Beyond Archimedes: A Pythagorean Theorem Solution to Pi as Lone Unknown Triangle Side

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***Abstract.* We explore the practical application of isoperimetric inequality ($L^2 \geq 4\pi A$) to classical methods of circle measurement. Exampling Archimedes' n-gon approach, we compare it to a real-world kinematic scenario of a unit diameter circle rolling on a flat plane surface. Using annular geometry, we demonstrate that π can be derived algebraically by solving for the linear distance its centre travels per full revolution. Unlike exhaustive methods involving non-circular figures, our annular approach begins with isoperimetric equality by deriving a right triangle (with π its lone unknown side) & applying the Pythagorean theorem to it. This algebraic approach to π reveals unexpected yet significant connections between it and the golden ratio. We further explore more assumptions underlying 3.14159... discovering its embedment in an unbounded plane to be catastrophic & remedy with a bounded one. Finally, we close with a fresh new perspective on the notoriously unsolved Riemann Hypothesis problem. Our result suggests both a need for physical experimentation, as well as a need to re-evaluate the general reliability of non-circular methods in rigorously bounding and/or converging on the circle constant π.**

1 Introduction & Background

The 'isoperimetric inequality', a fundamental property of plane geometry, was rigorously proven in 1901 by A. Hurwitz [1]. It states that for any curved length L enclosing an area A, $L^2 \geq 4\pi A$, with equality holding only for perfect circles. Hurwitz's proof, using Fourier series applied to arbitrary rectifiable curves, subsumes Archimedes' n-gons, making them equivalent in isoperimetric contexts.

This inequality presents a compelling paradox: given its invariable application to all non-circular plane figures, how could Archimedes' method (or any classical method) ever converge to π? By critically examining Archimedes' n-gon approach in a kinematic environment, this paper aims to reconcile this apparent contradiction.

Despite the inequality's retroactive consequences for classical approaches to circle measurement, its implications seem largely unexamined. We posit this oversight stems from the classical assumption that as $n \to \infty$, the perimeter of the non-circular figure trivially approaches π. We demonstrate why this assumption, dating back to Archimedes, is fundamentally unsound.

Our kinematic model provides a clear mathematical foundation for circle measurement, verifiable through real-world experimentation with circular objects. This approach not only resolves the paradox inherent in classical methods, but also opens new avenues for understanding circular geometry in both abstract and practical contexts.

2 Scientific Process

We observe 'science' as an ongoing process of discovery via. inquiry through which *unknowns* become *knowns*. This process is fundamentally driven by our willingness to question our most deeply embedded assumptions, beliefs & conclusions hitherto held to be true.

Accordingly, we challenge the foundational assumptions underlying the many classical methods of circle measurement. We search for any conceivable circumstances—including any & all highly unbelievable ones—under which any one or more assumption(s) could potentially be *false*. This methodology of actively seeking falsification serves as a guiding principle for escaping debilitating misapprehensions.

3 Questioning Archimedes' Assumption of Entrapment

Over two millennia ago, in his highly influential work 'CIRCVLI DIMENSIO' (*trans.* 'Measurement of a Circle',) Archimedes made a tacit assumption in his third postulate [2]. We can extract this assumption in general form from numerical bounds he proposes: **3 1/7 > π > 3 10/71** with π assumed to be the circumference of a circle whose diameter is numerically 1 unit:

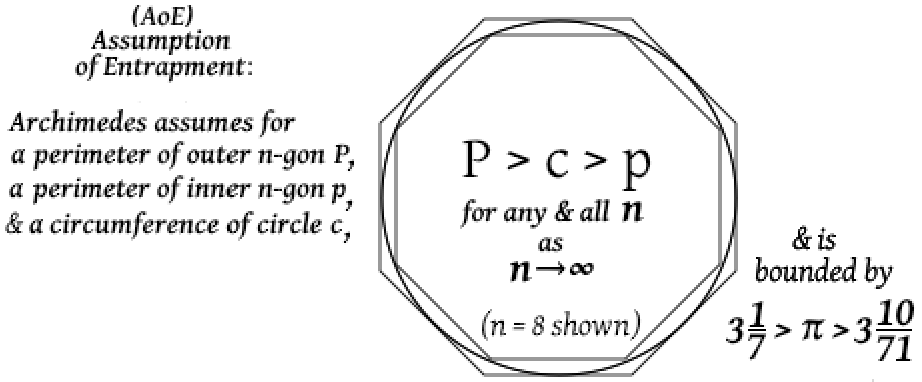

Figure 1: Archimedes assumes the perimeter of a circumscribed polygon must always be numerically greater than the circumference of the circle it contains. This assumption was made without rigorous regard for isoperimetric inequality, following it remained unproven for millennia thereafter. With the inequality now proven, it begs for a retroactive application(s) of it to any & all classical methods.

The AoE presents a significant challenge to isoperimetric inequality. This discrepancy becomes particularly apparent when considering the behaviour of the centre of each figure (both circle & non-) as they roll along a flat plane surface.

By critically examining the AoE in light of both the isoperimetric inequality and a real-world kinematic scenario, we lay the groundwork for a more rigorous approach to circle measurement.

3.1 A Kinematic Scenario

For a unit diameter circle rolling on a flat plane, its centre travels a linear distance of precisely π per revolution. Crucially, owing to its radius r being constant, the centre translates linearly along the x-axis without any vertical displacement along the y-axis. This contrasts sharply with that of any non-circular figure.

The distinction has profound implications for circle measurement and derivations of π which do not assume a kinematic frame of reference involving fluid motion:

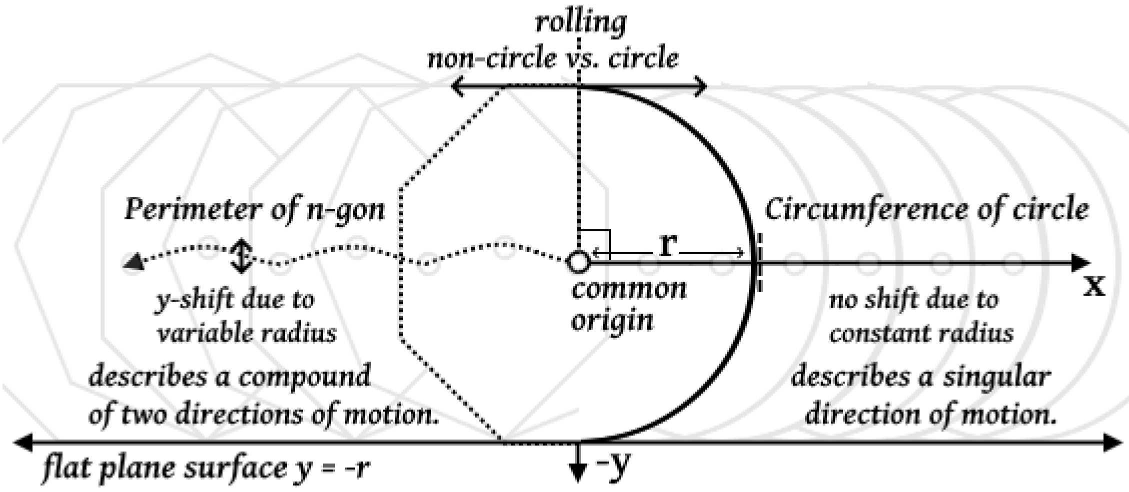

Figure 2. A sketch conveys isoperimetric inequality from a different point of view. For only a perfect circle does its centre translate linearly along the x-axis. In contrast, for any non-circular figure, its centre translates along both x- & y-axes due to the variability of its circumradius.

A circle's constant radius ensures its centre translates linearly, while an n-gon's varying radius produces non-linear wave motion. We term the latter 'y-shift': any non-zero translation(s) orthogonal to primary motion. This y-shift is inherent to all non-circular shapes & is never fully neutralized (even as $n \to \infty$) with there always being some non-zero residual shift.

Allowing $n \to \infty$ eschews exact divisions like π, π/2, π/4 *etc.* for an infinitely complex approximation $\pi/n \to \infty$, obscuring the circle's fundamental nature as a finite curve. The persistent y-shift causes a critical disconnect between limit-based circle definition and its intrinsic geometric and kinematic properties. Because y-shift persists as $n \to \infty$, each evaluation inevitably introduces isoperimetric inequality.

