cos} = mc^2\sqrt{\Lambda}}$

Fig. 5. Law of universal gravitation = Newton's law of gravity + law of cosmological force.

The application of the law of universal gravitation (Fig. 4 and Fig. 5) is preferred if the large mass M is known with acceptable accuracy.

## 9. Extended Law of universal gravitation without using the gravitational constant G.

The equivalent formula of the law of gravitational interaction between two bodies (2) makes it possible to present the extended law of universal gravitation without using the gravitational constant G and large mass M (Fig. 6).

$$\begin{cases} F_K = \dfrac{mR^3}{T^2 r^2} \\ F_{Cos} = mc^2\sqrt{\Lambda} \end{cases}$$

Fig. 6. Extended Law of universal gravitation without using the gravitational constant G.

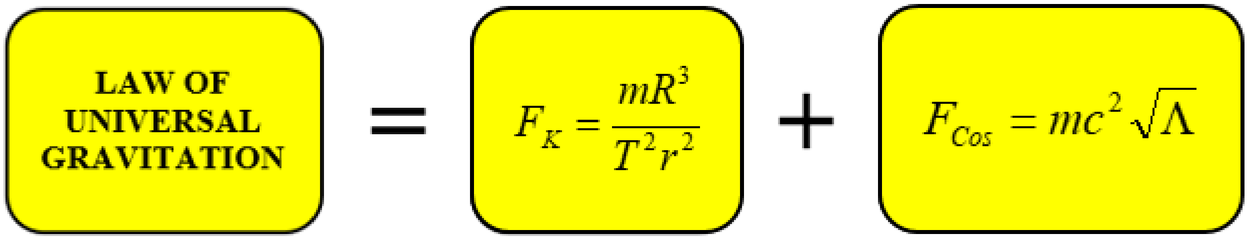


Fig. 7. Law of universal gravitation = Law of gravitation of two bodies without using the gravitational constant G + law of cosmological force.

The application of the law of universal gravitation, shown in Fig. 6 and Fig. 7, is preferable if distances and periods are known with greater accuracy. In the observable universe, distances and periods of rotation of bodies are known with greater accuracy than masses. Therefore, the law of universal gravitation without using the gravitational constant G is more preferable.

## 10. Fraction of the cosmological force $F_{Cos}$ = (mc^2)√Λ in the extended law of universal gravitation

The fraction of the cosmological force $F_{Cos}$ in the extended formula of the law of universal gravitation depends on the distance and masses of interacting bodies. On small scales, the additional cosmological force $F_{Cos}$ is much smaller than the Newtonian force.For example, on the Earth, $F_{Cos}$ is 10^9 times smaller than the Newtonian force. At small distances, the Newtonian force $F_N$ makes up the bulk of the force of universal gravitation. With increasing mass of interacting bodies and distance, the share of Newton's force in the law of universal gravitation decreases, while the share of the cosmological force increases. On the scale of galaxies, the cosmological force Fcos becomes commensurate with the Newtonian force.

At the value of $M/r^2$ = 15.720....kg/m$^2$

$$\mathbf{M/r^2 = (c^2\sqrt{\Lambda})/G = 15.720...\ kg/m^2} \qquad (7)$$

the cosmological force ($F_{Cos}$ = (mc^2)√Λ) is equal to the Newtonian force ($F_N$ = GMm/r^2). The same constant (15.7202... kg/m2), but with much greater accuracy, is given by the following combination of electron constants:

$$\mathbf{m_e/\alpha r_e^2 = 15{,}7202729...\ kg/m^2} \qquad (8)$$

The constant (15.7202...kg/m2) leads to a mass density value of $18.3520\cdot10^{-26}$ kg/m$^3$ for the observed Universe.

On the scale of the Universe, the cosmological force is enormous. At large distances it exceeds the Newtonian force. The $F_{Cos}$ cosmological force has a theoretical limit equal to the Planck force $F_P$ = c^4/G = 1,21027 • 10^44 N. At large distances, the cosmological force $F_{Cos}$ makes up the main share of the universal gravitational force.

## 11. Conclusion

For more than 300 years Newtonian dynamics remained an incomplete mathematical model. It lacked the law of cosmological force to complete it. The force of gravitational interaction between two bodies is only part of the law of universal gravitation. The rest of the total force is the force of gravitational interaction of bodies with the mass of the universe. Newton's gravitational interaction formula does not "see" the additional component of the law of universal gravitation. The force not accounted for by Newton's law is a significant part of the gravitational force. At large distances and for large masses, it exceeds the Newtonian gravitational force. Therefore, to call Newton's law of gravitation the law of universal gravitation is an exaggeration.

To overcome the limitations of Newton's law of gravitation, Newtonian dynamics is supplemented with a new law of gravitation, the law of cosmological force. The two laws of gravitation - Newton's law of gravitation and the law of cosmological force form the extended law of universal gravitation. Two constants act as coupling constants in the new law of universal gravitation: Newtonian constant of gravitation G and cosmological constant Λ ($\Lambda = 1.36285... \bullet 10^{-52}$ $m^{-2}$). The extended law of universal gravitation gives the full force of gravitational interaction both on small scales and on the scale of the Universe.

Good old classical gravitation has not yet said its most important word in science. In its extended version it is able to give the solution of problems of astrophysics and cosmology without expecting (and instead of) quantum gravity. "*From quantum physics - forward to new classical...*": this is the future paradoxical vector of development of physics and cosmology.

## 12. Conclusions

1. Besides the forces described by Newtonian dynamics in the Universe there is a gravitational force, which is observed in experiments, but does not follow from Newton's law of gravitation. The formula of Newton's law of gravitation gives the force of interaction of two bodies, but "does not see" the additional cosmological force of gravitational interaction of bodies with the mass of the Universe. Newton's law of gravitation formula is only a part of the law of universal gravitation.

2. The extended law of universal gravitation is obtained, which has no limitations inherent in Newton's law of gravitation.

3. The extended law of universal gravitation includes two laws of gravitation - Newton's law of gravitation $F_N$ = GMm/r^2 and the cosmological force law $F_{Cos}$ = (mc^2)√Λ.

4. The extended law of universal gravitation provides a complete description of gravitational interaction on both small scales and the scale of the universe.

5. An equivalent formula of the law of universal gravitation is obtained using the cosmological constant Λ without using the gravitational constant G. The equivalent formula of the law of universal gravitation is preferred for large distances and large masses.

6. The extended law of universal gravitation provides an explanation of the Galaxy rotation curve and the Pioneer anomaly without involving the "dark matter" hypothesis.

7. Unaccounted for by Newton's law of gravitation, the additional force of gravitational interaction with the mass of the universe turned out to be exactly equal to the force from the supposed "dark matter" halo.

8. Good old classical gravitation has not yet said its most important word in science. In its extended version it is able to give the solution of problems of astrophysics and cosmology without expecting (and instead of) quantum gravity.

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