A fallacy in estimating the age of the universe

Samuel Meng
Research School of Physics, Australian National University, Canberra, ACT 2601
email: xianming.meng@anu.edu.au

Abstract:

The James Webb space telescope reveals much earlier and bigger galaxies than we expected, indicating inadequacies in our current understanding of the universe. This paper reviews the current practice in estimating the age of the universe and uncovers a problem: the luminosity distance is incorrectly related to the proper distance. This mistake causes the underestimation of the comoving distance, and thus of the expansion and the age of the universe. By linking the luminosity distance to the light travel distance, this paper provides a rectified formula for estimating the age of the universe.

Key words: proper distance, luminosity distance, comoving distance, ΛCDM model

1. Introduction

The Early Release Observations (ERO) data taken by James Webb space telescope (JWST) surprised astronomers and physicists to the extent of 'panic' (Witze, 2022; Ferreira et al, 2022). Ferreira et al found there are about 10 times relative higher number of disk galaxies than seen by the Hubble Space Telescope at the redshifts of at $z > 1.5$. Adams et al (2022) found four $z > 9$ galaxies which have not previously been identified, with one object at $z = 11.5$, and another a close pair of galaxies. Naidu et al (2023) found two remarkably luminous galaxies at $z \approx 10–12$. Atek et al (2023) found two galaxies have a red shift z=16, which indicates they are only 250 million years after the Big Bang. Yan et al, (2023) indicated they had detected galaxies with redshift up to z=20, which will push the age of earliest galaxy even closer to the Big Bang. Witze (2022) summarized the findings from the ERO data as: so many galaxies in the early universe, which are very young (or far away from us) and surprisingly close to each other; and some of which are already complex and massive, and rich in chemical elements.

The surprising findings from the JWST data highlight our inadequate understanding of universe. Since the galaxies can be much earlier, bigger, more complex, and closer to each other, it is most likely that the formation of this galaxies has started much earlier. Given current belief that at least 1 billion years are needed to form a galaxy, the findings from the JWST data point to a much older universe. Currently the common approach to the findings from the JWST data is to revise the number of years for galaxy formation and thus keep the estimated the age of universe intact. Through reexamining the current practice in estimating the age of the universe, this paper identifies a problem that leads to an underestimation. The paper also proposes a rectified equation to eliminate the estimation bias.

2. The cause of underestimating the age of the universe

Astronomers have used two methods to estimate the age of the universe. One is through estimating the age of the oldest stars known as globular clusters (e.g. Krauss and Chaboyer, 2003). It is generally agreed that this method provides only a lower boundary for the age of the universe for two reasons (Cheng, 2005). First, star formation starts during the decelerated expansion after the Big Bang (i.e. the inflation epoch), so this approach excludes the time before the star formation. Second, since observations are limited by the instruments and technology used today, it is most likely that the oldest stars we observed today are not the oldest stars in the universe, so the oldest star age may also be underestimated.

The other method is to estimate the age of the universe based on astronomical survey data and cosmological models. This method is comprehensive, includes time right after the Big Bang, and can obtain estimates for a number of cosmological parameters. However, the estimates from this approach crucially rest on the assumptions used for estimation.

Interestingly, the estimates from the second method are close to those from the first method. On the surface, this ‘consistency’ seems comforting. However, considering that the first approach is very likely to underestimate the age of universe, we must conclude that the ‘consistent’ results tend to suggest that the second approach may also has an underestimation issue.

The current cosmological model – the ΛCDM model is well tested and supported by observations, so we are not going to challenge the model here. Instead, we examine the assumptions added when applying this model.

A full ΛCDM model has many variables to be calibrated by observation data. For simplicity, we illustrate a simple version for a flat universe filled with radiation, matter (including baryonic matter, cold dark matter and hot dark matter), and dark energy:

$$H(t)^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_M a(t)^{-3} + \rho_R a(t)^{-4} + \rho_\Lambda\right) = H_0^2(\Omega_M a(t)^{-3} + \Omega_R a(t)^{-4} + \Omega_\Lambda) \quad (1)$$

Where H is the Hubble parameter, H_0 the Hubble constant for the current epoch, a the scale factor, $\dot{a}$ the derivative of a, w_i the parameter for the equation of state, ρ_i density of mass/energy; Ω_M, Ω_R and Ω_Λ are the density ratio (compared with critical density ρ_c) for matter, radiation, and dark matter, respectively.

