Invariant polynomial

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Abstract:

for centuries, mathematicians have been studying polynomials, especially the zeros of polynomials. the theory of Galois states that we cannot find a general formula for solving equations greater than 4.

in this article I study the invariant polynomials of degrees 6,8 and 10.

when we make a variable change to these polynomials, they become two-square. which allows us to solve equations of higher degree.

Let be the polynomial P(x) of six degree , with real or complex coefficients.

$$P(x)=\lambda_1 x^6+\lambda_2 x^5+\lambda_3 x^4+\left(\frac{2\lambda_2\lambda_3}{3\lambda_1}-\frac{5\lambda_2^3}{27\lambda_1^2}\right)x^3+\lambda_3 x^2+\left(\frac{\lambda_2\lambda_5}{3\lambda_1}-\frac{\lambda_2^3\lambda_3}{27\lambda_1^3}+\frac{\lambda_2^5}{81\lambda_1^4}\right)x+\lambda_7$$

This polynomial is **invariant**, because if we make a change of variable $x=X+t$

We obtain a polynomial:

$$P(X)=k_1X^6+k_2X^5+k_3X^4+\left(\frac{2k_2k_3}{3\lambda_1}-\frac{5k_2^3}{27\lambda_1^2}\right)X^3+k_3X^2+\left(\frac{k_2k_5}{3\lambda_1}-\frac{k_2^3k_3}{27\lambda_1^3}+\frac{k_2^5}{81\lambda_1^4}\right)X+k_7$$

Example 1 :

$$P(x)=x^6+3x^5-2x^4-9x^3-4x^2+x-2$$

$$\lambda_1=1;\lambda_2=3;\lambda_3=-2;\frac{2\lambda_3\lambda_2}{3}-\frac{5\lambda_2^3}{27}=-9;\frac{\lambda_2\lambda_5}{3\lambda_1}-\frac{\lambda_2^3\lambda_3}{27\lambda_1^3}+\frac{\lambda_2^5}{81\lambda_1^4}=-4;\lambda_5=1;\lambda_7=-2$$

let's do the variable change $x=t-2$ we obtain:

$$P(t-2)=t^6-9t^5+28t^4-33t^3+2t^2+21t-12$$

$$k_1=1;k_2=-9;k_3=28;\frac{2k_2k_3}{3}-\frac{5k_2^3}{27}=-33;k_5=2$$

$$\frac{k_5k_2}{3}-\frac{k_3k_2^3}{27}+\frac{k_2^5}{81}=21;k_7=-12$$

If we make the change of variable: $x = X - \frac{\lambda_2}{6\lambda_1} = X - \frac{1}{2}$, Or: $t = T - \frac{k_2}{6k_1}$

We obtain a two-square polynomial.

$$P(X) = X^6 - \frac{23}{4}X^4 + \frac{59}{16}X^2 - \frac{165}{64}$$

<u>**Example 2:**</u>

$$\lambda_1 = 1; \lambda_2 = 3; \lambda_3 = -2; \lambda_5 = 4; \lambda_7 = -5$$
$$P(x) = x^6 + 3x^5 - 2x^4 - 9x^3 + 4x^2 + 9x - 5$$

Let's do the variable change: $x = X - \frac{1}{2}$

$$P(X) = X^6 - 5.75X^4 + 11.6875X^2 - 7.578125$$

<u>**Example 3**</u>:

$$\lambda_1 = 3; \lambda_2 = 18; \lambda_3 = -4; \lambda_5 = 2; \lambda_7 = 8$$
$$P(x) = 3x^6 + 18x^5 - 4x^4 - 136x^3 + 2x^2 + 324x + 8$$

Let's do the variable change: $x = X - 1$

$$P(x) = 3x^6 - 49x^4 + 251x^2 - 205$$

<u>**Invariant Polynomial P(x) of 10 degree ,with real or complex coefficients**</u>

$$Q(x)=x^{10}+\lambda_2x^9+\lambda_3x^8+(\frac{4\lambda_3\lambda_2}{5}-\frac{30\lambda_2^3}{5^3})x^7+\lambda_5x^6+(\frac{3\lambda_5\lambda_2}{5}-\frac{14\lambda_3\lambda_2^3}{5^3}+\frac{126\lambda_2^5}{5^5})x^5$$
$$+\lambda_7x^4+(\frac{2\lambda_7\lambda_2}{5}-\frac{5\lambda_5\lambda_2^3}{5^3}+\frac{28\lambda_3\lambda_2^5}{5^5}-\frac{255\lambda_2^7}{5^7})x^3+\lambda_9x^2+$$
$$(\frac{\lambda_9\lambda_2}{5}-\frac{\lambda_7\lambda_2^3}{5^3}+\frac{3\lambda_5\lambda_2^5}{5^5}-\frac{17\lambda_3\lambda_2^7}{5^7}+\frac{155\lambda_2^9}{5^7})x+\lambda_{11}$$

If we make the change of variable: $x=X-\frac{\lambda_2}{10}$

We obtain a two-square polynomial.

<u>Invariant Polynomial P(x) of 12 degree ,with real or complex coefficients</u>

$$Q(x)=x^{12}+\lambda_2x^{11}+\lambda_3x^{10}+(\frac{5\lambda_3\lambda_2}{6}-\frac{55\lambda_2^3}{6^3})x^9+\lambda_5x^8+(\frac{4\lambda_5\lambda_2}{6}-\frac{30\lambda_3\lambda_2^3}{6^3}+\frac{396\lambda_2^5}{6^5})x^7$$
$$+\lambda_7x^6+(\frac{3\lambda_7\lambda_2}{6}-\frac{14\lambda_5\lambda_2^3}{6^3}+\frac{126\lambda_3\lambda_2^5}{56}-\frac{1683\lambda_2^7}{6^7})x^5+\lambda_9x^4+$$
$$(\frac{2\lambda_9\lambda_2}{6}-\frac{5\lambda_7\lambda_2^3}{6^3}+\frac{28\lambda_5\lambda_2^5}{6^5}-\frac{255\lambda_3\lambda_2^7}{6^7}+\frac{3410\lambda_2^9}{6^9})x^3+\lambda_{11}x^2+$$
$$(\frac{\lambda_{11}\lambda_2}{6}-\frac{\lambda_9\lambda_2^3}{6^3}+\frac{3\lambda_7\lambda_2^5}{6^5}-\frac{17\lambda_5\lambda_2^7}{6^7}+\frac{155\lambda_3\lambda_2^9}{6^9}-\frac{2073\lambda_2^{11}}{6^{11}})x+\lambda_{13}$$

If we make the change of variable: $x=X-\frac{\lambda_2}{12}$

We obtain a two-square polynomial