Consequently, all non-circular methods for bounding or estimating π are invalid. This insight, emerging from kinematic analysis of fluid motion, leads us to approach the circle not as composed of an infinite number of points, but as a binary: one static point and one rotating point with the latter maintaining a constant radius from the former. This kinematic conception ensures radial constancy and isoperimetric equality, replicating the translation of a circle's centre without any y-shift associated.

Considerations of primary (x-axis) & secondary (y-shift) motion of a plane figure's centre fundamentally challenges conventional π estimation approaches, underscoring the critical importance of kinematic considerations in geometrical analysis.

4 Foundations of Isoperimetry

In establishing our geometric constructions, we explicitly state our primary assumptions while consciously avoiding the classical assumptions of *calculus, trigonometry, limits, infinitesimals,* & *infinities*. Though widely useful in other applications, these tools carry assumptions (akin to the AoE) made prior to rigorous considerations of isoperimetric inequality.

Our foundation rests on the following carefully chosen principles:

1. Euclidean Geometry: We adopt the basic axioms of Euclidean geometry, including the parallel postulate, the concept of congruence, and the properties of straight lines.
2. Properties of Basic Shapes: We assume the standard properties of squares (four equal sides, four right angles) and circles (all possible points equidistant from a centre point.)
3. Euclidean 3-Space: While our constructions occur in a 2D plane, we acknowledge Euclidean 3-Space as a valid continuation of our plane. Consequently, all 2D relations presented are conformal to their 3D Euclidean extensions.
4. Bounded Continuity: We assume continuity only within bounded regions. Specifically, a bounded region is any squarely enclosed region of known size relative to a given unit of 1 clearly represented on the plane. This bounded approach observes the circle constant π as squarely contained by only a single discrete unit of perimeter 4 & area 1. Unbounded continuity unbinds the finite region(s) outside the circle from inside the square containing it, with the area of the latter bound to be no more or less than 1.
5. Measure: Given a clearly represented unit of 1 on the plane, we assume the existence of a consistent measure of length, area, and volume. Integers (1, 2, 3... *etc.,*) their respective roots ($\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$... *etc.*) & their powers (along with ratios of them) may all describe discrete geometric lengths, areas, and/or volumes if & when so equated.
6. Basic Equations: We accept as valid the equations: $2\pi r$ for the circumference of a circle, πr^2 & $\pi(R^2 - r^2)$ for the area of a circle & annulus *resp.,* as well as the Pythagorean theorem $a^2 + b^2 = c^2$.

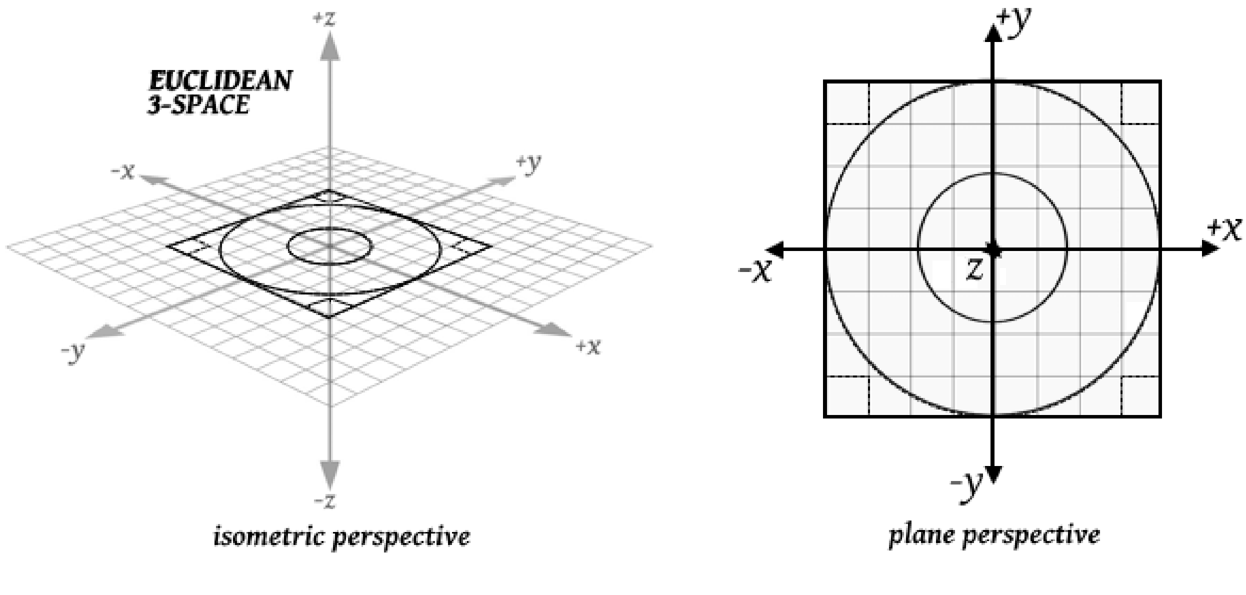

Figure 3. Euclidean 3-Space (left) is shown as a continuation of 2-Space (right).

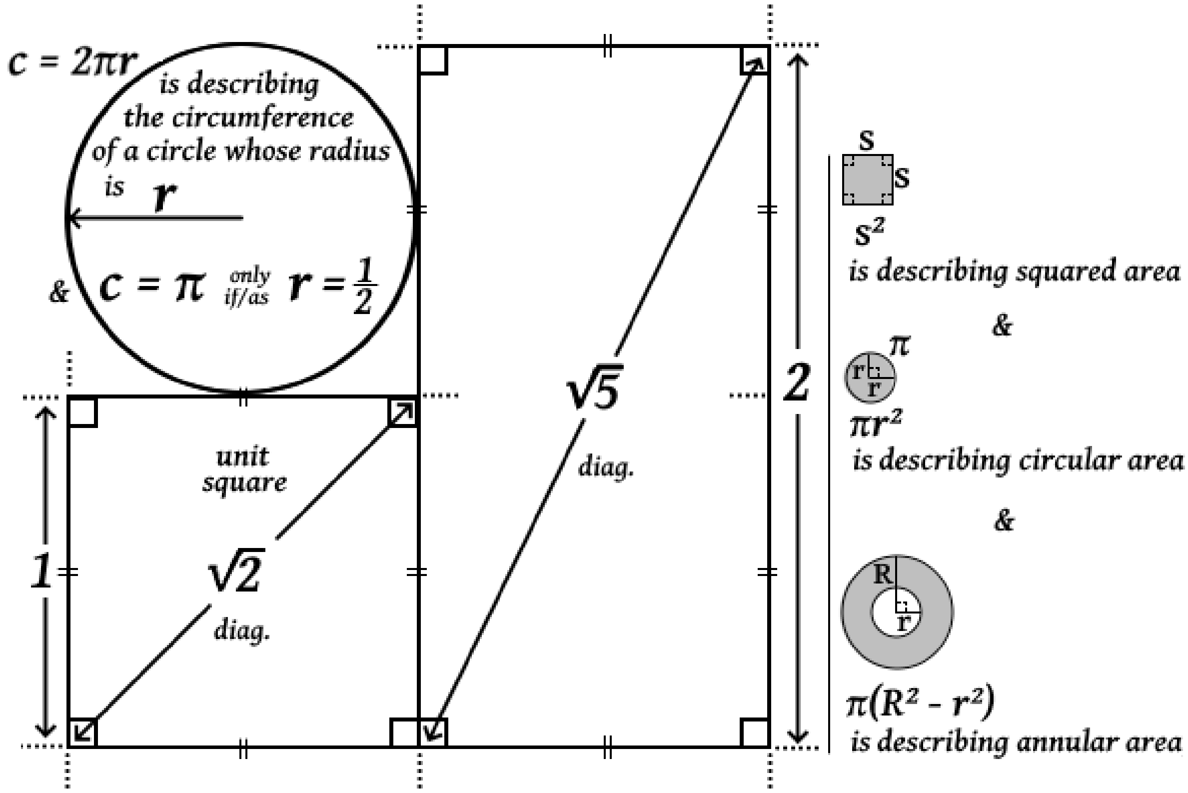

Figure 4. A summary of geometric assumptions as a foundation for isoperimetry. Each rectangle is composed of two right triangles whose diagonal lengths are known via. the Pythagorean theorem.