To calibrate parameters in the above equation, we need estimate the Hubble constant. This involves estimating the recessional speed through measuring redshift of light from distant galaxies, and estimating the distance from the galaxy to the earth through measuring light intensity or other measurements, so distance estimation is crucial for model calibration.

Due to the expansion of the universe, the distance between the galaxy of interest and the earth keep increasing, so we need to define distance in a dynamic system. In cosmology, it is a common practice to use comoving coordinate system to obtain proper time and proper distance. As the universe expands, the proper distance also expands accordingly. The proper distance of a light ray emitted as time t_{em} and received currently at t_0 can be expressed as:

$$d_p(t_{em}, t_0) = \int_{t_{em}}^{t_0} \frac{cdt}{a(t)} = \int_{a_{em}}^{1} \frac{cda}{a^2(t)H(t)} \quad (2)$$

By dividing the distance the light travelled in each epoch by its scale factor, the above equation transforms the light travel distance to the current epoch, i.e. the distance of current position of light source to current observer.

Another type of distance commonly used in astronomy is luminosity distance, which is the distance measured by light intensity. It is widely agreed that the luminosity distance is related to proper distance by redshift. A redshift z necessitates that the energy of each photon be reduced by $(1+z)^{-1}$ and that the photon numbers in given time is also reduced by $(1+z)^{-1}$, so the light intensity is reduced by $(1+z)^{-2}$. As a result, it is commonly accepted that the luminosity distance d_L in an expanding system is the proper distance enlarged by (1+z):

$$d_L(t_{em}, t_0) = (1+z)d_p(t_{em}, t_0) = (1+z)c\int_{a_{em}}^{1} \frac{da}{a^2(t)H(t)} \quad (3)$$

The redshift is determined by the time of photon emissions and the rate at which the universe expands. Due to the continuous expanding of the universe, the later emitted photon always travels in a slightly more expanded space left behind by the earlier emitted photon, so the gap between these two photons gradually increases, causing redshift. A formal proof of this redshift found in many cosmological textbooks shows the following relationship between redshift and the scale factor:

$$1 + z = \frac{\lambda}{\lambda_{em}} = \frac{1}{\mathrm{a(t)}}, or \ \ a(t) = \frac{1}{1+\mathrm{z}} \tag{4}$$

Plugging eq. (4) into eq. (1), we have:

$$H(z)^2 = H_0^2(\Omega_M(1+z)^3 + \Omega_R(1+z)^4 + \Omega_\Lambda) \tag{5}$$

Plugging eq. (4) and (5) into eq. (3), we can obtain:

$$d_L(\mathrm{z}) = (1+z)c\int_0^z \frac{dz}{H_0^2(\Omega_M(1+z)^3 + \Omega_R(1+z)^4 + \Omega_\Lambda)} \tag{6}$$

