The soliton model of elementary particles

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abstract:

In this paper we show that a unification of gravity and inertia, as it comes out of a correct implementation of Mach‘s principle, leads to elementary particles being solitons in the gravitational field (or more general: a unified field including the gravitational one). We show how the properties of elementary particles then give rise to the phenomenology of special relativity as well as quantum mechanics in the usual classical framework and in flat, 3 dimensional euclidean space. The soliton nature of the elementary particles naturally gives rise to elementary quantum phenomena, like their wave-particle duality, the uncertainty principle, De Broglie relations $E=\hbar\omega$ and $P=\hbar k$. A formula for h can in principle be obtained. This opens up a possibility to explain the origin of quantum mechanics. At the same time, also the special relativistic phenomena like length contraction, time dilation, the relativistic energy-momentum relation and the apparent constancy of the speed of light can be explained. The speed of light is just an apparent constant when measured with co-moving rulers and clocks, provided by the elementary particles themselves. It obeys the usual vector addition, just like all other velocities and vectors do, too. Ultimately, mass itself can be explained as entirely of gravitational origin, as the field-energy of the soliton, which will also yield an explanation for the energy-mass equivalence. No additional scalar field like the Higgs field is needed.

1. Introduction:

Theories implementing Mach‘s principle and giving a unified description of gravity and inertia [1-5] turn out to have some far reaching consequences. This especially concerns the nature of mass and therefore also the nature of the elementary particles themselves. It turns out, that such theories suggest, that mass is entirely of (gravitational) field origin and that elementary particles are solitons in the gravitational field (or more general: a unified field). This we want to prove and discuss in this paper. We will show, that all basic phenomena of special relativity, as well as quantum mechanics, can be derived from such a theory in a classical framework and in flat, 3 dimensional space. No additional assumptions have to be postulated. However, we work with model equations here and just show how in principle the phenomena emerge from the theory. The exact field equations of the unified field remain to be found.
Before we start our discussion, we will give a brief introduction in what a soliton is. A reader already familiar with this concept is invited to skip to the next section. A soliton is a localised, self-reinforcing, stable propagating wave packet which preserves its shape under collision. It owes its stability to an exact balance between dispersion and non-linearity. Solitons are therefore an intrinsically non-linear phenomenon; only non-linear partial differential differential equations have soliton solutions. They exhibit both wave and particle properties and are therefore a natural candidate for elementary particles, which immediately resolves the problem of the wave-particle duality. An example for an equation admitting soliton solutions is the Korteweg-De Vries equation:

$$\frac{\partial u}{\partial t}+6u\frac{\partial u}{\partial x}+\frac{\partial^3 u}{\partial x^3}=0 \qquad (1.1)$$

It describes surface waves in shallow water. The 2nd term is the non-linearity, the 3rd one is the dispersion. This equation admits soliton solutions of the form

$$u(x,t)=\frac{v}{2}\cdot sech^2\left(\frac{\sqrt{v}}{2}\cdot(x-vt-x_O)\right) \quad (1.2)$$

Here, v is the velocity of the soliton, x_0 its initial "position". This wavepacket propagates form-preserving, even when it collides with other solitons. It is localised and stable: The envelope decays exponentially away from the "position" of the wave $x=vt+x_0$. One can already see though, that the wave is not exactly localised in this „position", but distributed around it. We will later see, that this naturally leads to quantum phenomena when used to describe elementary particles.

2. The Consequences of the unification of gravity and inertia:

As a consequence of Mach's principle and the resulting unification of gravity and inertia, inertial mass has its origin in the gravitational mass and the gravitational field [1-5]. In concrete terms, in [5], we found that the inertial mass of a particle k given by:

$$(m_k)_{inert}=\frac{2\varphi_k}{c^2}\cdot(m_k)_{grav} \quad (2.1)$$

Here, φ is the gravitational potential:

$$\varphi_k=\sum_{j\neq k}\frac{m_j}{r_{kj}} \quad (2.2)$$

Thus, inertial mass is no longer an independent property of particles, but a consequence of their gravitational mass and the gravitational field. This, in turn implies that the concept of mass only makes sense in relation to the gravitational field. Unlike in standard theories, where (inertial) mass has a meaning even in the absence of any (gravitational) field. Therefore, a unification of gravity and inertia puts mass and gravitational field hierarchically on the same level: There is no gravitational field without mass, but also the concept of mass is meaningless without a gravitational field, since the only thing it does is quantifying the interaction with this very field.
This leads to the obvious conclusion that mass is a special manifestation of the gravitational field itself. Or, to put it in other words: The elementary particles are a special, localised, form-preserved propagating energy „clump" in the gravitational field, which even retains its shape under collisions. Those are exactly the properties which a soliton has. One can therefore conclude that elementary particles are solitons in the gravitational field. The mass of a particle is then just given via the energy-mass equivalence[1]):

$$m=\frac{E}{c^2} \quad (2.3)$$

where E is the field energy concentrated within the „clump". Such a soliton approach unifies the concepts of fields and particles: There is no longer particles moving in fields, or fields generated by particles, but the particles are part of the field itself, namely the soliton. The particle is the core- or nearfield of the soliton, while what is usually referred to as „field" is actually the far field of the soliton.

3. Particles as solitons in the gravitational field:

Let us look now sketch how this would work and demonstrate how all the phenomenology of special relativity and Quantum mechanics comes out. For simplicity reasons, we just work with the

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1) We will in section 7 actually prove this equation by showing that indeed the term E/c^2 plays the role of the inertial mass in Newton's law in the framework of soltion theory, without introducing any notion of mass a priori.

simplest relativistic field equation for the gravitational field in a 1 dimensional toy model. The correct, full 3 dimensional (non-linear) equations remain to be found.
Consider therefore the relativistic equation of the gravitational field:

$$\Box\varphi = 4\pi G\rho \tag{3.1}$$

Assume now mass as a part of the gravitational field itself. We can then write above equation in the form:

$$\Box\varphi = -V'(\varphi) \tag{3.2}$$

where V is some function of the gravitational field φ. This equation admits travelling wave solutions of the form:

$$\varphi(x,t) = \varphi(\gamma(x - vt)) \tag{3.3}$$

where γ is the Lorentz-factor and φ is an envelope satisfying:

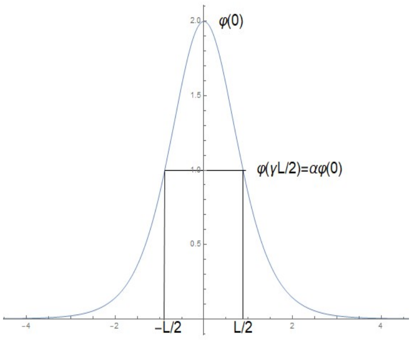
$$\varphi'' = -V'(\varphi) \tag{3.4}$$

We now want to show how this idea allows to explain all special relativistic as well as most basic quantum phenomena as properties of the elementary particles (the solitons) in a pure classical framework and in Euclidean space.

4. The special relativistic phenomena:

4.1 Length contraction:

We start with length contraction. The elementary particles provide elementary rulers. Their length is characterised by a certain α (e.g. 1/e) decay of the envelope φ of the soliton, as is shown in the picture below.



We have:

$$\varphi(\gamma L/2) = \alpha\varphi(0) \;\;\Leftrightarrow\;\; L = \gamma^{-1} 2\varphi^{-1}(\alpha\varphi(0)) \tag{4.1}$$

For a resting soliton we then have:

$$L = 2\varphi^{-1}(\alpha\varphi(0)) =: L_0 \tag{4.2}$$

and for a moving one:

$$L=\gamma^{-1}L_0=\sqrt{1-\beta^2}\,L_0 \qquad (4.3)$$

Since all macroscopic rulers are composed of elementary rulers, they inherit this behaviour.