These foundations are carefully chosen to provide a simple yet rigorous foundation in addressing the isoperimetric inequality.

5 Geometric Construction & Constraints

While Archimedes' approach to circle measurement began by inscribing and circumscribing n-gons of $n = 4$ & then increasing n beyond 4, our method diverges significantly. Mindful of the isoperimetric inequality, we both begin and end with $n = 4$ such that $n(\pi/4) = \pi$. This constraint limits our use of plane figures to four-sided or fewer.

Our approach adheres strictly to $n = 4$ in order to observe the containment of $\pi/4$ as owing to one of four sides of a unit square. We posit that $n = 4$ is not merely sufficient but requisite to entirely reconcile the circle. Crucially, $n = 4$ allows $\pi/4$ to be numerically contained to a single side of a right triangle. Instead of assuming there is such a thing as an "infinitely-sided" figure in reality, we approach & treat the circle as a discretely four-sided object whose centre translates linearly a length of $\pi/4$ per quarter revolution. Our goal is to isolate π on one unknown side of a right triangle thus enabling an algebraic solution for π using the Pythagorean theorem. This approach observes the constraint $n = 4 = 8r$ for $r = 1/2$ as being the perimetric boundary of the unit square containing the circle whose circumference is π.

We begin without assuming an x- and y-axis, but justify their introduction based on the properties of a perfect square. Given that both diagonals of any perfect square intersect orthogonally at its centre, we establish our axis accordingly:

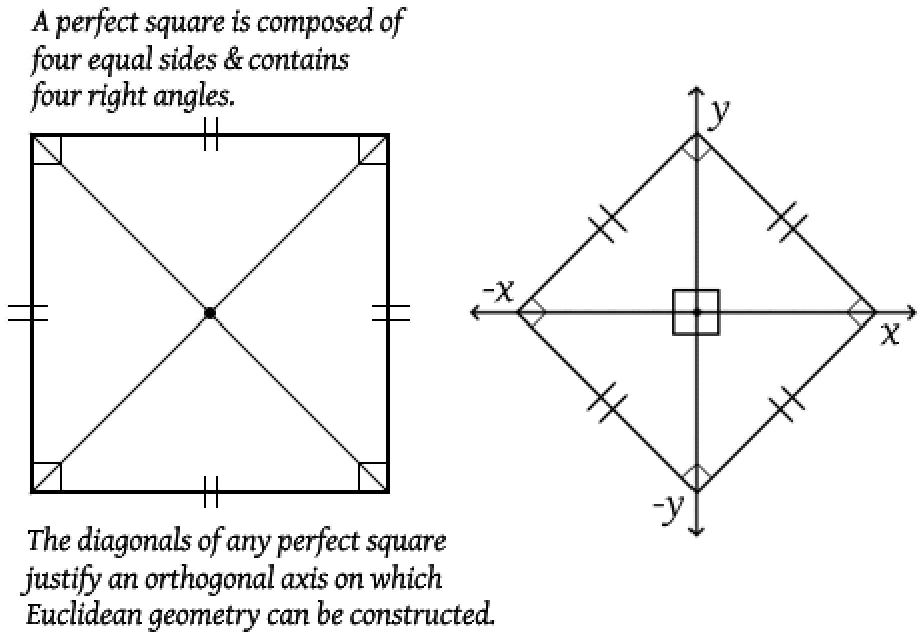

Figure 5. An orthogonal x- & y-axis is justified according to the diagonals of a perfect square.

This construction forms the foundation for our subsequent analysis. By adhering strictly to n = 4 and leveraging the properties of right triangles, we aim to provide a more rigorous and geometrically consistent method for understanding circular geometry.

5.1 Unit Square Geometry

The axis we derived is rooted in the diagonals of a perfect square, inheriting the characteristic four right angles of that square. Until now, we abstained from numerals because the numerical size of the square was irrelevant. The ratio of any square's side to its diagonal is a constant $1/\sqrt{2} = \sqrt{2}/2$ regardless of the square's size. We find this important ratio depicted inside both the unit square & circle (as a chord) expressed as a finite numerical length $1/\sqrt{2} = \sqrt{2}/2$:

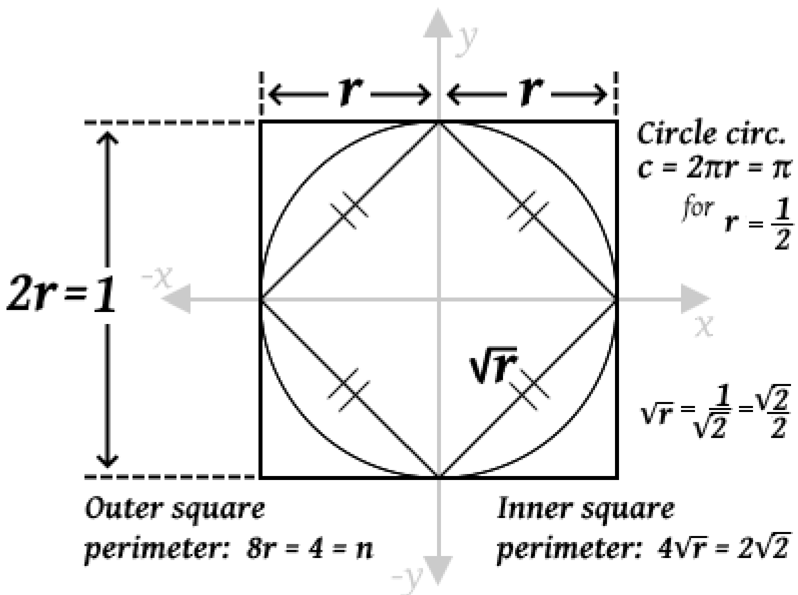

Figure 6. A unit diameter circle is shown with inscribed & circumscribed squares, the former of which has a side length ($\sqrt{r} = 1/\sqrt{2}$) equal to the square root of the radius of π.

5.2 ULANE: The Necessarily 'Real Element'

We express perimeters in terms of r because r is numerically known to be 1/2 and intrinsically tied to the circle whose circumference we aim to algebraically solve for. Mathematically, we observe an implicit $2r / (\pi/4) = 8r/\pi = 4/\pi$ relation. This ratio directly compares the perimeter of a unit square $4 = n$ to the circumference of the largest possible circle it contains $c = \pi$.

A critical observation emerges: the side length of the inner square $\sqrt{r} = 1/\sqrt{2}$ is equal to the square root of the radius of π. This relationship has profound implications:

- The inner square's four vertices all fall directly on the circle's circumference at four orthogonally related points, dividing π into four equal quarters,
- These four vertices represent four *real,* tangible points on the outer circumference of a *real* objective circle the mathematical model is meant to describe, &
- The size of this square is absolute with respect to a given unit.

