Determination of Solar Granule Temperature and Sunspot Basins by Quantum Gravity Theory

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Abstract: Matter wave has been generalized on a planetary scale as a quantum gravity theory, and applied to solar science. It was found that celestial size is the consequence of interference of the generalized matter waves, the sunspot cycle 11 years represents the beat-period of the interference. By the quantum gravity theory, the mean density of convection zone is determined as 3.3e-3 kg/m3, the mean temperature of the granule layer is calculated out to be 8689°K. The solar surface winds are investigated, the sunspot basins locate at 0°N ~ 30°N on the northern hemisphere, where the wind speeds almost equal to zero.

1. Introduction

Today, quantum gravity has grown into a vast area of research in many different approaches. Many directions have led to significant advances with various appealing ideas [1,2,3]. The concept of generalized relativistic matter wave and its applications were proposed and investigated in the author's early papers [4,5], the present paper continues to discuss the generalized relativistic matter wave and its applications on a planetary scale.

Newton’s first law states: in the absence of external forces, an object in motion continues in motion with a constant velocity. For introducing the generalized matter wave, the Newton’s first law of motion is amended, which states: when an object in 1D motion enters 2D space, the suddenly increasing dimension produces uncertainty that needs to be described by matter wave, as shown in Figure.1(a).

Figure 1  (a) When an object in 1D motion enters 2D space, the suddenly increasing dimension produces uncertainty that needs to be described by matter wave. (b) Force produces the uncertainty.

For electrons, that uncertainty is described by the de Broglie matter wave [6,7,8]. In the solar system, the sun attracts each planet to enter 2D orbits consecutively, deviating its inertial
1D motion consecutively, the gravitational force produces the uncertainty which needs to be described by a generalized matter wave on a planetary scale, as shown in Figure 1(b).

Consider a particle on the planetary scale, its generalized relativistic matter wave is given by the path integral

\[ \psi = \exp \left[ i \frac{\beta}{c^3} \int_0^x (u_1 dx_1 + u_2 dx_2 + u_3 dx_3 + u_4 dx_4) \right] . \]  

(1)

where \( u \) is the 4-velocity of the particle, \( \beta \) is the ultimate acceleration determined by experiments. The constant \( \beta \) has replaced the Planck constant in this quantum gravity theory so that its wavelength becomes a length on the planetary-scale. The early papers \([4,5]\) have discussed its applications to planetary science, showing that celestial size is the consequence of interference of the generalized matter waves.

The present paper shows that this quantum gravity theory based on the amended Newton’s first law provides a mechanism which allows us to calculate solar activity, such as solar size, the mean density of convective zone, solar granule temperature, sunspot period, sunspot basin locations, etc.

2. Calculation of the mean density of the convective zone

Similar to the Bohr model of hydrogen atom, the orbital circumference is \( n \) multiple of the wavelength of the planetary-scale relativistic matter wave, as shown in Figure 2(a). According to Eq. (1), consider a planet, we have

\[
\begin{align*}
\frac{\beta}{c^3} \int L v_i dl &= 2\pi n \\
v_i &= \frac{GM}{r}
\end{align*}
\]

\[
\Rightarrow \sqrt{r} = \frac{c^3}{\beta \sqrt{GM}} n, \quad n = 0, 1, 2, \ldots .
\]  

(2)

![Figure 2](image)

This orbital quantization rule only achieves a half success in the solar system, as shown in Figure 2(b), the Sun, Mercury, Venus, Earth and Mars satisfy the quantization equation; while other outer planets fail. But, since we only study quantum gravity effects among the Sun, Mercury, Venus, Earth and Mars, so this orbital quantization rule is good enough as a foundational quantum theory. In Figure 2(b), the blue straight line expresses a linear regression
relation among the quantized orbits, so it gives $\beta=2.956391\times10^10$ (m/s$^2$) by fitting the line. The quantum numbers $n=3,4,5,...$ were assigned to the solar planets, the sun was assigned a quantum number $n=0$ because the sun is in the central state.