4.2 Relativistic kinetic energy and momentum:

The field energy derived from equation (3.2) is given by the expression:

$$E=\int_{-\infty}^{\infty}\frac{1}{2}\cdot\left(\frac{1}{c^2}\left(\frac{\partial\varphi}{\partial t}\right)^2+\left(\frac{\partial\varphi}{\partial x}\right)^2\right)+V(\varphi)\,dx \qquad (4.4)$$

For a soliton at rest, one calculates from this, using the field equation (3.2):

$$E=2\int_{-\infty}^{\infty}V(\varphi(x))\,dx=:E_0 \qquad (4.5)$$

For a moving soliton, one obtains:

$$E=2\gamma\int_{-\infty}^{\infty}V(\varphi(z))\,dz=\gamma E_0 \qquad (4.6)$$

Using the mass definition of the soliton $m_0=E_0/c^2$, one obtains the formula for the relativistic kinetic energy.
The flux-density is given by the formula:

$$p=-\int_{-\infty}^{\infty}\frac{\partial\varphi}{\partial t}\frac{\partial\varphi}{\partial x}\,dx \qquad (4.7)$$

and evaluates to:

$$p=\frac{E_0}{c^2}\gamma v=m_0\gamma v \qquad (4.8)$$

which is the well known formula for the relativistic momentum. Equation (4.6) and (4.8) together yield the relativistic energy-momentum relation:

$$E=\sqrt{(m_0c^2)^2+c^2p^2} \qquad (4.9)$$

4.3 Time-dilatation:

To describe time dilation, oscillatory soliton solutions are needed, the so called „breathers“. The internal oscillations of the elementary particles then provide elementary clocks. To demonstrate how this works, we follow Günther [6] and consider a toy model, the Sine-Gordon equation:

$$\Box\varphi=-\frac{1}{d^2}\sin(\varphi) \qquad (4.10)$$

Here, d is some length parameter. This equation has a breather solution given by[2]):

$$\varphi(x,t)=4\arctan\left(\frac{\sin\left(\gamma\frac{r}{d}\left(ct-\frac{v}{c}x\right)\right)}{\cosh\left(\frac{\sqrt{1-r^2}}{d}\gamma\,\omega\,(x-vt)\right)}\right) \qquad (4.11)$$

2) This solution still fulfills the relations (4.6) and (4.8) for the energy and momentum, just with a slightly different rest mass. Also, the envelope still shows the same contraction as was discussed in 4.1. Therefore all previously derived results apply in the same way to the breather solution, too.

r is some dimensionless parameter of order of unity. For a breather at rest, the envelope of this solution oscillates with a frequency of

$$\omega=\frac{r\,c}{d}=:\omega_0 \quad (4.12)$$

An elementary clock at rest therefore ticks with a frequency ω_0. For a moving breather, the envelope at the „position“ $x=vt$ oscillates with a frequency

$$\omega=\omega_0\sqrt{1-\beta^2} \quad (4.13)$$

Again, since all macroscopic clocks are composed of elementary clocks, they inherit this behaviour. This also shows, that particles have to be a specific type of soliton, namely a breather. We will later see, that this demand is further strengthened by the approach to Quantum phenomenae.

4.4 Apparant constancy of the speed of light:

The speed of light is only apparently constant, when measured with comoving rulers and clocks, but otherwise obeys the normal velocity addition. This was shown in a similar way also by Günther [6] in the context of the Sine-Gordon model.
Consider therefore a test section of rest-length x_0. The time needed for a light signal to travel back and forth with the test section at rest is:

$$\Delta t_0=\frac{2\Delta x_0}{c_0} \quad (4.14)$$

The measured speed of light then is:

$$c=\frac{2\Delta x_0}{\Delta t_0}=c_0 \quad (4.15)$$

Consider now the test section moving with a velocity v. In this case, the time the light signal needs to travel back and forth is:

$$\Delta t_{\rightarrow}=\frac{\Delta x}{c_0-v} \qquad \Delta t_{\leftarrow}=\frac{\Delta x}{c_0+v}$$

$$\Delta t_0=\Delta t_{\rightarrow}+\Delta t_{\leftarrow}=\frac{2\Delta x}{c_0}\frac{1}{1-\beta^2} \quad (4.16)$$

