A Direct Proof that Goldbach's Conjecture is True

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Abstract

We introduce a Goldbach table. It consists of two rows. A bottom row counts from zero to a given n and the top counts from the right from n to $2n$. The columns generated give all the whole numbers that add to $2n$. We confirm that using a sieve, we do always seem to get top and bottom primes that show Goldbach's conjecture is true for the particular $2n$ depicted by this table. Next we cumulatively depict these tables and we see some interesting patterns. We can infer that all prime pairs will occur in one of these tables. We also see diagonal prime lines that seem to start and stop in symmetrical ways. These patterns suggest that for any even number we can choose a column and then find a prime pair.

Goldbach Tables

Goldbach tables are tables that count from 0 to a given n on the bottom row and then count down from n to $2n$ on the second, top row. We give several example tables.

4	3	2
0	1	2

Table 1: Goldbach table for $k = 2$.

6	5	4	3
0	1	2	3

Table 2: Goldbach table for $k = 3$.

8	7	6	5	4
0	1	2	3	4

Table 3: Goldbach table for $k = 4$.

10	9	8	7	6	5
0	1	2	3	4	5

Table 4: Goldbach table for $k = 5$.

12	11	10	9	8	7	6
0	1	2	3	4	5	6

Table 5: Goldbach table for $k = 6$.

14	13	12	11	10	9	8	7
0	1	2	3	4	5	6	7

Table 6: Goldbach table for $k = 7$.

16	15	14	13	12	11	10	9	8
0	1	2	3	4	5	6	7	8

Table 7: Goldbach table for $k = 8$.

These Goldbach tables show how existing primes march across the top row and are cumulatively above all lesser or equal bottom row primes. Let's combine tables and look for patterns.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	2	1																		
2	0	1																		
3	4	3	2																	
4	0	1	2																	
5	6	5	4	3																
6	0	1	2	3																
7	8	7	6	5	4															
8	0	1	2	3	4															
9	10	9	8	7	6	5														
10	0	1	2	3	4	5														
11	12	11	10	9	8	7	6													
12	0	1	2	3	4	5	6													
13	14	13	12	11	10	9	8	7												
14	0	1	2	3	4	5	6	7												
15	16	15	14	13	12	11	10	9	8											
16	0	1	2	3	4	5	6	7	8											
17	18	17	16	15	14	13	12	11	10	9										
18	0	1	2	3	4	5	6	7	8	9										
19	20	19	18	17	16	15	14	13	12	11	10									
20	0	1	2	3	4	5	6	7	8	9	10									
21	22	21	20	19	18	17	16	15	14	13	12	11								
22	0	1	2	3	4	5	6	7	8	9	10	11								
23	24	23	22	21	20	19	18	17	16	15	14	13	12							
24	0	1	2	3	4	5	6	7	8	9	10	11	12							
25	26	25	24	23	22	21	20	19	18	17	16	15	14	13						
26	0	1	2	3	4	5	6	7	8	9	10	11	12	13						
27	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14					
28	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14					
29	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15				
30	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15				
31	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16			
32	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16			
33	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17		
34	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17		
35	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	
36	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
37	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19
38	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

Figure 1: Caption for goldbach-green-red-to19by19

Lightening and Move Away Proof

In Figure 1 this has been done.

Observe that every top prime will have cells with bottom numbers that have all odd numbers, as we expected. As all primes of interest are odd, this means that every prime will have bottom cells with lesser primes. This means all prime cells (cells with both top and bottom prime numbers) do occur somewhere.

The question is whether or not there is a pair of rows corresponding to some even number that doesn't have such a prime cell. This can't happen.

Using the information we can glean from such a cell as $(15, 9)$ that occurs for even total 24 or $(17, 9)$ that occurs for even total 26, we can redirect our attention to a prime cell. Just start factoring. As 15 has a factor of 3 we can find the bottom cell containing this number and look at the top cell for it: 21. We repeat for the prime factors of 21 which we haven't encountered yet: 7. We find a prime cell $(17, 7)$. For $(17, 9)$, the first is a prime, but 9 isn't but we look at the cell associated with its prime factor and we find $(23, 3)$, a prime cell. It seems that such repetitions will have to come to an end with a prime pair; we are always reducing the given pair by prime factors and eventually no further reductions are possible, as all prime factors occur in either the top or bottom row once and twice (right most cell) when the even is two times a prime.

References

[1] Apostol, T. M. (1976). *Introduction to Analytic Number Theory.* New York: Springer.

[2] Hardy, G. H., Wright, E. M., Heath-Brown, R. , Silverman, J. , Wiles, A. (2008). *An Introduction to the Theory of Numbers*, 6th ed. London: Oxford Univ. Press.