In the solar interior, if the coherent length of the relativistic matter wave is long enough, its head may overlap with its tail when the particle moves in a closed orbit, as shown in Figure 2(a). Consider a point on the equatorial plane, the overlapped wave is given by

$$\psi = \psi(r)T(t)$$

$$\psi(r) = \psi_0(1 + e^{i\delta} + e^{2i\delta} + ... + e^{i(N-1)\delta}) = \psi_0 \frac{1-\exp(iN\delta)}{1-\exp(i\delta)} .$$

(3)

$$\delta(r) = \frac{\beta}{c^3} \int_{l} (v_l) dl = \frac{2\pi\beta\omega r^2}{c^3}$$

where $N$ is the overlapping number which is determined by the coherent length of the relativistic matter wave, $\delta$ is the phase difference after one orbital motion, $\omega$ is the angular speed of the solar self-rotation. The above equation is a multi-slit interference formula in optics, for a larger $N$ it is called as the Fabry-Perot interference formula.

The generalized relativistic matter wave function $\psi$ needs a further explanation. In quantum mechanics, $|\psi|^2$ equals to the probability of finding an electron due to Max Born’s explanation; in astrophysics, $|\psi|^2$ equals to the probability of finding a nucleon (proton or neutron) averagely on an astronomic scale, we have

$$|\psi|^2 \propto \text{nucleon-density} \propto \rho .$$

(4)

According to the multi-slit interference formula, the solar radius corresponds to the first minimum of $|\psi|^2$ nearby the center, therefore

$$N\left(\frac{2\pi\beta\omega r^2}{c^3}\right) = 2\pi \quad \Rightarrow \quad N = \frac{\lambda}{2\pi r} = 650; \quad \lambda = \frac{2\pi c^3}{\beta\omega r} .$$

(5)

Therefore we estimate that the coherent length equals to one wavelength $\lambda$, the overlapping number is 650 on solar equatorial circumference.

Sun’s angular speed at its equator is known as $\omega=2\pi/(25.0524\times3600)$ (s$^{-1}$). Its mass 1.9891e+30 (kg), well-known radius 6.95e+8 (m), mean density 1408 (kg/m$^3$), the constant $\beta=2.956391e+10$ (m/s$^2$). According to the $N=650$, the matter distribution of the $|\psi|^2$ is calculated in Figure 3, it agrees well with the general description of star’s interior. When the overlapping number $N$ is known, for instance the solar $N=650$, the celestial radius is given by

$$N\left(\frac{2\pi\beta\omega r^2}{c^3}\right) = 2\pi \quad \Rightarrow \quad r = \sqrt{\frac{c^3}{\beta\omega N}} = 6.949671e+8(m) .$$

(6)

This is the celestial radius formula, which predicts the Sun with a radius of 6.949671e+8(m) with a relative error of 0.005% in Figure 3, where the solar radius is defined as the distance from the center to the convective zone top, it is a high-accuracy prediction.
The nucleon distribution $|\psi|^2$ in the Sun is calculated in the radius direction. (b) A cross-section of the Sun showing the regions referred to in the text.

Another method to determine the overlapping number $N$ follows from the multi-slit interference optics, the overlapping number $N$ is estimated by

$$N^2 = \frac{|\psi(0)|_{\text{multi-wavelet}}}{}^2 \simeq \frac{\rho_{\text{core}}}{\rho_{\text{gas}}} .$$

The solar core has a mean density of 1408 (kg/m$^3$), the surface of the sun is comprised of convection zone with a mean density of 2e-3 (kg/m$^3$) reported [10]. According to the above equation, suppose that $\rho_{\text{core}}=\rho_{\text{mean}}$, $N=650$, we obtain the gas density on the solar surface

$$\rho_{\text{gas}} = \frac{\rho_{\text{core}}}{N^2} \approx 3.3 \times 10^3 (kg/m^3) .$$

Calculation gives a correct estimation for the mean density of the surface gas, identified as the mean density of the convection zone.