In addition, the test section is also contracted, according to (4.3) by $\Delta x=\sqrt{1-\beta^2}\Delta x_0$ and the time measured with a comoving clock is reduced by $\Delta t=\sqrt{1-\beta^2}\Delta t_0$, according to (4.13). This yields for the measured speed of light

$$c=\frac{2\Delta x_0}{\Delta t}=\frac{2\Delta x_0}{\sqrt{1-\beta^2}\Delta t_0}=\frac{2\Delta x_0}{(1-\beta^2)\frac{2\Delta x_0}{c_0}\frac{1}{1-\beta^2}}=c_0 \quad (4.17)$$

This concludes our section about the special relativistic effects. As we have seen, all the phenomenology of special relativity comes out as a consequence of the soliton properties. We emphasize again, that no Minkowski space is needed. No change in time or space occurs in a moving frame of reference. It is just the elementary particles which give rise to all the relativistic phenomena in 3 normal dimensional euclidean space.

5. Quantum phenomena:

5.1 Uncertainty relation:

The soliton model naturally gives rise to quantum phenomena like the wave particle duality and the uncertainty principle. Since solitons are localised waves, which exhibit particle properties, particles naturally posses wave and particle properties when described as solitons. Further, uncertainty arises naturally due to the bandwidth theorem for signals. Suppose a signal (in this case a soliton) is given by a function $u(x,t)$. We can define an expectation value for some quantity g analogous to quantum mechanics by[3]):

$$\langle g\rangle=\frac{\int_{-\infty}^{\infty} u(x,t)g(x)dx}{\int_{-\infty}^{\infty} u(x,t)dx} \tag{5.1}$$

and the mean square deviation for position and wavenumber variables in the well known way as:

$$\Delta x^2=\langle(x-\langle x\rangle)^2\rangle \tag{5.2}$$

$$\Delta k^2=\langle(k-\langle k\rangle)^2\rangle \tag{5.3}$$

Then, the bandwidth theorem states the inequality:

$$\Delta x^2\cdot\Delta k^2\geq\frac{1}{4} \tag{5.4}$$

This is exactly the Heisenberg uncertainty principle, apart from the De Broglie relation $p=\hbar k$, which we show in the next paragraph how they come out of the soliton model. It comes out naturally just from the soliton nature of the particles. If one defines the mean square deviations for angular frequency and time in the same way as done above for k and x as, then one also gets the second uncertainty relation:

$$\Delta\omega^2\cdot\Delta t^2\geq\frac{1}{4} \tag{5.5}$$

This is again equivalent to the Heisenberg uncertainty relation for energy and time, apart from the relation $E=\hbar\omega$, which we will also derive in the next paragraph.

5.2 De Broglie relations:

As was shown by Enz [7], the De Broglie relations are included in the Sine-Gordon breather. We will show later, that this is not restricted to the Sine-Gordon breather, but a consequence of the Lorentz-symmetry of the underlying field equation when applied to a breather solution. Therefore, they are provided by any relativistic field equation which posses breather solutions.
To derive the relations, we first recall that De Broglie‘s idea [8,9] was that every particle possesses an internal oscillation, which in his rest frame is given by:

$$\psi=a\exp(i\omega t) \tag{5.6}$$

with $\omega=\frac{E}{\hbar}=\frac{m_0 c^2}{\hbar}$ and a constant amplitude a. For a moving particle, equation (5.6) then reads:

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3) We can identify $u=|\psi|^2$, then eq. (5.1) is exactly the same expectation value as in quantum mechanics, just the normalisation constant is not included in u, but written separately in the denominator (since solitons are not normalised)

$$\psi = a \exp\left(\frac{i}{\hbar}(Et - px)\right) \quad (5.7)$$

with $E = E_0 \gamma$ and $p = \frac{E}{c^2} v$. In addition to the ψ wave, De Broglie postulated a corresponding wave u which has an amplitude varying in space. This wave should have a large amplitude near the classical position of the particle, which decays rapidly further away from it. The u wave should be a solution to some non-linear wave equation, which De Broglie didn't specify further. De Broglie considered only the set of both waves u and ψ as a complete description of Quantum particles, the common wavefunction ψ alone being incomplete.
If we now look again at the breather solution (4.11) of the Sine Gordon equation:

$$\varphi(x,t) = 4 \arctan\left(\frac{\sin\left(\gamma \frac{r}{d}\left(ct - \frac{v}{c}x\right)\right)}{\cosh\left(\frac{\sqrt{1-r^2}}{d} \gamma \,\omega\,(x - vt)\right)}\right) \quad (5.8)$$

we can easily see that the periodic function describing the internal oscillation is given by:

$$\sin\left(\gamma \frac{r}{d}\left(ct - \frac{v}{c}x\right)\right) \quad (5.9)$$