For numerical units:

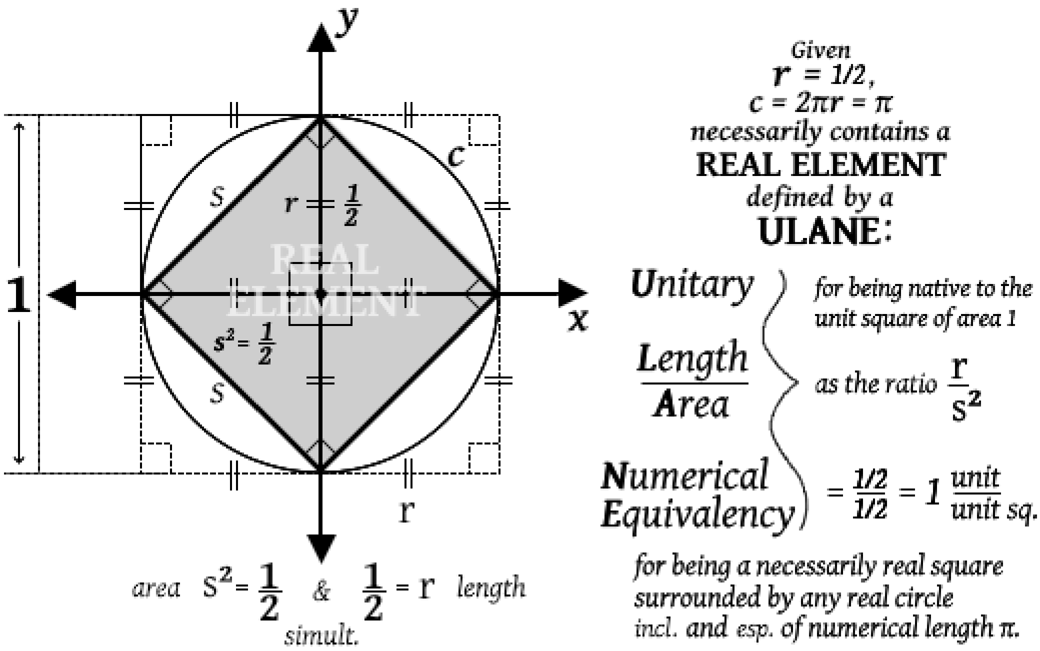

Figure 7. The 'real element' is defined by a numerical ratio of $r/s^2 = 1/2 / 1/2 = 1$ unit/unit *sq.* & termed a unitary length/area numerical equivalency 'ULANE' accordingly.

The ULANE is crucial because it acknowledges underlying geonumerical agreements between π and the area of the largest possible square it contains: because π has a constant radius of length 1/2, it must also necessarily surround the square of numerical area 1/2.

The 'real element' therefore reflects an underlying reality: any real objective circle composed of any real material(s) necessarily contains a real objective square whose vertices are four real points on the circle's circumference. We therefore treat these vertices as real points owing to a necessarily real square geometrically contained by π. This concept of what constitutes 'real' allows us to use the vertices to determine precisely where they fall on a flat plane. First, we finish analyzing the circle by comparing its circ. length to the area it contains:

5.3 A Self-Referential Pi

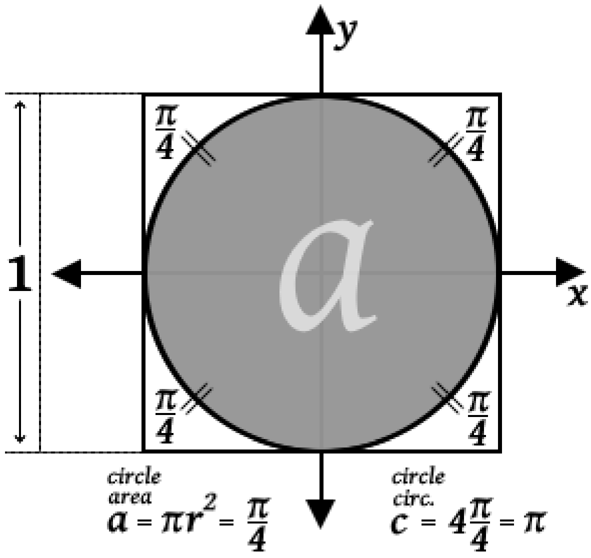

Figure 8. Circumferential length π contains π/4 area implying π is *self-referential*.

Given the area of a circle described by $a = \pi r^2 = \pi/4$ whose $r = 1/2$, we find its area to be numerically equivalent to one quarter of its own circumference π/4. As with the radius of π ($r = 1/2$) being numerically equal to the area of the largest possible square π contains ($s^2 = 1/2$), its circumference is numerically equal to four times *all* area it contains. This means given length π/4 represented in any one quadrant of four, it has a numerically equivalent area $a = \pi/4$ simultaneously represented on the same plane.

5.3.1 An Annular Pi

If we associate the area $a = \pi/4$ with one quarter revolution of the circle π/4, we may know if & when the circle has revolved once by multiplying the area by $n = 4$ & by placing $4a = \pi$ outside the circle in the form of an annulus. We may then represent the area associated with one full revolution such to satisfy $\pi/\pi = 1$ unit/unit *sq.* thereby satisfying ULANE. We use the area of an annulus equation $\pi(R^2 - r^2)$ to find the radius of the major which contains $4a = \pi$ between it & the minor of known $r = 1/2$. We equate it to π & solve for the unknown:

$$\pi(R^2 - r^2) = \pi$$

& given $r = 1/2$ is known,

$$(R^2 - (1/2)^2) = \pi/\pi$$

$$R^2 = 1 + 1/4$$

$$R^2 = 5/4$$

$$R = \sqrt{5}/2$$

5.3.2 A Golden Ratio

We recognize this ratio $\sqrt{5}/2$ from the golden ratio. This result implies the uniform width of the π annulus (simultaneously representing both π length & π area) is $w = R - r = \sqrt{5}/2 - 1/2 \approx 0.618...$ the inverse (or conjugate) of the golden ratio referred to as phi $\Phi = \sqrt{5}/2 + 1/2 \approx 1.618...$ but what is the golden ratio conjugate doing hiding in/as the π annulus' width?

The arc length π we wish to solve for is numerically equal to the annulus' area, with the width needed to capture that area equivalent to the golden ratio conjugate. The quadratic $x^2 - x = 1$ describes a geometric progression involving the radii of this annulus: we find x = r + R, r – R for r = 1/2 and R = √5/2. This quadratic defines the unit in terms of the golden ratio.

5.3.3 The Golden Annulus of Pi (GAP)

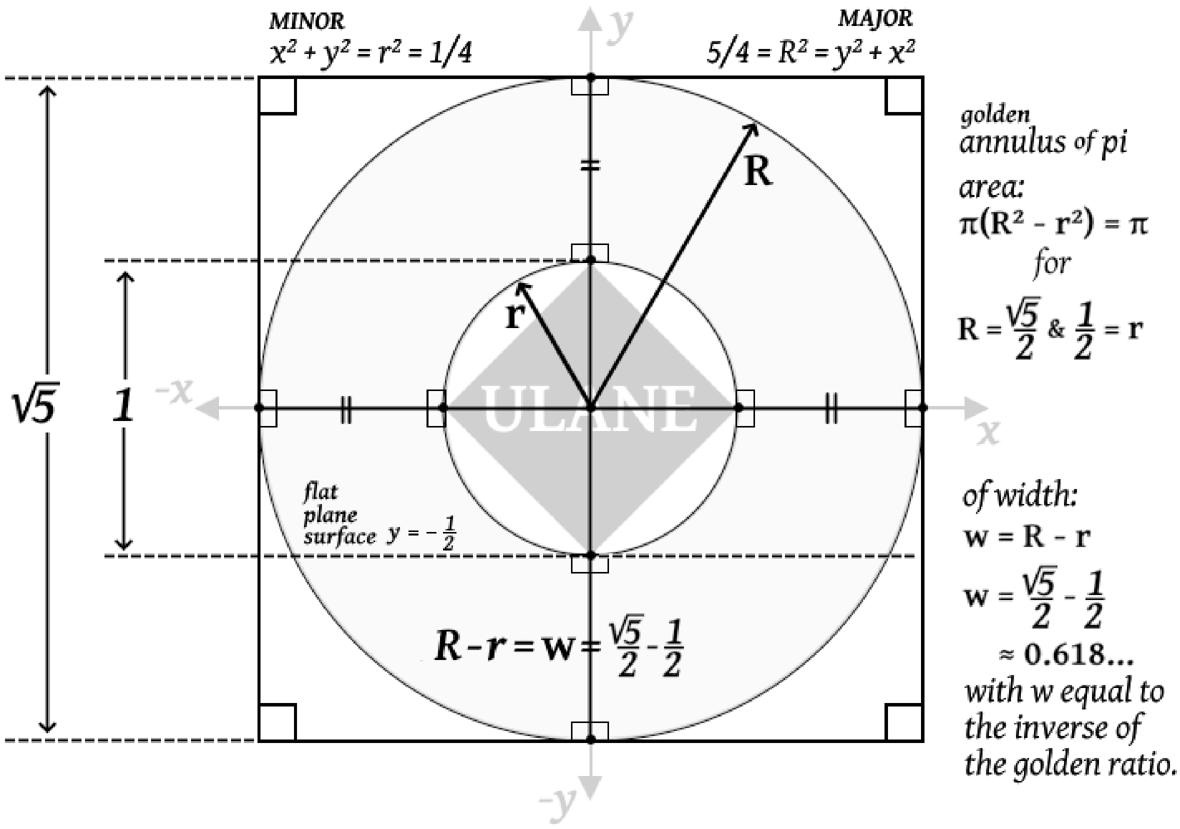

Figure 9. The golden annulus of pi (GAP) whose radii r + R sum to Φ & whose width w = 1/Φ = Φ - 1. After w sweeps each of four quadrants, area π is contained. Given this clear geometric relationship between the two universal constants Φ & π, this annulus embodies a major 'gap' in our understanding.