3. Interference between photosphere and granule layer

In the frame of reference fixed to the solar system, the sun rotates at the origin with the angular speed $\omega$, consider unit volume of gas in the solar photosphere at the latitude angle $\lambda$, which contains $n$ molecules moving at the velocity $V=\omega r + V_{\text{wind}} + \nu$, where $r$ is the solar radius,
\( \mathbf{v} \) denotes the molecular velocity relative to the wind on the solar surface. Defining a set of local coordinates \((x,y,z)\) as shown in Figure 4(a), the collective generalized matter wave of the gas in the volume element can progresses only along the \(x\)-axis, while fails to develop in the \(y\)-axis and \(z\)-axis. Therefore, the collective matter wave per unit volume is given by

\[
\psi_{gas} = \sum_{j=1}^{n} \exp \left( \frac{i \beta}{c^3} \int_{0}^{x} \frac{V_j}{\sqrt{1 - V_j^2 / c^2}} \, dl - \frac{i \beta}{c^3} \int_{0}^{t} \frac{c^2}{\sqrt{1 - V_j^2 / c^2}} \, dt \right)
\]

\[= \sum_{j=1}^{n} \exp \left( \frac{i \beta}{c^3} \int_{0}^{x} V_j \, dl - \frac{i \beta}{c^3} \int_{0}^{t} c^2 \left(1 + \frac{V_j^2}{2c^2} \right) dt \right) \]

\[V_j = \mathbf{\omega} \times \mathbf{r} + \mathbf{v}_{wind} + \mathbf{v}_j\]

![Figure 4](image)

(a) traces of 5 molecules in the gas  
(b) global winding for spheroidization

Figure 4 (a) traces of 5 molecules in the gas element. (b) The shell’s matter wave winds up the solar spheroid on the north hemisphere, the global winding waves make the shell to become spheroidization.

\[
\psi_{gas} = \sum_{j=1}^{n} \exp \left[ \frac{i \beta}{c^3} \int_{0}^{x} \left( \mathbf{\omega} \times \mathbf{r} + \mathbf{v}_{wind} \right) \cdot d\mathbf{l} - \frac{i \beta}{c^3} \int_{0}^{t} c^2 \left(1 + \frac{\left( \mathbf{\omega} \times \mathbf{r} + \mathbf{v}_{wind} \right)^2}{2c^2} \right) dt \right]
\]

\[= \sum_{j=1}^{n} \left[ 1 + \frac{i \beta}{c^3} \int_{0}^{x} \mathbf{v}_j \cdot d\mathbf{l} - \frac{i \beta}{c^3} \int_{0}^{t} c^2 \left(2 \mathbf{\omega} \times \mathbf{r} + 2 \mathbf{v}_{wind} \right) \cdot \mathbf{v}_j + \frac{\mathbf{v}_j^2}{2c^2} \right] dt + O^2 + ...
\]

where the integral path takes on the collective mean path \(d\mathbf{l}\) of the gas in the volume element. Let \(v_{rms}\) denote the root-mean-square speed of the molecules, \(m\) expresses the mean mass of the molecules, \(k\) is the Boltzmann constant, \(T\) the temperature. Using the well-known knowledge
\[
\sum_{j=1}^{n} v_j = 0;
\]
\[
v_{\text{rms}}^2 = \frac{1}{n} \sum_{j=1}^{n} v_j^2;
\]
\[
v_{\text{rms}} = \sqrt{\frac{3kT}{m}}.
\]

we have
\[
\psi_{\text{gas}} = \exp \left[ \frac{i \beta}{c^3} \int_0^\infty (\omega r + v_{\text{wind}}) dl - \frac{i \beta}{c^3} \int_0^\infty \left( \frac{1 + \frac{\omega r + v_{\text{wind}}}{2}}{2c^2} \right) dt \right]
\]
\[
\left[ n - n \frac{i \beta}{c^3} \int_0^\infty c^2 \frac{v_{\text{rms}}^2}{2c^2} - O^2 + ... \right].
\]
\[
\approx n \exp \left[ \frac{i \beta}{c^3} \int_0^\infty (\omega r + v_{\text{wind}}) dl - \frac{i \beta}{c^3} \int_0^\infty c^2 \left( 1 + \frac{\omega r + v_{\text{wind}}}{2c^2} \right) dt \right]
\]

where the wind speed \( v_{\text{wind}} \) is defined as in the eastward direction. To meet the relativistic covariance, the collective matter wave is finally improved as
\[
\psi_{\text{gas}} = n \exp \left[ \frac{i \beta}{c^3} \int_0^\infty \sqrt{\left( \omega r + v_{\text{wind}} \right)^2 + v_{\text{rms}}^2} dl \right]
\]
\[
- \frac{i \beta}{c^3} \int_0^\infty c^2 \left( 1 + \frac{\omega r + v_{\text{wind}}}{2c^2} \right) dt \right].
\]

In fact, the eastward wind \( v_{\text{wind}} \) term also can absorb up any inadequacy in the approximation used, in other words, the eastward wind \( v_{\text{wind}} \) term represents all resident quantum effects that dose not be specified by the temperature-dependent \( v_{\text{rms}} \) term.