From this we can read of the frequency and wavenumber as:

$$\omega = \frac{rc}{d} \gamma \quad (5.10)$$

$$k = \frac{r}{dc} v \gamma \quad (5.11)$$

On the other hand, the energy and momentum of the breather can be calculated using equations (4.4 & 4.7), yielding:

$$E = 16 d \sqrt{1-r^2} \gamma = E_0 \gamma \quad (5.12)$$

$$p = \frac{E_0}{c^2} v \gamma \quad (5.13)$$

From equations (5.10 & 5.11) and (5.12 & 5.13), we find the De Broglie relations:

$$E = \hbar \omega \quad (5.14)$$

$$p = \hbar k \quad (5.15)$$

with Planck's constant h given by:

$$\hbar = \frac{E_0 d}{rc} \quad (5.16)$$

It is very interesting to compare this expression with the „coincidence“

$$\hbar \sim m_p r_p c = \frac{E_p r_p}{c} \quad (5.17)$$

Since $r \sim 1$, one can see that (5.16) and (5.17) have the same structure. In (5.16), E_0 plays the role of a fundamental energy (the rest energy of the breather solution) and d is a fundamental length scale, entering via the field equation (4.10). Equation (5.17) suggests, that both these roles in the complete theory are played by the „radius“ and the rest energy of the Proton (or the Neutron, respectively). It is remarkable and non-trivial, that already in the Sine-Gordon model, h depends on

the rest-energy of a particle (the breather). It is the same fact, that is found in the „coincidence“ (5.17).
Finally, we can show that the De Broglie relations are not just coincidentally included in the Sine-Gordon breather, but that all breather solutions to relativistic field equations exhibit them. They are a Therefore, we first establish that a resting breather has the general form:

$$\varphi(x,t)=\varphi(\psi(\frac{ct}{d})u(\frac{x}{d})) \qquad (5.18)$$

where ψ is a function periodic in time in the sense of De Broglie and u is the corresponding function of spatially variable amplitude. d is again some length parameter. Now, ψ has some oscillation frequency ω. If we now consider the same breather moving, due to the Lorentz-symmetry of the underlying field equation, we have:

$$\varphi(x,t)=\varphi(\psi(\frac{\gamma}{d}(ct-\frac{v}{c}x))u(\frac{\gamma}{d}(x-vt))) \qquad (5.19)$$

From this, we can read off:

$$\omega=r\frac{c}{d}\gamma$$

$$k=\frac{r}{dc}v\gamma$$

where r is again a dimensionless constant. Recalling $E=E_0\gamma$ and $P=m_0v\gamma$ we obtain again the relations (5.14 & 5.15) with h given by (5.16) (the parameters r and d are, of course, different in general here).
It is interesting to notice, that the reason why breather solutions to relativistic field equations exhibit the De Broglie relations, is the structure of the Lorentztransformations, under which these equations are invariant. More precisely, the transformation of time

$$t\rightarrow\gamma(t-\frac{v}{c^2}x)$$

If the field equations would only exhibit Galilean symmetry, time would transform as $t\rightarrow t$ and therefore (5.19) would yield neither of the De Broglie relations (5.14 & 5.15). This is even more astonishing, since the De Broglie relations are already fundamental in non-relativistic quantum mechanics. It shows again, how quantum mechanics and relativistic phenomena have a common origin in the soliton model.