The orthogonality of w with respect to each of the four lengths π/4 is extremely important. If the minor circle were resting on a flat plane y = - r, w is a known length orthogonal to that plane. Crucially, both the circumference of the circle & w passes through the vertices of the real element termed ULANE. This configuration not only unifies π and Φ but also provides a tangible, measurable link between abstract mathematical constants and physical space. Specifically, assuming the surface area of a sphere is $4\pi r^2 = \pi$, the annulus $\pi = \pi(R^2 - r^2)$ for a common r = 1/2 geometrically surrounds it with an equal amount of surface area. This could & would not be possible if not for w.

(Before proceeding, we wish to state the π annulus & its relation to the golden ratio Φ stands on its own merits independent of any & all considerations of what follows. Regardless of what π numerically is, one need not commit to a numerical value to see the radii of its annulus is based on the proportions of the golden ratio Φ. This relation is deserving of further inquiry if even 3.14159... is assumed unaffected by isoperimetric inequality.)

5.3.4 Figuring Out Isoperimetric Equality

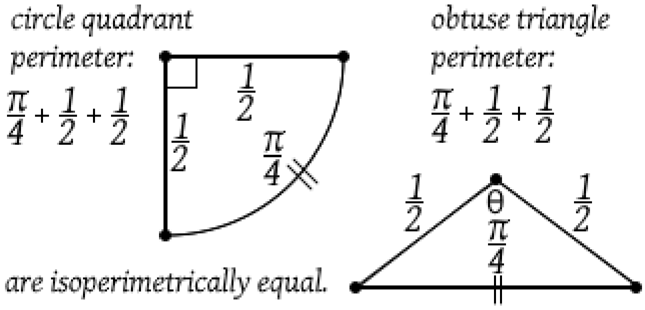

Figure 10. A circle quadrant is flattened to become an obtuse triangle of unknown apex angle whose perimeter is equivalent to that of the circle quadrant. The base length is equivalent to the distance travelled by the centre of a unit diameter circle per quarter revolution.

Our objective is to geometrically contain π length to one side of a right triangle whose other sides are known, enabling an algebraic solution for π. The obtuse triangle of isoperimetric equality (perimeter π/4 + 1/2 + 1/2) binds π/4 by the same 2r = 1 constriction, neither less nor more because the perimeter containing π is bound to be 4 = 8r = n.

5.3.5 Extensions of Isoperimetric Equality

Given the width of the annulus, we begin with it as a known length in known relation to π length & areas. We then plot π/4 as two possible orthogonal extensions of w (both linear & curved) noting the precise point at which the linear representation of π terminates is unknown (q' shown below:)

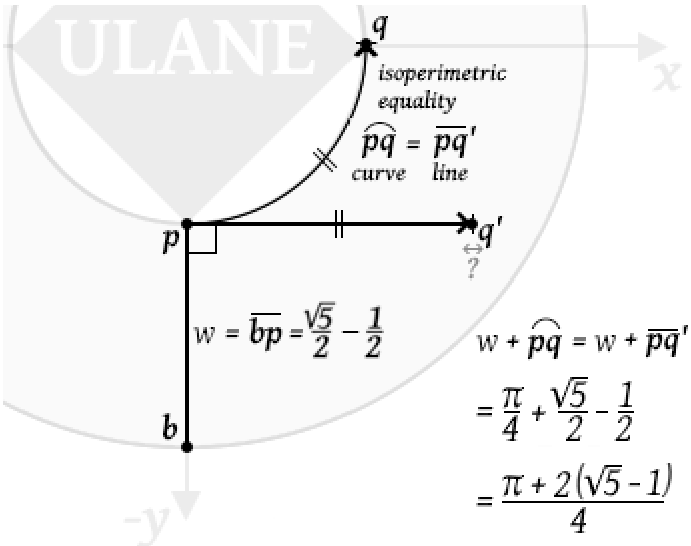

Figure 11. Linear and curved representations of π/4 plotted as extensions of known length w, with q' at an unknown location along the plane. Determining point q' is critical.

5.3.6 A Critical Line & Question

Importantly, points p & q (& therefore q') are all real points corresponding to the vertices of the geometric square π length surrounds. Because q is a real point on the circumference, it will at some point contact the plane which we represent as q'. Accordingly, knowledge of the precise location of q' along y = -1/2 is critical.

Is it possible to find the precise location of q'?

5.3.7 A Double-Bind

Recall curve pq is contained & therefore bound by two orthogonal radii each r = 1/2. If this is true for arc length π/4 & the annulus' quadrant area is numerically equivalent to this, the annulus must also be bound by the very same numerical constriction 2r = 1. As with curve π/4, we similarly bind the linear representation of π/4 by extending length 2r = 1 from the outer extreme of the annulus (b) back towards the plane, binding q' of line pq' to 2r = 1:

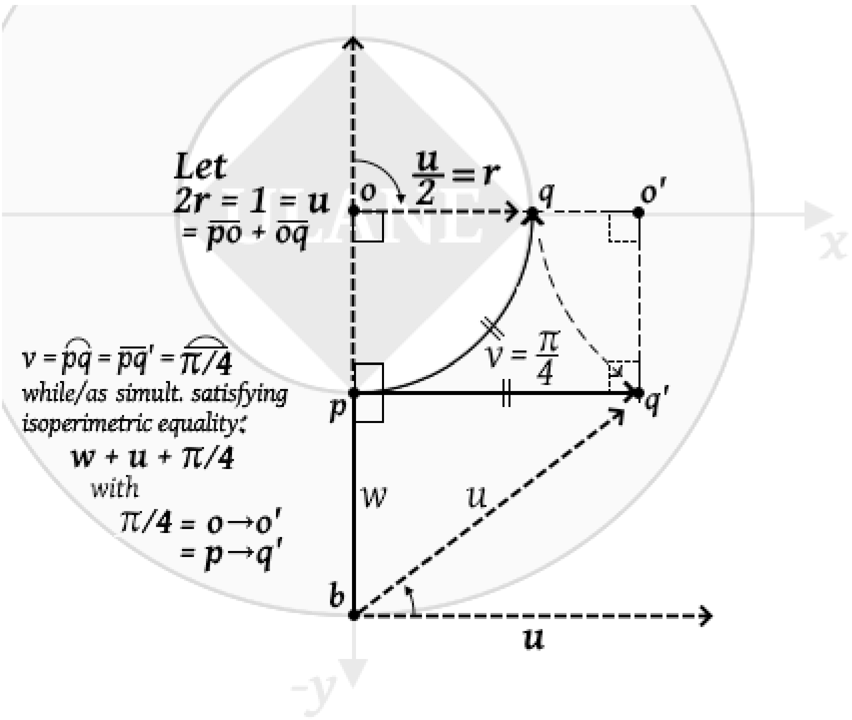

Figure 12. By extending u from either end of the annulus' width w, it is possible to simultaneously enclose both π/4 curve (by providing for a right-angled 2r = 1 inside the circle) & line (by closing a right triangle outside the circle) – a novel "double-bind". The height of the right triangle represents the absolute furthest linear distance p (therefore o) can translate along the x-axis until bound to end at q' by length u = 1 from b. This approach satisfies isoperimetric equality via. w + u + π/4.