The sun has experienced a spheroidization process for a long term evolution before it becomes a spheroid. The solar shell is defined as the granule layer to bear the gas in the photosphere as shown in Figure 3(b), because of \( \rho = \rho(r) \), the solar granule layer as the shell is with the spherical symmetry. The shell's matter wave winds up the solar spheroid in a way illustrated in Figure 4(b) on the north hemisphere, the global winding matter waves make the shell to become spheroidization. Thus, the granule layer has a global orbital matter wave with the spherical symmetry which originates from the equatorial regions, is given by
\[
\psi_{\text{shell}} = \sqrt{\rho} \exp \left[ \frac{i \beta}{c^3} \int_0^\infty \left( \frac{\omega r^2 + v_{\text{rms, shell}}^2}{2} \right) dl - \frac{i \beta}{c^3} \int_0^\infty c^2 \left( 1 + \frac{\omega r^2 + v_{\text{rms, shell}}^2}{2c^2} \right) dt \right].
\]

Obviously, there is an interference between the shell \( \psi_{\text{shell}} \) and the gas \( \psi_{\text{gas}} \) at any point on the solar surface. The interference produces a beat phenomenon at latitude angle \( \alpha \) as
\[
|\psi |^2 = |\psi_{\text{shell}} + C|\psi_{\text{gas}}|^2 = 1 + C^2 + 2C \cos \left( \frac{2\pi}{\lambda_{\text{beat}}} \int_0^\infty dl - \frac{2\pi}{T_{\text{beat}}} t \right)
\]
\[
\frac{2\pi}{T_{\text{beat}}} = \frac{\beta}{2c^3} \left[ \omega^2 r^2 + v_{\text{rms, shell}}^2 - (\omega r + v_{\text{wind}})^2 - v_{\text{rms, gas}}^2 \right].
\]
\[
\frac{2\pi}{\lambda_{\text{beat}}} = \frac{\beta}{c^3} \left( \sqrt{\omega^2 r^2 + v_{\text{rms, shell}}^2} - \sqrt{(\omega r + v_{\text{wind}})^2 + v_{\text{rms, gas}}^2} \right)
\]
where \( C \) is the coupling constant, absorbed the \( \rho \) and \( n \), the \( v_{\text{wind}} \) expresses the eastward wind speed.
4. Determination of the solar granule temperature

In the solar convection zone, its constituents are 91.2% hydrogen by number and 0.087% helium by number, the mean mass of the molecules in thermal motion is estimated by

\[ m \approx 0.912m_{\text{hydrogen}} + 0.087m_{\text{helium}}. \]  

(16)

From the inner boundary of the convective zone to the visible surface, the temperature decreases from 1500000°K to 5778°K [11]. As we know, the granule layer as the shell would interfere with the gas in the photosphere to produce the sunspot cycles of 11 years period, according to the beat formula, we have

\[ T_{\text{beat}} \approx \frac{4\pi c^3}{\beta (v_{\text{rms,shell}} - v_{\text{rms,gas}})^2} = 11 \text{ (years)}. \]  

(17)

Assume that the temperature of the gas in the photosphere is 5800°K, we obtain the mean temperature of the granule layer as the shell at the calm equator region, these are

\[ T_{\text{gas}} \approx 5800 \, ^{\circ}\text{K}; \quad T_{\text{shell}} = T_{\text{granule}} \approx 8689 \, ^{\circ}\text{K}; \]

\[ v_{\text{rotation}} = \omega r = 2017 \, (m/s) \quad \text{(solar rotation)}; \]

\[ v_{\text{wind}} = 0 \quad \text{(calm)} \]  

(18)

The results agree well with the experimental observations [11]. This gravitational beat phenomenon of \(|\psi|^2\) turns out to be a nucleon density oscillation that undergoes to drive the sunspot cycle evolution.