5.3 Discrete energy levels:

The discrete nature of the energy levels of bound states results from the modes of the trapped breathers in a potential, similar to the modes of a standing wave in classical physics. As an example of how this works, we consider a Sine-Gordon breather trapped in an infinite potential well of width L. The breather is reflected at the walls of the well, resulting in a second breather, travelling in the opposite direction with the same velocity and phase shifted by π. Thus, the entire solution is a two breather solution to the Sine-Gordon equation, which can be found analytically. This solution has to fulfill the boundary conditions $x(0)=x(L)=0$ for all times t, as well as satisfy the Sine-Gordon equation (4.10). The explicit calculations are carried out in a separate paper [10], as they are a bit more technical. The basic idea is similar to how a standing wave occurs: A superposition of two waves travelling in opposite direction. Demanding that the solution fulfills the boundary conditions, one obtains:

$$kL = n\pi \tag{5.20}$$

for the wavenumber of the breather, as defined by equation (5.11). The solution is then a breather oscillating back and forth between both ends of the potential well. The condition (5.20) exactly agrees with the quantisation condition derived from quantum mechanics. If we combine this with the De Broglie relation (5.15) and plug both in the relativistic energy-momentum relation (4.9), we obtain:

$$E_n = \sqrt{(m_0 c^2)^2 + c^2 \hbar^2 \left(\frac{n\pi}{L}\right)^2} \tag{5.21}$$

Those are the well known energy levels for the infinite potential well obtained from the Klein-Gordon equation. To show that also for more sophisticated potentials in general the energy levels agree with those of the Klein-Gordon equation, remains a task to be done. Of course, this is to be expected from the full theory.

6. Self-energy, the Coulomb-Singularity and „renormalisation“:

Due to the particles not being points, but spread out distributions in space, the soliton model immediately removes the problem of infinite self energies, as well as the unphysical 1/r singularity in the potentials, be it gravitational or electromagnetic. As is well known, both problems can be tracked down to particles being described as points. We will restrict ourselves here to the gravitational potential, but all arguments apply to the electromagnetic as well. The energy of a mass distribution in a field is given by:

$$E = -\frac{1}{8\pi G} \int_{\mathbb{R}^3} (\vec{\nabla}\varphi)^2 \, dx \tag{6.1}$$

For a point particle, the gravitational potential φ reads

$$\varphi = G\frac{m}{r} \tag{6.2}$$

which, if plugged into (6.1), yields an infinite energy. One can see, that this infinite self energy is a consequence of the likewise unphysical singularity of the 1/r field, which in turn is a result of point particles being the source. This, one can easily see by plugging the Dirac-distribution into the equation for the potential:

$$\varphi = -G \int_{\mathbb{R}^3} \frac{\rho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d^3\vec{r}\,' \tag{6.3}$$

On the other hand, one can easily see that the potential (6.3), as well as its derivative, remain finite for $r \to 0$ for any density which remains finite in this limit, too. Now, a particle described by a soliton, obviously fulfills this requirement. Therefore, the singularity in the potential as well as the resulting infinite self energy are removed. Indeed, as mentioned at the beginning, since the solitons are part of the field, they contain the field generated by the particle. Thus, the energy calculated by equation (4.4) contains both the combined energy of the particle (the core field) and its (far-) field. As we saw, it yields a well defined, finite value for its rest mass. No renormalisation is needed.

7. Soliton interaction:

Of course, one does not just want to recover the phenomenology of special relativity and Quantum mechanics, but also the known behaviour of particles in classical physics. As stated at the beginning, it is well known that solitons exhibit particle behaviour in the sense that they propagate and collide shape-perserving. Of course, one also wants to have soliton dynamics. This interaction between solitons via their fields is included in the soliton solutions themselves. More precisely, in the multi-soliton solutions of the field equations. To demonstrate this, we look again at the Sine-Gordon equation and its two-soliton solution[4]):

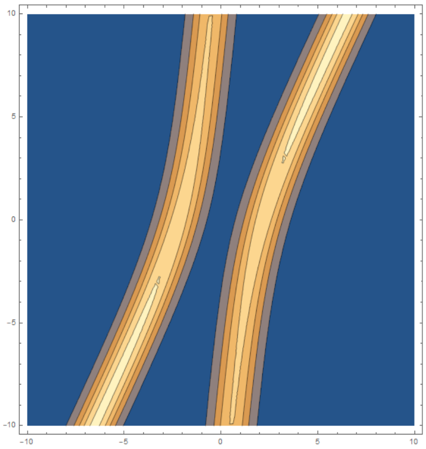
$$\varphi(x,t)=4\arctan\left(\alpha\frac{\sinh\left(\frac{z_1+z_2}{2}\right)}{\cosh\left(\frac{z_1-z_2}{2}\right)}\right) \qquad (7.1)$$