5.3.8 No Pi Left Behind

For being the perimeter of the square containing π, we posit 8r = 4 = n is the correct numerical bound of π. Because the annulus' width w can not take any other value so long as r = 1/2, the only two remaining lengths are π/4 & its perimetric bound of 1 per quadrant. We argue this annular approach rigorously entraps π leaving no pi left behind.

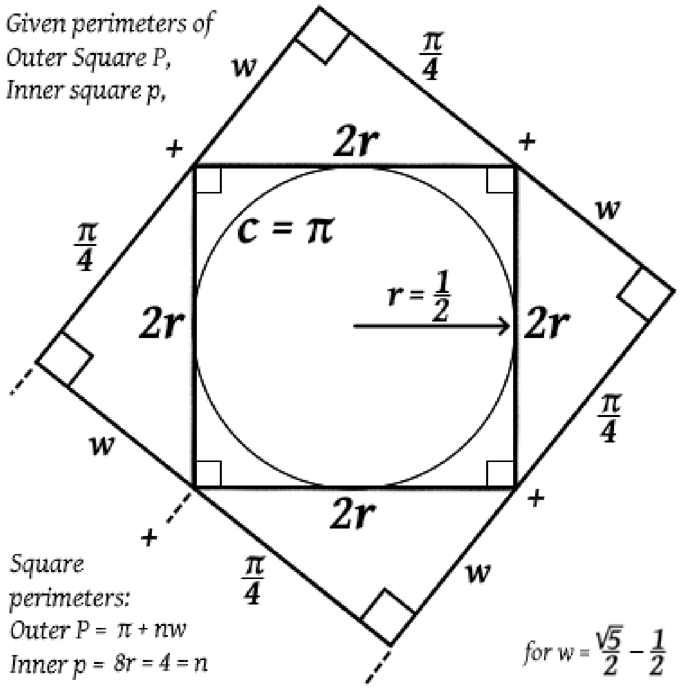

Figure 13. π is shown both circularly & linearly by squarely containing a unit diameter circle using the four right triangles whose sides w and π/4 all commonly observe 2r = 1 in/as the hypotenuse.

5.3.9 An Algebraic Pi

We use the Pythagorean theorem on the right triangle to algebraically solve for π:

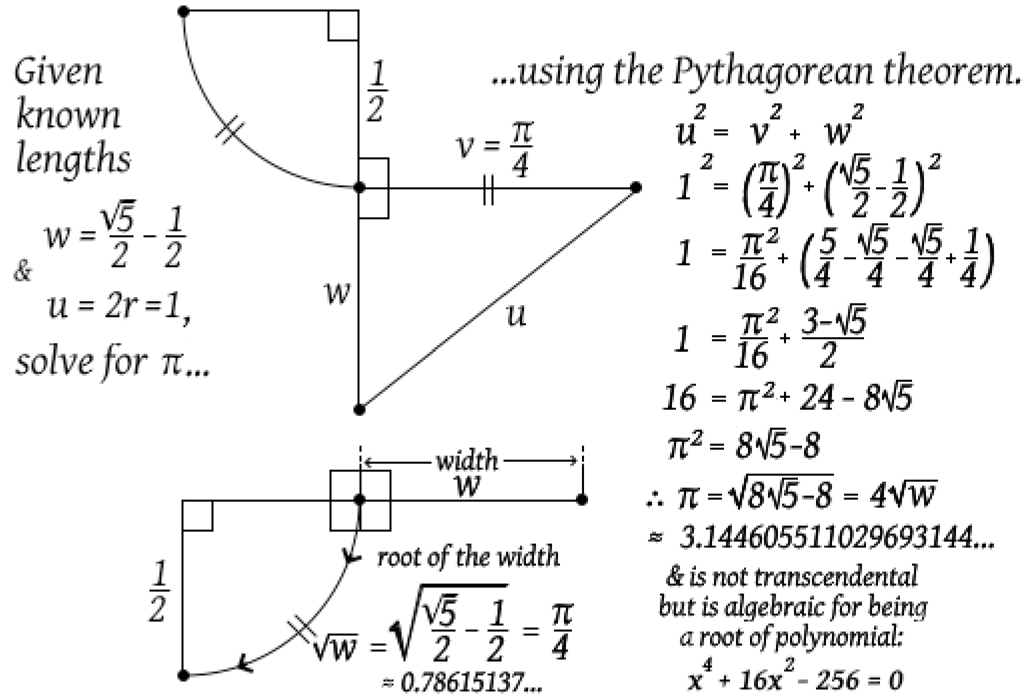

Figure 14. By applying the Pythagorean theorem to the right triangle derived from the π annulus, we find $\pi = 4\sqrt{w}$ for $w = R - r = \sqrt{5}/2 - 1/2$ (with $\pi = 4/\sqrt{\Phi}$ being an equivalent expression) both suggesting π is based on the geometric properties of the golden ratio $\Phi = R + r = \sqrt{5}/2 + 1/2$.

Here is a summary of our annular approach:

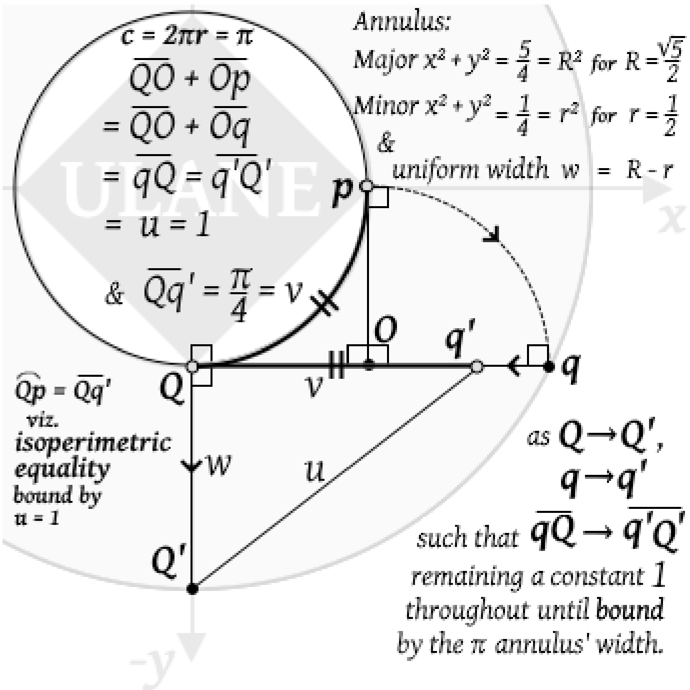

Figure 15. Shows p arcs to q placing line Qq = 1 before the circle. If/as the circle revolves, Q of Qq translates down (-) the y-axis all while dragging q towards q' (with u intact throughout) until capturing the requisite width of π's annulus. This isolates π/4 to one side v. In physics, v = s/t for space s, time t & speed or velocity v. In our model, centre o translates π space per unit time.

6 Acknowledgements & Argumentation(s)

We acknowledge that our annular result exceeds both 3.14159... and Archimedes' proposed upper-bound of 3 1/7.

Absent concern for isoperimetric inequality, one may assume Archimedes' bounds hold true to reality. One may readily dismiss $4\sqrt{w} \approx 3.1446...$ for being *out-of-Archimedes'-bounds*, so-to-speak. Were isoperimetric inequality still not a proven property of plane geometry, Archimedes' approach would still provide safe refuge, and proponents of $4\sqrt{w}$ would be referred back to his work. Fortunately, mathematicians' own undying efforts brought $L^2 \geq 4\pi A$ to its present status of proven.

Accordingly, after careful examination of Archimedes' assumption in a kinematic scenario, we find that Archimedes' bounds are *invalid.* We find that when the radius r of a figure is not a constant, its numerical perimeter begins to describe a compound motion spread about multiple axes, rather than a single axis. Dissatisfied with this, we rejected $n \to \infty$. Instead, we let n = 4, discovered a 'GAP' in our understanding (the golden annulus of pi) & used the π annulus' width $w = R - r$ and n = 4 of the square containing π to first bind it and then solved for π algebraically.