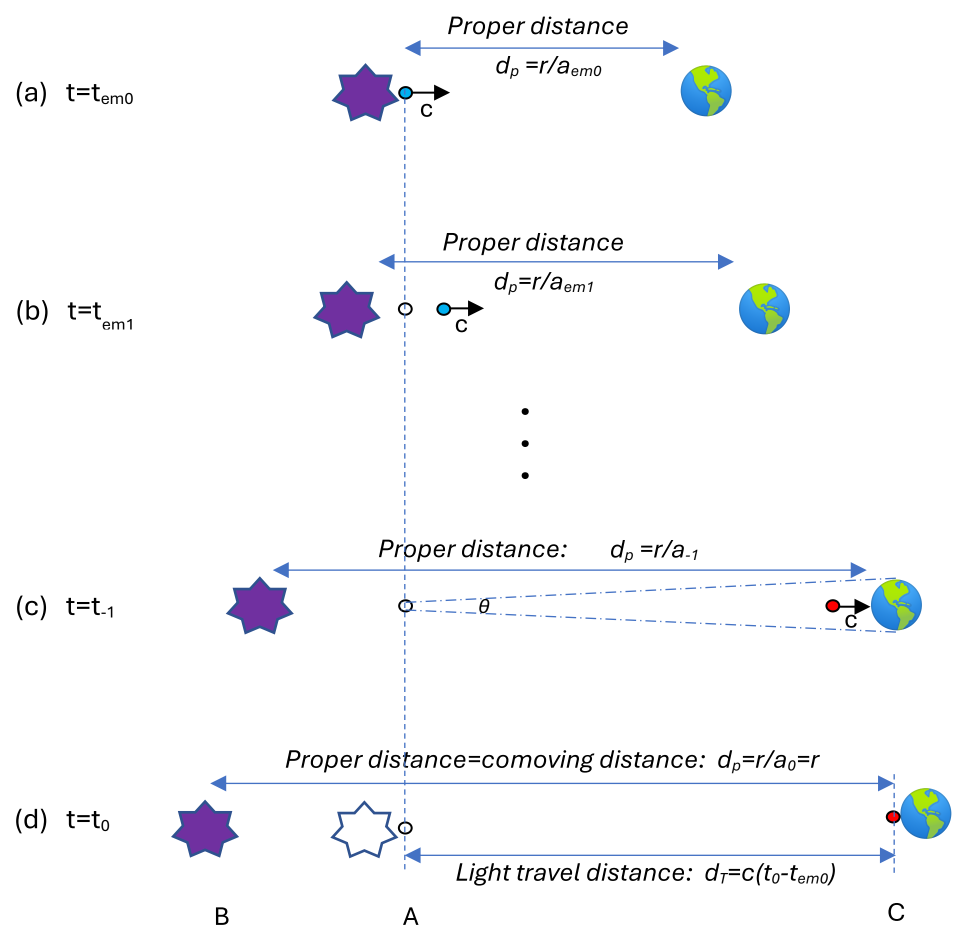
With observed data for redshift z and the luminosity distance d_L for different galaxies, we can use eq. (6) to calibrate the parameters Ω_M, Ω_R and Ω_Λ). With calibrated model, we can calculate the age of the universe t_0 by using eq. (1) to obtain:

$$t_0 = \int_0^{t_0} \frac{\dot{a}dt}{\dot{a}} = \int_0^1 \frac{da}{aH} \tag{7}$$

The estimation process looks vigorous, but eq. (3) based on common wisdom is problematic, because it ignores the difference between the comoving distance (proper distance at the current epoch) and the light travel distance, as shown in Fig.1.

In panel (a), as the star starts to emit a photon at the time of t_{em0}, the proper distance between the star and the earth is: $d_p=r/a_{em0}$, where r is the comoving distance, a_{em0} is the scale factor at t_{em0}.

Fig. 1 Illustration of proper distance and light travel distance



At the time t_{em1} shown in panel (b), the photon travels away from its initial position marked by the dashed line, and the star and the earth both move outwards due to expansion of the universe. As a result, the proper distance increases to $d_p=r/a_{em1}$, a_{em1} is the scale factor at t_{em1}.

The expansion of universe and photon travel continue. In panel (c), the photon travels very close to the earth in time t_{-1}, the proper distance increases to $d_p=r/a_{-1}$, a_{-1} is the scale factor at t_{-1}.

In panel (d), the photon arrives at the earth and is detected by an astronomer. The proper distance increases to $d_p=r/a_0$, where a_0 is the scale factor at t_0. Since the scale factor at t_0 is 1, the proper distance at t_0 equals to the comoving distance r. The light travel distance AC can be calculated as $d_T=c(t_0-t_{em0})$, which is much smaller than the proper distance BC.

The distance the astronomer is trying to measure is the light travel distance AC. What the astronomer has detected at time t_0 is the photon emitted by the star at time t_{em0}, so the star seen by the astronomer is the image of the star at the position A when the star emitted this photon. As a result, the luminosity distance the astronomer measured is the light travel distance but, as we discussed earlier, in consider of the effect of red shift and photon flux change, this distance needs to be augmented by (1+z).