The beat wavelength \(\lambda_{\text{beat}}\) is too long to observe, only the beat period is easy to be observed. Since we suppose that the wind is calm at the equatorial regions, the beat phenomenon at the equator becomes

\[ v_{\text{wind}} \approx 0 \quad \Rightarrow \quad \frac{\lambda_{\text{beat}}}{2\pi r} \approx 736 \]  

\[ |\psi|^2 \approx 1 + C^2 + 2C \cos \left( \frac{2\pi}{T_{\text{beat}}} t \right) \]  

(19)

This \(|\psi|^2\) oscillation is understood as the expansion and contraction of the granule mass density on an astronomical scale, which squeezes out the sunspots periodically.

5. Solar granule cell under destructive interference condition

The beat wavelength \(\lambda_{\text{beat}}\) is a function of latitude angle \(A\) on the north hemisphere, where the rotational circumference is \(2\pi R = 2\pi r \cos(A)\), we calculate out
For example, \( A=30^\circ \text{N} \), one beat wavelength winds up the local rotational circumference by \( N=746 \) cycles, as illustrated in Figure 5(a), the overlapped beat-wave at any point of the circumference should take the sum over all overlapped wavelets, similar to the Fabry-Perot interference formula in optics, that is

\[
\psi = \psi_0 \left( 1 + e^{i\delta} + e^{i2\delta} + e^{i3\delta} + \ldots + e^{i(N-1)\delta} \right) = \frac{1 - \exp(iN\delta)}{1 - \exp(i\delta)} \psi_0 .
\]  

(21)

Where \( \delta \) is the phase difference of one cycle. If the beat-wave runs in a solid matter as shown in Figure 5(b), then \( \delta=2\pi/N \) at everywhere on the circumference, where the orbit is under the condition of destructive interference for \( |\psi(\delta)|^2=0 \) everywhere, as illustrated in Figure 5(c). But gas has fluidity which leads the soft matter to converge to its favorite positions, causing a deformation of the \( \delta \) distribution on the circumference, as illustrated in Figure 5(d). Corresponding to the fluctuation of the \( \delta \) distribution, the granule mass density varies according to the density curve in Figure 5(c), some \( |\psi(\delta)|^2 \neq 0 \) being close to \( |\psi(\delta)|^2=0 \) repeatedly. Thus, granule cells appear on the top surface of convection zone.

Figure 5 (a) Wave winds around the sun. (b) On an orbit of solid matter, \( \delta=2\pi/N \) at everywhere on the circumference. (c) The overlapped beat-wave at any point of the circumference should take the sum over all
overlapped wavelets, similar to the Fabry-Perot interference formula in optics. (d) Gas has fluidity which causes a deformation of the $\delta$ distribution on the circumference.

Granules are small-scale cellular regions in the convection zone top, close to or in the photosphere, 500-1500 km in diameter, with several million covering the sun's entire surface except where are sunspots. They are convection regions of hot plasma that rise from the bottom of the convection zone and generally last for 5-20 min before cooling and turning dark near the edges and beginning to sink. Therefore, the destructive interference of the beat-waves plays the role of a knife through $|\varphi(\delta)|^2=0$ to cut out the boundaries of the granule cells. The size of a granule cell mainly depends on the fluid properties of the hot plasma gas.

In the above calculation, although this seems to be a rough model, there is an obvious correlation between solar radius, solar rotation, solar density, temperature distribution and the ultimate acceleration $\beta$.

6. Wind distribution on the solar surface

As we have known, the beat period $T_{\text{beat}}=11$ years dominates the low latitude regions and middle latitude regions, in these regions winds are used to maintain the constant period, the beat period formula is rewritten as

$$\text{Wind distribution on the solar surface}$$

$$\text{As we have known, the beat period } T_{\text{beat}}=11 \text{ years dominates the low latitude regions and middle latitude regions, in these regions winds are used to maintain the constant period, the beat period formula is rewritten as}$$
$$v_{\text{wind}} = \sqrt{\frac{\omega^2 r^2 - 4\pi c^2}{\beta T_{\text{beat}}} + (v_{\text{rms\_shell}}^2 - v_{\text{rms\_gas}}^2)} - \omega r \cos A \quad (22)$$

To note that the sunspot basin locates at $0^\circ\text{N} \sim 30^\circ\text{N}$ on the northern hemisphere, where the wind speeds almost equal to zero except sunspot vortices. It was found that the granule temperature may influence the sunspot basin location, for instance, equatorial $T_{\text{granule}}=8650^\circ\text{K}$, keeping on the constant beat period $T_{\text{beat}}=11$ years, the required winds as a function of latitude angle $A$ is calculated out in Figure 6(a), as indicated by the blue line, the sunspot basin with calm wind is at $A=17^\circ\text{N}$.