$$z_i=\gamma_i(x-v_i t-x_i) \qquad i=1,2 \qquad (7.2)$$

Here, v_1, v_2 are the velocities of the two solitons, x_1, x_2 their positions at t=0. α is a parameter depending on both velocities of the solitons in a bit more sophisticated way. The solution (7.1) describes two solitons propagating and eventually colliding. For simplicity reasons, we shall just consider the non-relativistic case. The picture doesn‘t change qualitatively, if the full relativistic case is considered.
In this case, both solitons follow a trajectory given by:

$$x_{\pm}(t)=\frac{v_1+v_2}{2}t\pm d\,\mathrm{arsinh}\left(\sqrt{4\frac{c^2}{v_{12}^2}\cosh^2\left(\frac{v_{12}}{2d}t\right)-1}\right) \qquad (7.3)$$

Here, „+“ is the right soliton, „-“ the left one, $v_{12}=v_1-v_2$. It is plotted in the following x-t-diagram of the energy density:



After the collision, the slower soliton takes over the velocity of the faster soliton and vice versa. They behave the same way two particles of same mass behave, when they collide. With one important difference: The collision is not like a collision of two solid balls, but a continuous transfer of momentum occurs via the fields of the particles. Indeed, eq. (7.3) describes an accelerated motion of both solitons, despite their „velocities“ v_1, v_2 entering the solution (7.1) being constant. Those velocities are actually the initial and terminal velocities of both solitons before and after the interaction. The dynamical interaction in between, due to the fields generated by the two solitons, is encoded in the multi-soliton solutions.

<u>Newton‘s law of motion:</u>

We can show that the above interaction obeys Newtons law of motion

4) We discuss the so called Kink-Kink solution. Since there are two types of solitons in the Sine Gordon equation, Kinks and Antikinks, there is also the Kink-Antikink solution. But it doesn‘t exhibit the known behaviour of classical particles upon collision (both solitons run through each other), therefore we chose the Kink-Kink solution here.

$$F = ma \tag{7.4}$$

when the solitons are separated far enough from each other. Physically, this means that the particles (the core fields of the solitons) are separated far enough so that their distance fulfills $r \gg r_p$, where r_p is the „size“ of the particle. Consequently, in this case, the particles are only interacting with their far fields. This corresponds to the limit, where core interactions between the particles can be neglected, which is especially the case in classical mechanics, but even quantum mechanics, as long as what is usually termed as „strong force“ can be neglected.
To show (7.4), we first note that the centre of mass of the two solitons moves uniformly. Indeed, from (7.3), it can be seen that

$$\frac{1}{2}(x_+ + x_-) = \frac{v_1 + v_2}{2} t \tag{7.5}$$

For the relative separation of the solitons we have:

$$r = x_+ - x_- = 2d \operatorname{arsinh}\left(\sqrt{4\frac{c^2}{v_{12}^2}\cosh^2\left(\frac{v_{12}}{2d}t\right) - 1}\right) \tag{7.6}$$

Differentiating this once with respect to time, solving (7.6) for r and plugging it into the result, one obtains[5]):

$$\left(\frac{dr}{dt}\right)^2 + 4c^2 \frac{1 - \left(\frac{v_{12}}{c}\right)^2}{\sinh\left(\frac{r}{2d}\right)^2} = v_{12}^2 \tag{7.7}$$

In the far field r>>d we have $\sinh\left(\frac{r}{2d}\right) \approx \frac{1}{4}\exp\left(\frac{r}{d}\right)$ and thus

$$\left(\frac{dr}{dt}\right)^2 + 16c^2 \exp\left(\frac{-r}{d}\right) = v_{12}^2 \tag{7.8}$$