One may still be uncertain if the luminosity distance should be related to the light travel distance AC or to the comoving distance BC. Panel (c) provides a re-assurance. The luminosity distance is determined by the light intensity (or flux) at the telescope on (or orbiting) the earth. As shown in panel (c), it is the angle θ formed by the light rays the star emitted at the time t_{em0} that determines the light intensity at the telescope. The closer distance from the earth to the initial position of the photon, the larger θ, and thus the more energy into the fixed area of the telescope lens. The expansion of the universe causes the star moves away from the initial position, but this movement has no effect on the detected light intensity on the earth. The earth's moving away from the initial position of the star does affect the angle θ, but this increased distance is already included in the light travel distance.

If one insists that the star never move away from the initial position and that it is the observer that move away, the consequence is that the light travel distance is the same as the proper distance. This claim is untenable. In panel (b), when the photon travels to the new position, the expansion of the universe must cause the expansion of the space at both sides of the photon (i.e. the distance the photon has traveled, and that the photon is going to travel). If so, the star has to move away from the initial position. A mathematic calculation can dismiss this claim categorically.

The light travel distance can be calculated as:

$$d_T(t_{em}, t_0) = \int_{t_{em}}^{t_0} c dt = c \int_{t_{em}}^{t_0} \frac{\dot{a} dt}{\dot{a}} = c \int_{a_{em}}^{1} \frac{da}{a(t)H(t)} \tag{8}$$

The light travel distance shown in eq. (8) is different from the proper distance in eq. (2), so there is no way the two distances are the same. The only possibility for them to equal is that a(t)=1 (so $a(t)=a(t)^2$), but this contradicts the fact that the universe is expanding and that $a(t) \leq 1$.

In short, it is the light travel distance, rather than the comoving distance, that is directly related to luminosity distance. However, in eq. (3), we view the measured luminosity distance as the proper distance at t_0 (i.e. comoving distance) augmented by (1+z), so we effectively reduce the proper distance to light travel distance, and thus underestimate the comoving distance. Fitting these underestimated distances into the cosmological model, we inevitably underestimate the expansion of the universe and thus the age of universe.

Next, we derive a correct formula for estimation and show how the underestimation could occur. The luminosity distance should be the light travel distance augmented by (1+z), so from eq, (8) we can derive:

$$d_L(t_{em},t_0) = (1+z)d_T(t_{em},t_0) = (1+z)c\int_{a_{em}}^{1} \frac{da}{a(t)H(t)} \tag{9}$$

The only difference between eq. (3) and (9) is that there is an extra term 1/a(t) in the integrant in eq. (3). Since $a(t) \leq 1$, for given luminosity distance from observation data, eq. (4) will overestimate H(t). For given data on z and thus a(t), this overestimated H(t) will lead to an overestimation of Ω_M, and Ω_R in eq.(1). From eq. (7), it is obvious that an overestimation of H(t) leads to an underestimation of the age of the universe.

To correct the mistake in estimation, what we need to do is simply replace the eq.(3) with the eq.(9) and re-estimate the parameter. Namely, plugging eq. (4) and (5) into eq. (9), we can obtain:

$$d_L(\mathrm{z}) = c\int_0^z \frac{dz}{H_0^2(\Omega_M(1+z)^3+\Omega_R(1+z)^4+\Omega_\Lambda)} \tag{10}$$

Compared eq. (10) with eq.(6), we find that existing estimation reduced the proper distance by a factor of (1+z), leading to underestimation of the age of the universe. Using eq. (10) we can calibrate Ω_M, Ω_R and Ω_Λ, then we can use eqs. (1) and (7) to estimate the age of the universe.

3. Conclusion

This paper reviews the current practice in estimating the age of the universe and uncovers a problem: the luminosity distance is incorrectly related to the proper distance. This mistake causes the underestimation of the comoving distance, and thus of the expansion and the age of the universe. By linking the luminosity distance to the light travel distance, this paper provides a rectified formula to for estimating the age of the universe.

Data availability statement: All data generated or analysed during this study are included in this published article.