In the north polar regions, the solar geomagnetic field flips per 22 years, which corresponds to a beat phenomenon with the period of $T_{\text{beat}}=22$ years in the polar regions, where the $v_{\text{rms}}$ term also decreases dramatically due to the polar singularity. It is estimated, the $v_{\text{rms\_shell}}^2 - v_{\text{rms\_gas}}^2$ term decreases to 39% relative to that at the equatorial regions. Keeping on the constant beat period $T_{\text{beat}}=22$ years on the north polar regions, the required winds as a function of latitude angle $A$ is calculated out in Figure 6(a), as indicated by the brown line.

The red poly-line indicates the prediction of the eastward winds on the north hemisphere. Taking into account of longitude winds and radial winds, Figure 6(b) shows the 3D wind distribution on the solar surface (granule layer).
7. Sunspot basins

In the Figure 6, the latitudes \( A=17^\circ N \) and \( A=73^\circ N \) have zero winds, called as the first constructive ridge and second constructive ridge respectively. A ridge is defined as a calm latitude sandwiched by eastward winds and westward winds. The first constructive interference ridge, where the shear action of the winds will produce a lot of vortexes if the winds are disturbed by the convective activity. The pregnancy of a sunspot needs three steps, as shown in Figure 7 (a).

Step1, during an active season, the stronger radiation outputs at higher altitudes over the warm surface and releases the \( v_{tem} \) into the gas at lower altitudes; consequently, the strengthened easterlies make a distortion to the first constructive interference ridge, as shown in Figure 7 (a).

Step2, day by day, the distortion develops into an extent that it is going to separate from the mother-like first constructive interference ridge hit strongly by the westerlies, as shown in Figure 7 (a).

Step3, finally, the distorted constructive interference ridge grows up to become an isolated baby ring which is recognized as a new sunspot, counter-clockwise in the northern hemisphere, as shown in Figure 7 (a), similar to the famous large vortex on the surface of Jupiter.

Pregnancy mechanism of sunspots likes tropic cyclones on the Earth. Actually, the sunspot basins latitude vs. time as shown in Figure 7 (b) tells us that sun’s constructive interference ridge swings between \( 0^\circ N \) and \( 30^\circ N \) in the northern hemisphere, so do in the southern hemisphere.

![Figure 7](a) Three steps of the pregnancy of a sunspot. (b)Sunspot basins: latitudes vs. time.

Like tropic cyclones on the Earth [12], when the constructive interference ring of a newly born cyclone forms, its wavelength will adapt to the ring size as

\[
\frac{1}{hM_{cyclone}} \int_{L} v_{r}dl = 2 \pi n; \quad n = 1,2,...
\]

\[
\psi = \exp\left[ \frac{i}{hM_{cyclone}} \int_{0}^{r} (u_{1}dx_{1} + u_{2}dx_{2} + u_{3}dx_{3} + u_{4}dx_{4}) \right]
\]

(23)
where the mass $M$ represents the total mass of the new cyclone, including the imagined dark mass which accounts for the latent heat released during its formation; the constant $h$ is the Planck-constant-like constant determined by experimental observations. Tropic cyclones concerns with the same quantum gravity theory, see ref. [12,13].

8. Conclusions

Matter wave has been generalized on a planetary scale as a quantum gravity theory, and applied to solar science. It was found that celestial size is the consequence of interference of the generalized matter waves, the sunspot cycle 11 years represents the beat-period of the interference. By the quantum gravity theory, the mean density of convection zone is determined as 3.3e-3 kg/m3, the mean temperature of the granule layer is calculated out to be 8689°K. The solar surface winds are investigated, the sunspot basins locate at $0^{\circ}N \sim 30^{\circ}N$ on the northern hemisphere, where the wind speeds almost equal to zero.

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