Our investigation uncovered a definite (ie. definable) geometric relationship(s) between the universal constants π and Φ, with the inverse of the latter being the requisite width of the former's annulus. This important geometrical figure merits further inquiry esp. as it may relate to any & all kinematic environments *incl*. and *esp*. physical reality.

Finally, we acknowledge that our numerical result for π is unexpected & are aware of the gravity of the proposition forthcoming.

Nevertheless, we both acknowledge & accept the Pythagorean theorem result. According to it, $\pi \neq 3.14159...$ in any & all kinematic environments. A need for physical experimentation is implied & therefore strongly encouraged.

7.0 In Defense of 3.14159...

We anticipate & address a common rebuttal: given a plane figure whose numerical surface area is known to be no less than $A \approx 0.785398...$ entirely smothering a circle whose area is purported (herein) to be no less than $C \approx 0.78615137...$ one argues:

Let $A \approx 0.785398...$ be the surface area of a non-circle smothering a unit diameter circle,
Let B be the numerical surface area of the unit diameter circle smothered, &
Let $C \approx 0.786151...$ be a circular squared area $\sqrt{(\sqrt{5}/2 - 1/2)}$

IF: $A > B$, how can $C = B$? One may observe $A > B$ for containing not less, but more surface area than B. Therefore, $C \neq B$.

The above argument, if endorsed, would fallaciously:

i. Assume an unbounded plane around an otherwise bounded figure (& thereby)
ii. Assume the use of a larger unit than natively contains the circle.

We discuss the argument & its fallacies throughout section 7.

7.1 To Be, Or Not to Be... Bound (Concerning i.)

In chapter 4 'Foundations of Isoperimetry', foundation 4 discusses 'bounded continuity', citing failure to bind the plane practically unbinds the area outside the circle from inside the square containing it. We elaborate on this further given its relevance to the common rebuttal above.

The circle constant π is contained by only a single unit square... not more. The circumferential length it describes does not ever once depart from the unit square & neither does the area it contains. Accordingly, to begin with more than a single unit to capture π is already a fallacy.

π is surrounded only by a finite amount of area before the unit containing it ends. In other words: π is natively bound to the unit square containing it & the exterior of the unit square is practically undefined. As far as π length is concerned, there is no radius beyond $r = 1/2$ & no space beyond $\sqrt{r} = 1/\sqrt{2}$ from π's centre. These lengths refer to the boundaries of the containing unit & are therefore absolute in magnitude with respect to both it & π length.

The point we wish to emphasize is: if π is not being captured in its native frame of reference with the numerical integrity of its boundary intact, what is being captured is not π. Instead, by assuming unbounded continuity of the plane surrounding π, one places no restrictions on how much unoccupied area surrounds the circle before the unit containing it terminates. We remedy this by proposing a bounded continuity:

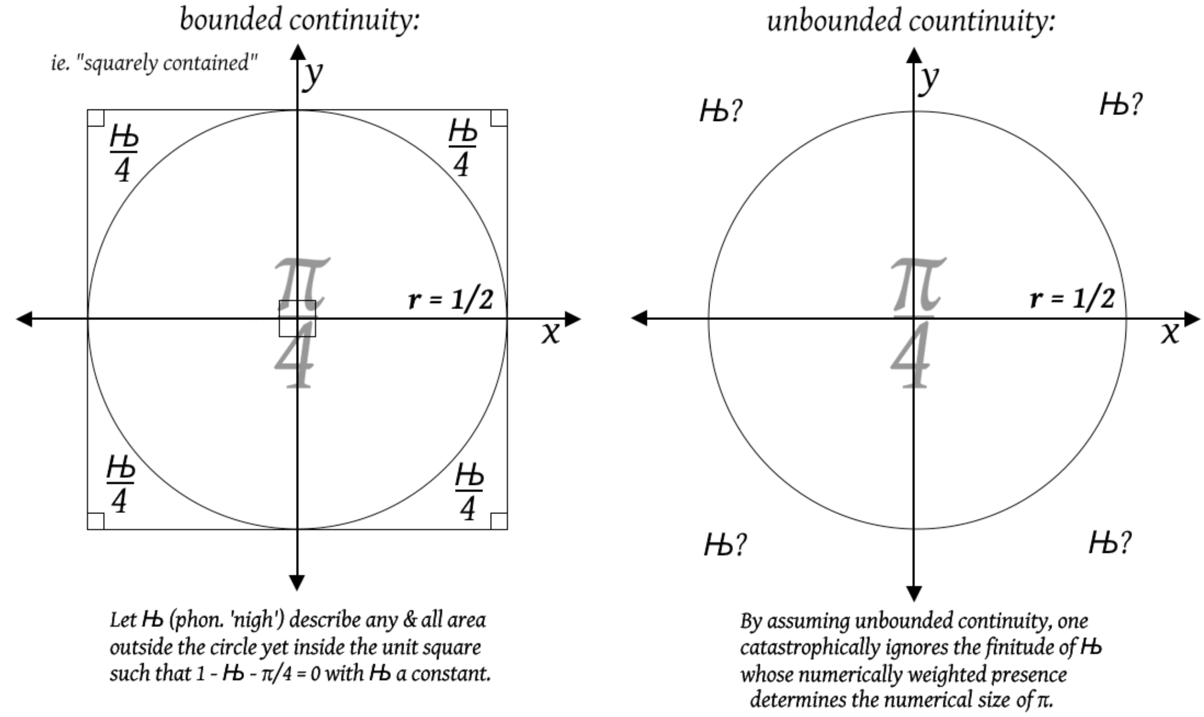

Figure 16. Њ describes the region(s) outside a unit diameter circle yet inside a unit square. Importantly: Њ is no less a universal constant than π is (!) The numerical value of the circle increases & decreases according to the presence and/or absence of Њ. Only for a unit diameter circle of a constant radius r = 1/2 bounded by n = 4 can the absolute numerical values of Њ & π be known.

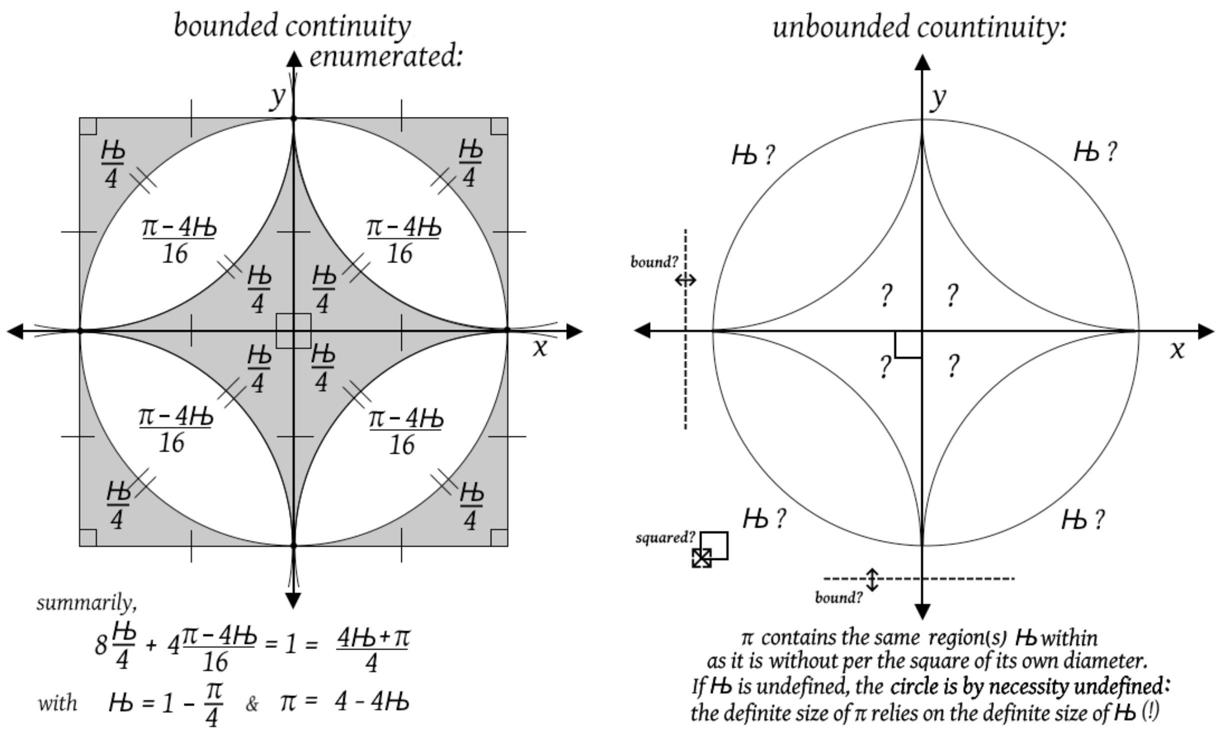

Figure 17. Neglect of Њ ignores the finite boundaries of the square containing the circle. Because the circle itself contains Њ while/as also being surrounded by Њ, to neglect Њ is to neglect π. We derive π = 4 - 4Њ implying: to underestimate Њ is to underestimate π following Њ is composed of a finite amount of π. Classical approaches *completely overlook* the practical need for bounded continuity.