References:

Adams, N J, C J Conselice, L Ferreira, D Austin, J A A Trussler, I Juodžbalis, S M Wilkins, J Caruana, P Dayal, A Verma et al, 2022, Discovery and properties of ultra-high redshift galaxies ($9 < z < 12$) in the JWST ERO SMACS 0723 Field, Monthly Notices of the Royal Astronomical Society, Volume 518, Issue 3, January 2023, Pages 4755–4766, https://doi.org/10.1093/mnras/stac3347

Atek, H., Marko Shuntov, Lukas J Furtak, Johan Richard, Jean-Paul Kneib, Guillaume Mahler, Adi Zitrin, H J McCracken, Stéphane Charlot, Jacopo Chevallard et al, 2023, Revealing galaxy candidates out to $z \sim 16$ with JWST observations of the lensing cluster SMACS0723, Monthly Notices of the Royal Astronomical Society, Volume 519, Issue 1, February 2023, Pages 1201–1220, https://doi.org/10.1093/mnras/stac3144

Aurich R., Lustig S., 2015, Early-Matter-Like Dark Energy and the Cosmic Microwave Background, Journal of Cosmology and Astroparticle Physics.

Bennett C. et al., 2012, Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results, The Astrophysical Journal Supplement Series, Volume 208, Number 2

Cheng, T., 2005, Relativity, Gravitation and Cosmology, Oxford University Press.

Daly, R. A., & Djorgovski, S. G. 2003, A Model-Independent Determination of the Expansion and Acceleration Rates of the Universe as a Function of Redshift and Constraints on Dark Energy, Astrophysical Journal, 597, 9-20

Daly, R. A., & Djorgovski, S. G. 2004, Direct Determination of the Kinematics of the Universe and Properties of the Dark Energy as Functions of Redshift, Astrophysical Journal, 612, 652-659.

Daly, R. A., & Djorgovski, G. 2007, A Nearly Model-Independent Characterization of Dark Energy Properties as a Function of Redshift, Nuclear Physics, 173, 19 – 23.

Daly, R. A., Djorgovski, S. G., FreemanUS, K. A., MoryUS, M. P., O'Dea, C. P., Kharb, P., & Baum, S. 2008, Improved Constraints on the Acceleration History of the Universe and the Properties of the Dark Energy, Astrophysical Journal, 677, 1 – 11.

Ferreira1, L., Nathan Adams, Christopher J. Conselice, Elizaveta Sazonova, Duncan Austin, Joseph Caruana, et al, 2022, Panic! at the Disks: First Rest-frame Optical Observations of Galaxy Structure at $z > 3$ with JWST in the SMACS 0723 Field, The Astrophysical Journal Letters, **DOI** 10.3847/2041-8213/ac947c

Komatsu1, E., J. Dunkley, M. R. Nolta, C. L. Bennett, B. Gold, G. Hinshaw, N. Jarosik, D. Larson, M. Limon8, L. Page et al, 2009, Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations, The Astrophysical Journal Supplement Series, Volume 180, Number 2, DOI 10.1088/0067-0049/180/2/330

Krauss, L. M., & Chaboyer, B. (2003). Age estimates of globular clusters in the Milky Way: Constraints on cosmology. Science, 299(5603), 65-69. https://doi.org/10.1126/science.1075631

Naidu, R., Pascal A. Oesch, Pieter van Dokkum, Erica J. Nelson, Katherine A. Suess, Gabriel Brammer, Katherine E. Whitaker, Garth Illingworth et al, 2022, Two Remarkably Luminous Galaxy Candidates at $z \approx 10-12$ Revealed by JWST, The Astrophysical Journal Letters, Volume 940, Number 1 DOI 10.3847/2041-8213/ac9b22

Perlmutter S., et al, 1999, Measurements of omega and lambda from 42 high redshift supernovae, Astrophysics Journal, 517, 565.

Riess A.G., et al., 1998, Observational evidence from supernovae for an accelerating universe and a cosmological constant, Astronomy Journal, 116, 1009.

Witze, A. (2022). Four revelations from the Webb Telescope About Distant Galaxies. Nature , 608(7921), 18–19. https://doi.org/10.1038/d41586-022-02056-5.

Yan, H., Zhiyuan Ma, Chenxiaoji Ling, Cheng Cheng, and Jia-Sheng Huang, 2023, First Batch of $z \approx 11-20$ Candidate Objects Revealed by the James Webb Space Telescope Early Release Observations on SMACS 0723-73, The Astrophysical Journal Letters, Volume 942, Number 1, DOI 10.3847/2041-8213/aca80c.