Now, the interaction energy between the two solitons is in the same limit given by [11]

$$V(r) = 32d \exp\left(\frac{-r}{d}\right) \tag{7.9}$$

Plugging this into (7.8), we can write it as

$$\frac{1}{2}\mu \dot{r}^2 + V(r) = \frac{1}{2}\mu v_{12}^2 = E \tag{7.10}$$

Here, $\mu = \frac{m_0}{2}$ is the reduced mass of the two solitons of mass m_0 , which is given by:

$$m_0 = \frac{8d}{c^2} = \frac{E}{c^2} \tag{7.11}$$

In the second equality, we plugged in the energy of a single soliton obtained by direct calculation via equation (4.4). Equation (7.10) is the known energy expression for the motion of two particles in the center of mass system in Newtonian mechanics. We thus have seen, that the two solitons indeed move according to Newton‘s law in their generated (far-) fields.

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5) It is also interesting to notice, that the 2nd term on the left side, which is the „interaction potential“ of the motion of the two particles, exhibits the same dependency on the relative velocities of the particles, as in the non-relativistic Machian theories [1-5].

Another important thing to notice is, that no notion of mass has been introduced a priori. In fact, we identified the expression (7.11) with the mass in Newton‘s law (via the reduced mass in the energy expression (7.10)). This is in fact a derivation of the formula

$$m=\frac{E}{c^2} \tag{7.12}$$

The expression playing the role of the inertial mass in Newton‘s law is E/c^2, where E is the entire field energy. This is also very similar to the expression (2.1) for the inertial mass of a particle derived from the Machian theories. It is also proportional there to the entire field energy φ divided by c^2.

8. N-particle model:

In non-relativistic Machian theories, as in principle also in Newtonian dynamics, the universe is described by an N-particle model. Many soliton equations posses *anayltic* N-soliton solutions, as well as N-breather solutions. The particles (the soliton core fields), their (far-) fields as well as their mutual interaction (and the resulting motion) is then described in a *single* solution to the underlying field equations, as was shown for the two soliton case in section 7. As an example, the multi soliton solution of the sine-Gordon equation is given by [12]:

$$\varphi=4\arctan\left(\frac{\Im(f)}{\Re(f)}\right)$$

$$f=W(\psi_1,\ldots,\psi_N)(X) \qquad \psi_i=\exp\left(\frac{1}{2}\left(\alpha_i X+\frac{1}{\alpha_i}T\right)+\delta_i\right)$$

$$X=\frac{x+ct}{2d} \qquad T=\frac{x-ct}{2d}$$

Here, W denotes the Wronskian with derivatives performed after X. $\Im,\Re$ are the real and imaginary part of f; X, T are the so called light cone coordinates and δ is a complex phase. N is the number of solitons, respectively particles.
It is to be expected, that the correct gravitational field equations, too posses such N-soliton solutions. They are then the relativistic generalisation of the N-particle model in classical mechanics.

9. Conclusions

We have shown that it is suggested by a unification of gravity and inertia that elementary particles are solitons in the gravitational field, or more general, a „unified“ field. We have shown, that from such a theory of elementary particles all the basic phenomena of special relativity and quantum mechanics can be derived in a classical framework, in flat 3 dimensional euclidean space. Such an approach enabled us to derive an expression for the unexplained constant h. Further, we were able to actually explain the apparent constancy of the speed of light, instead of having to postulate it. Other than in common theories, however, the speed of light is not actually constant, but only appears constant due to the changes of the elementary rulers and clocks in moving frames, provided by the elementary particles. The theory also yields actual physical explanations for effects like length contraction, time dilation, as well as for the mentioned quantum phenomena. The same applies to the energy mass equivalence, which in current theories can be derived, but no physical

explanation can be given for it. This also lead us directly to the origin of mass, which could be explained as entirely of field origin, as the energy of the soliton divided by c^2.
The above said leads us to the conviction, that a soliton theory of elementary particles, as was presented here, is the gateway to a unified field theory incorporating both relativity and quantum mechanics. It would therefore be worthwhile to direct research programs into this direction and search for suitable candidates for non-linear field equations with soliton solutions. Since special relativity is based on space and time actually changing, while in a soliton theory, it is the elementary particles themselves which are changing in flat, euclidean space, it is necessary to part ways with special relativity when pursuing the soliton approach further. The same, consequently, also applies to general relativity and quantum mechanics in its current formulation.

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