7.2 Overstating 1's Unit Size (Concerning ii.)

By assuming unbounded continuity & entirely smothering a circle with a non-, by construct one ignores the finite remainder of the unit square after π/4 is removed:

$$1 - \pi/4 = \text{Њ}$$

This remainder Њ is also a constant for owing itself to only other constants. Re-arrange:

$$-\text{Њ} + 1 = \pi/4$$

and consider this sequence in light of a smothering figure. If one begins with a deficit (-) of Њ, what would one have to add to it in order to arrive at π/4? According to the above, given 1, one must pay the deficit with it (– Њ + 1) to remain with π/4. Now consider the smothering figure: given *the same* deficit (– Њ), what would one have to add to it to *surpass* the circle & arrive at an even larger non-circle? It would require either more than one unit, or an even bigger unit than the one containing the circle. In either case, the unit is not really that big.

Due to the unrealistic assumption of an unbounded frame of reference, we are sorely let down by overstated units belonging to figures whose radii are hardly a constant.

7.3 Unsmothering Common Sense

When smothering a unit diameter circle with a larger figure on a bounded plane, we inevitably encroach upon the region Њ beyond $r = 1/2$. This alters the fundamental areal equation from πr^2 to πr^2 + excess -Њ, effectively diluting the circle's area over a larger surface.

This process doesn't add area to the circle—an impossibility for a shape of absolute size. Instead, it spreads the same area over a larger surface, implicitly redefining the unit of measurement. The isoperimetric inequality states that π represents the maximum possible circular area per unit square. To assume that 3.14159... remains unaffected is to practically disregard the profound implications of this inequality *esp.* in kinematic environments.

Smothering figures therefore *can* & *do* yield values less than $C \approx 0.78615137...$ while ostensibly encompassing "more" than the circle. This contradiction arises from the fallacious imposition of a larger geometric unit predicated on an assumed unbounded plane. In reality, this approach shifts the frame of reference by superimposing a larger plane defined by a larger unit—a plane fundamentally incompatible with the original circle. Consequently: the two figures are incommensurable, rendering isoperimetric comparisons between them invalid.

This realization exposes the flaw in traditional circle measurement techniques: they inadvertently alter the very unit they aim to measure, leading to a systematic underestimation of π. By recognizing the bounded nature of the circle and the absolute constraints of the unit square, we open the door to even more accurate and conceptually sound approaches to determining π.

The concept of "smothering" a circle and its implications for π measurement can further be made accessible through a simple analogy in physics. Just as expanding a container affects the pressure of a gas within it, enlarging the boundary of a circle's domain impacts its geometric properties. This parallel aims for an intuitive framework for understanding the practical consequences of traditional approximation methods.

7.4 Putting Pressure On Pi

If P were a pressure per squared unit (psu) inside a singular unit cube... according to the equation $P = B/A$ for force B & area A, what would happen to the psu if the cube's boundary $A = 1$ were to suddenly expand $A > 1$? Would the psu go *up*? Or *down*? Or *stay the same*?

If one is able to understand the psu goes *down* according to any expansion(s) of B's native domain of 1, one is able to directly apply this understanding to what is generally happening when smothering a circle. One is not capturing more area (or increasing the pressure) by smothering a circle, one is instead spreading the same area over a larger space. Following any departure from its native domain of $A = 1$, B is catastrophically no longer contained by the perimeter of the boundary B is defined by (!) recalling B necessarily requires $r = 1/2$ by definition: $\pi = c/d$ for diameter $d = 2r$ with $c = \pi$ only for $r = 1/2$. If $A \neq 1$, $r \neq 1/2$.

8.0 Pressing Unsolved Problems

The implications of an algebraic π are numerous and far-reaching—far too many to list. We instead turn our attention to one important unsolved problem in mathematics directly involving circle geometry: the Riemann Hypothesis. By examining this problem through the lens of our new understanding of π, we aim to uncover a new avenue(s) towards its resolution.

8.1 The Riemann Hypothesis Problem

In his 1859 paper 'Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse' (trans. 'On the Number of Primes Less Than a Given Magnitude') (Riemann, 1859) [3], Bernhard Riemann posits the real part of complex non-trivial zeros of the zeta function is always 1/2. This conjecture, known as the Riemann Hypothesis, has remained unproven for over 160 years, with profound implications for prime number theory and numerous other mathematical domains.

In light of the preceding, a new condition under which this problem ceases to exist is now known. We present it as an hypothesis in postulate form whose return is either 'true' or 'false'. Should the following hypothetical postulate be 'true', the problem utterly ceases to exist.

8.2 Riemann's Real Problem (Hypothesis)

The unsolved status of the Riemann Hypothesis problem is owing to an unrecognized inexactitude in the circle constant π (hitherto assumed to be approx. 3.14159...) itself.

According to the Pythagorean theorem, π is perimetrically *bound* to be *no less than* $4\sqrt{w} \approx 3.144605511029693144...$ for $w = R - r = \sqrt{5}/2 - 1/2$, the width of the annulus $\pi(R^2 - r^2) = \pi$

Specifically: the zeta function was and is ill-defined for being both constructed with (& constrained by) a numerically deficient circle constant π. The real problem arises from the unrecognized application of isoperimetric inequality to any & all non-circular plane figures used to derive 3.14159... We posit $4\sqrt{w}$ holds for any & all real-world experimentation(s) meticulously engineered and rigorously conducted of equal or parallel construct to the mathematical model provided.

9 Conclusion

Given time is a valuable commodity, we express unbounded appreciation for all readers' time.

The potential influence of isoperimetric inequality on Archimedes' bounds—and π—transcends mere mathematical curiosity; it's a logical consequence of the inequality itself. We find there is practical need for experimental tests to probe whether the approximation of 3.14159... is affected by isoperimetric inequality for having been arrived at using non-circles. If it has, we expect reality to be unaffected & for it to show some measurable non-trivial discrepancy.

Our own analysis culminating in $4\sqrt{w} \approx 3.1446...$ reveals a numerically small yet profoundly significant discrepancy requiring empirical investigations to elucidate. We advocate for a series of high-precision physical experiments to measure circular properties with unprecedented accuracy, aiming to discern between traditional π and the Pythagorean theorem's prediction.

We extend an invitation to mathematicians, physicists, and experimental scientists for collaborative design and execution of these critical experiments.

The Riemann Hypothesis, a cornerstone problem in mathematics, exemplifies the multitude of issues potentially resolved should experiments reliably demonstrate 3.14159... to be unsupported. We assert that carefully conducted experimentation can and will demonstrate the latter.

Addressing the elephant in the room: while the notion of 3.14159... as being detached from reality seems inconceivable, we emphasize that our position stems directly from the Pythagorean theorem. According to it, 3.14159... is numerically detached from the very reality *it* (the theorem) purports to describe. We maintain: before dismissing the possibility, let empirical reality arbitrate through rigorous scientific experimentation and observational analysis.

Finally, in the interest of scientific progress: we must always remain open to a remote possibility that even our most fundamental assumptions concerning even the most fundamental of constants may yet harbor human oversight(s). Should this be the case here, the cost of *not* experimenting would be inconceivably *beyond measure*.

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